Lecture 11: Detectors and Descriptors

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Stanford Vision Lab

What we will learn today?

- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector
- Scale invariant region selection
  - Automatic scale selection
  - Laplacian-of-Gaussian detector
  - Difference-of-Gaussian detector (Problem Set 3 (Q2))
  - Combinations
- Local descriptors
  - An intro

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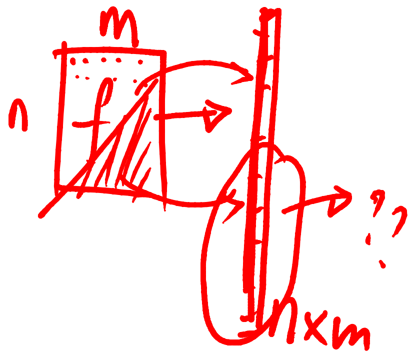
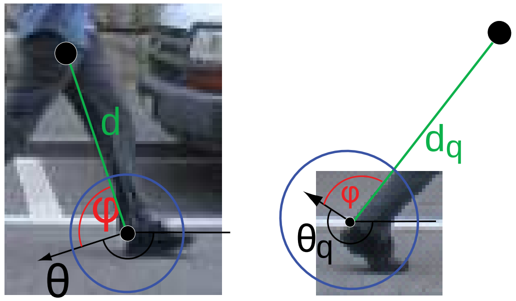
Recall Faces



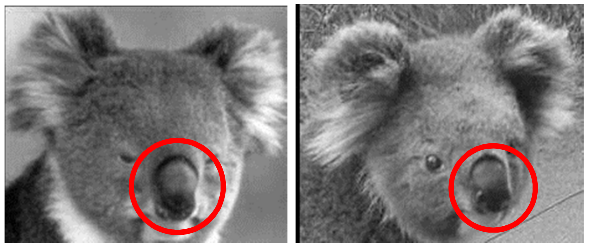
Motivation

- Global representations have major limitations
- Instead, describe and match only local regions
- Increased robustness to
  - Occlusions

  - Articulation



  - Intra-category variations



Application: Image Matching

by Diva Sian

by swashford

Slide credit: Steve Seitz

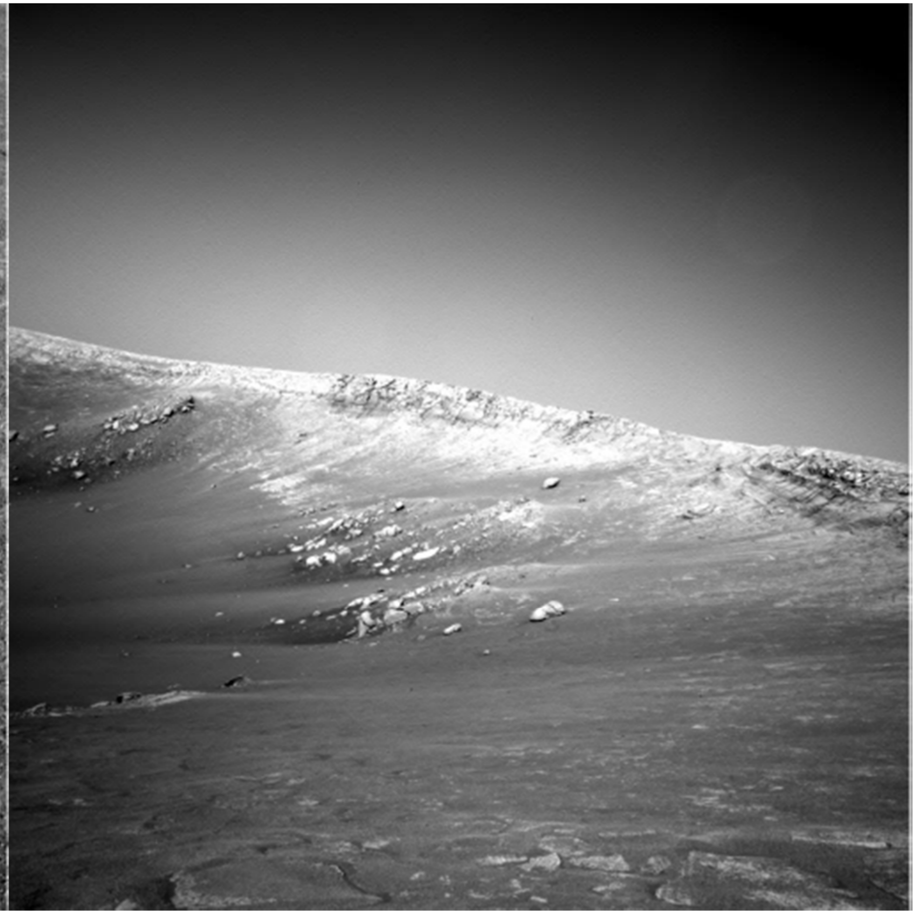
Harder Case

by Diva Sian

by scgbt

Slide credit: Steve Seitz

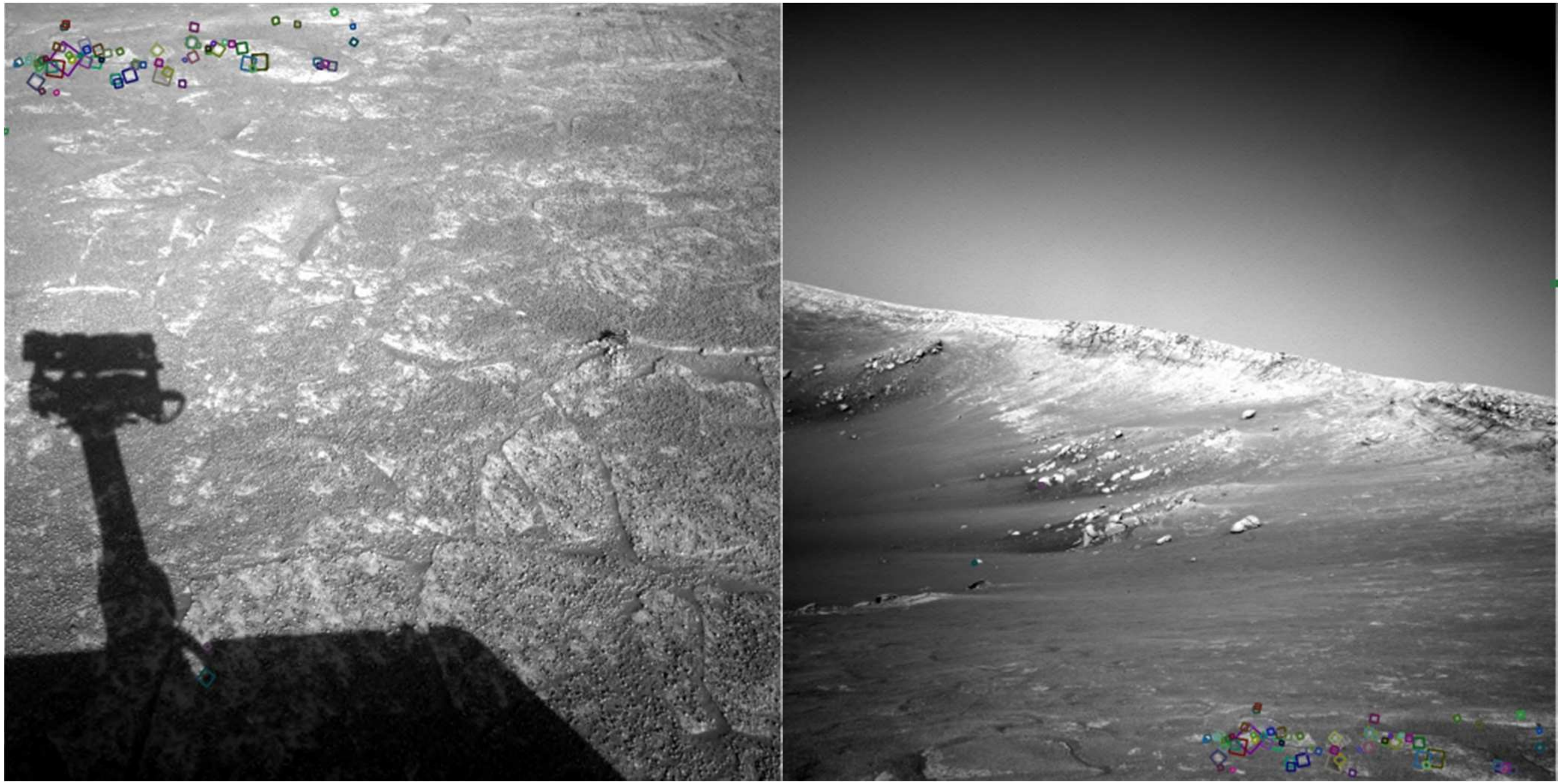
Harder Still?

NASA Mars Rover images

Slide credit: Steve Seitz

Answer Below (Look for tiny colored squares)



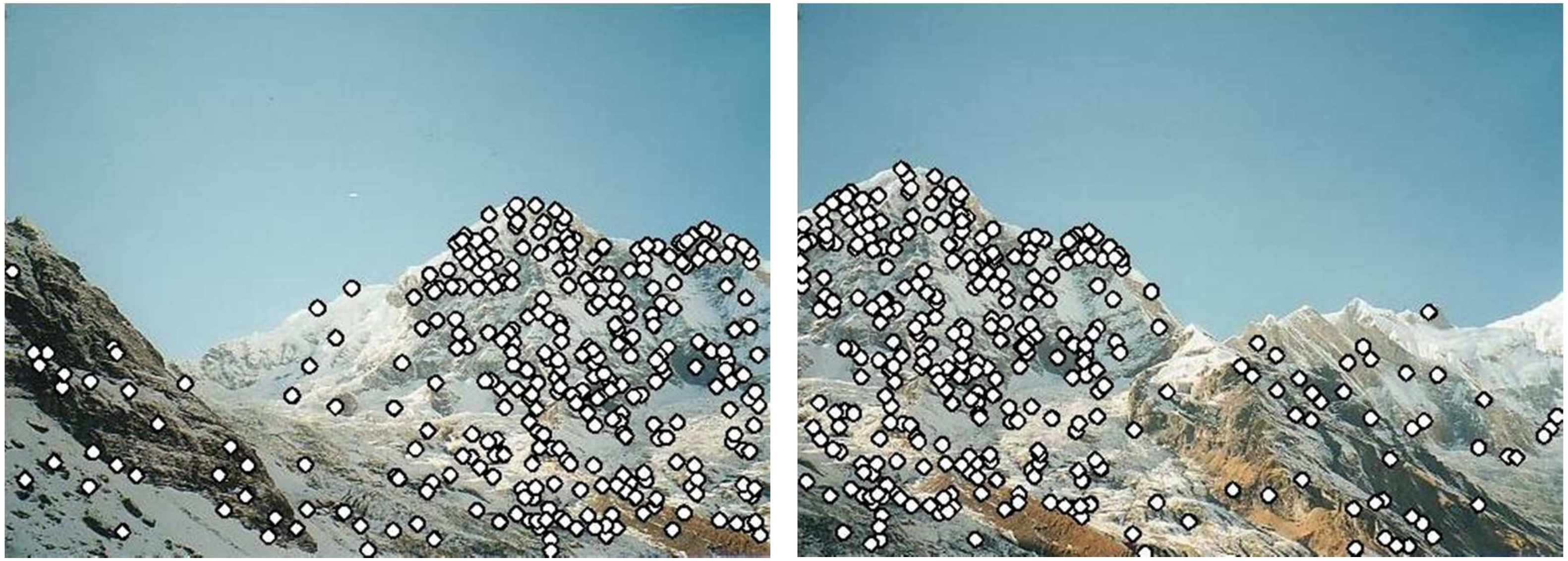
NASA Mars Rover images with SIFT feature matches
(Figure by Noah Snavely)

Slide credit: Steve Seitz

Application: Image Stitching

Slide credit: Darya Frolova, Denis Simakov

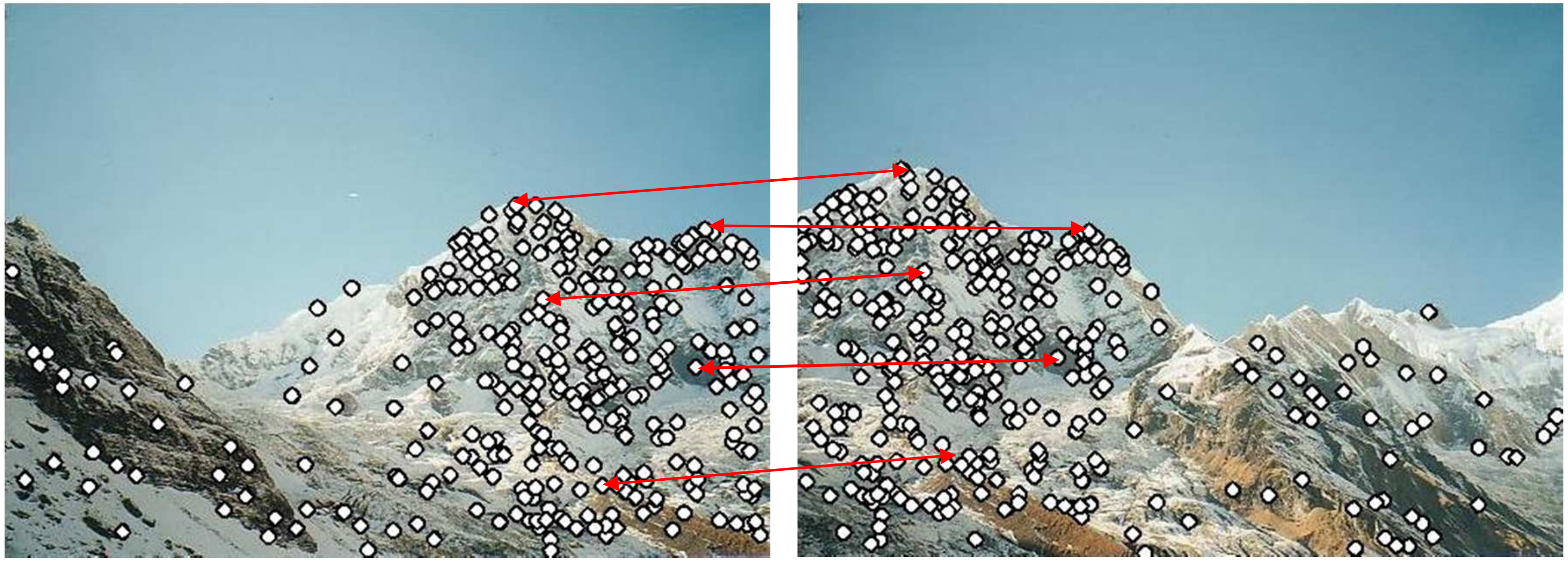
Application: Image Stitching



- Procedure:
  - Detect feature points in both images

Slide credit: Darya Frolova, Denis Simakov

Application: Image Stitching



- Procedure:
  - Detect feature points in both images
  - Find corresponding pairs

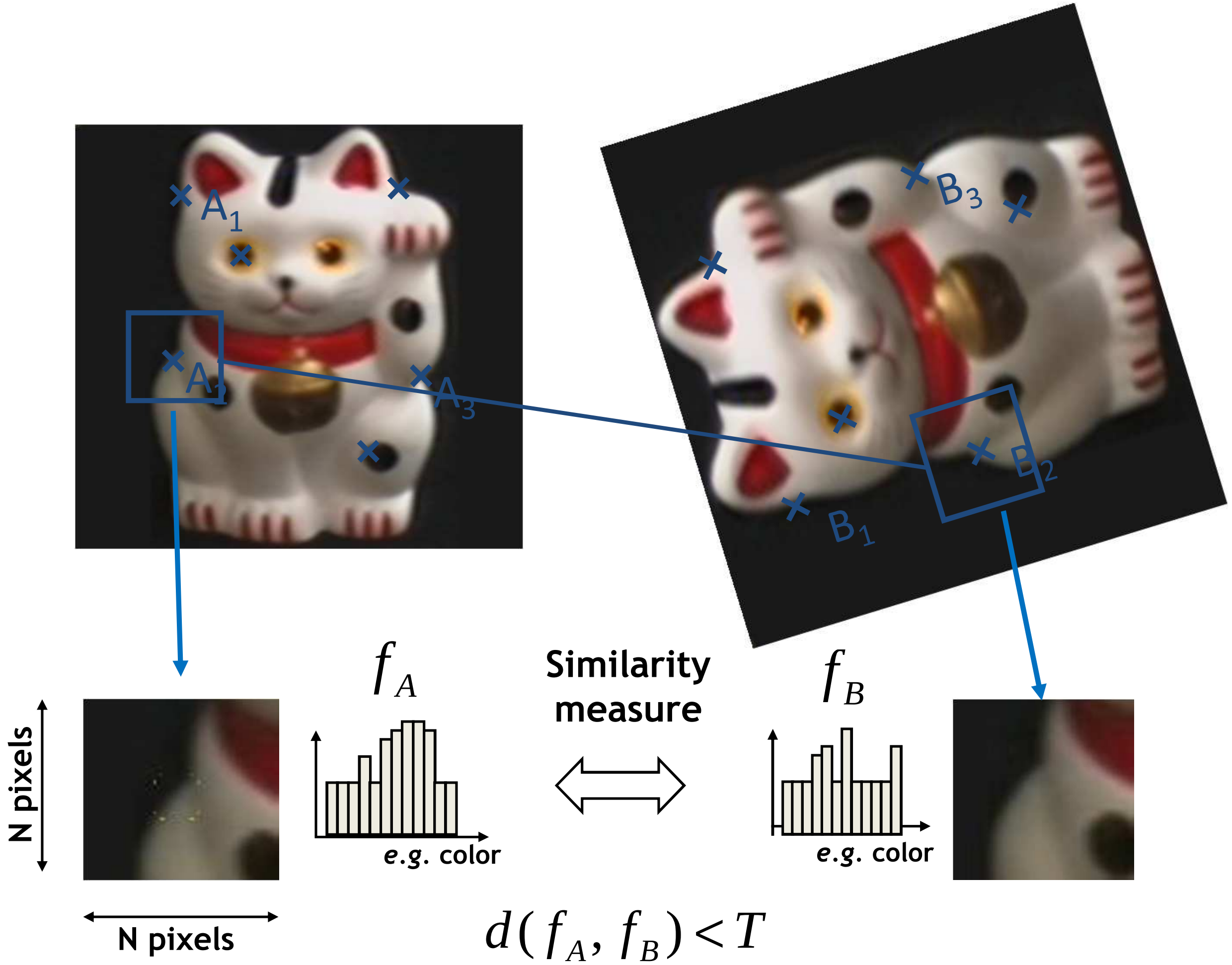
Slide credit: Darya Frolova, Denis Simakov

Application: Image Stitching

- Procedure:
  - Detect feature points in both images
  - Find corresponding pairs
  - Use these pairs to align the images

Slide credit: Darya Frolova, Denis Simakov

General Approach



1. Find a set of distinctive key-points
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors

Slide credit: Bastian Leibe

Common Requirements

- Problem 1:
  - Detect the same point *independently* in both images

No chance to match!

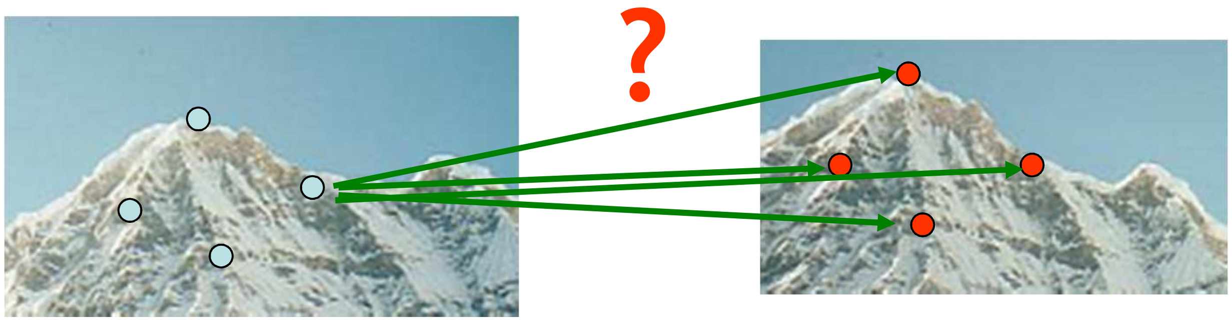
This lecture (#11)

We need a repeatable detector!

Slide credit: Darya Frolova, Denis Simakov

Common Requirements

- Problem 1:
  - Detect the same point *independently* in both images
- Problem 2:
  - For each point correctly recognize the corresponding one



Slide credit: Darya Frolova, Denis Simakov

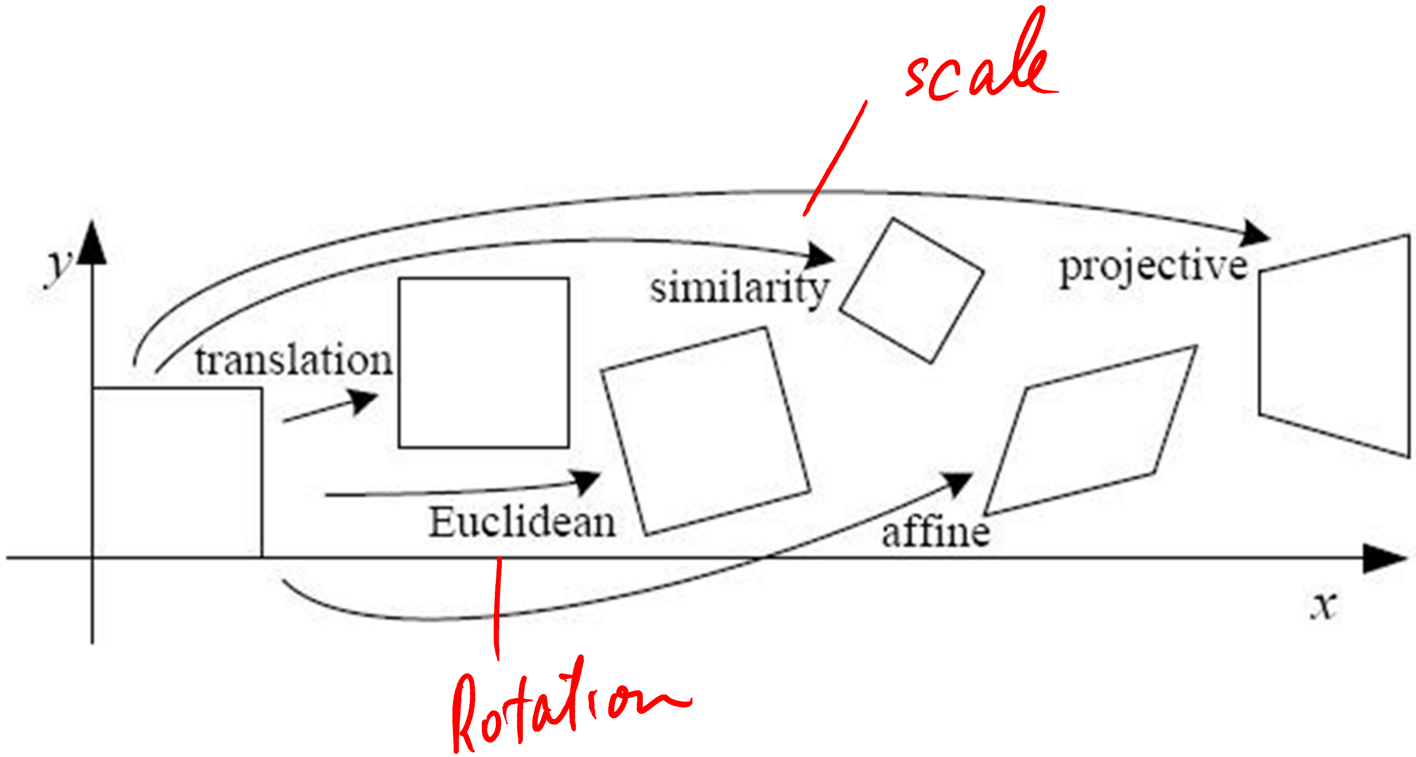
Next lecture (#12)

We need a reliable and distinctive descriptor!

Invariance: Geometric Transformations

Slide credit: Steve Seitz

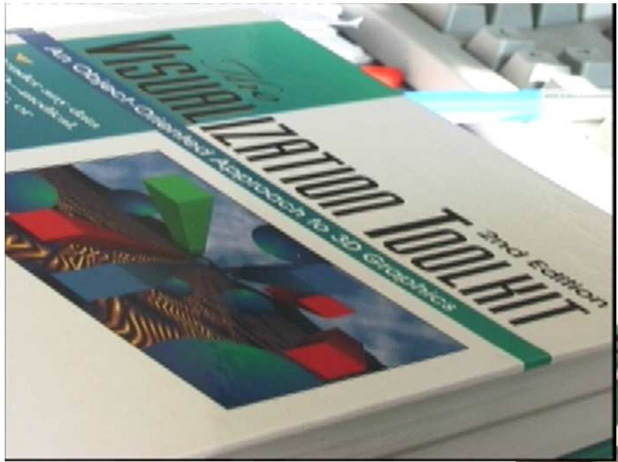
Levels of Geometric Invariance

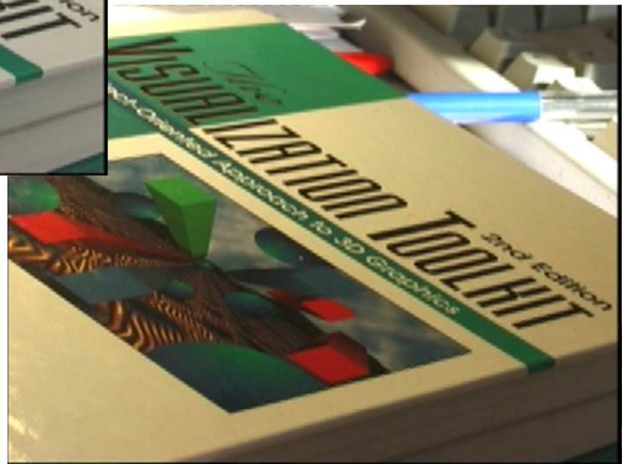


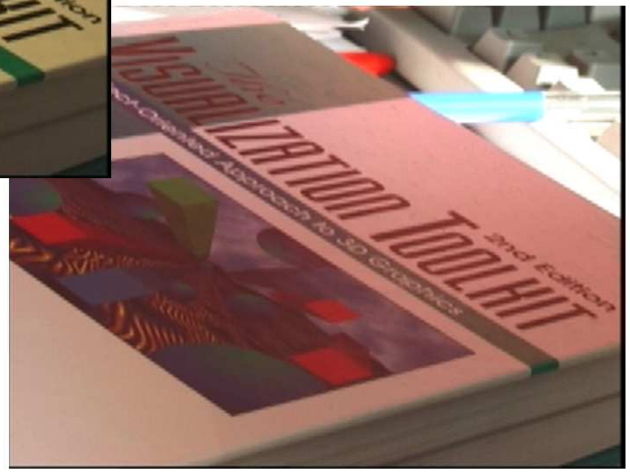
Slide credit: Bastian Leibe

Invariance: Photometric Transformations

illumination







- Often modeled as a linear transformation:
  - Scaling + Offset

Slide credit: Tinne Tuytelaars

Requirements

- Region extraction needs to be repeatable and accurate
  - Invariant to translation, rotation, scale changes
  - Robust or covariant to out-of-plane (≈affine) transformations
  - Robust to lighting variations, noise, blur, quantization
- Locality: Features are local, therefore robust to occlusion and clutter.
- Quantity: We need a sufficient number of regions to cover the object.
- Distinctivenes : The regions should contain "interesting" structure.
- Efficiency: Close to real-time performance.

Slide credit: Bastian Leibe

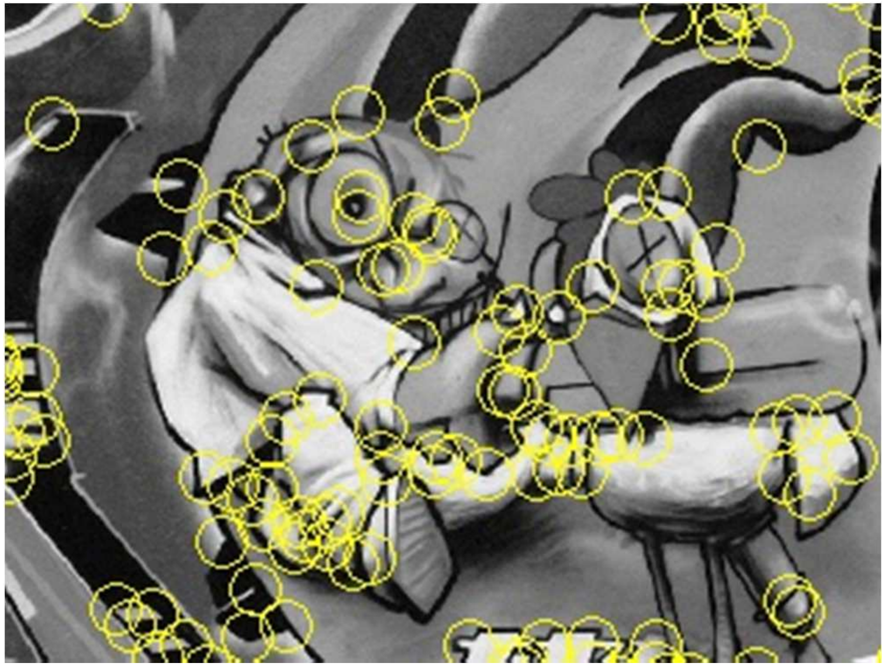
Many Existing Detectors Available

- Hessian & Harris [Beaudet ‘78], [Harris ‘88]
- Laplacian, DoG [Lindeberg ‘98], [Lowe ‘99]
- Harris-/Hessian-Laplace [Mikolajczyk & Schmid ‘01]
- Harris-/Hessian-Affine [Mikolajczyk & Schmid ‘04]
- EBR and IBR [Tuytelaars & Van Gool ‘04]
- MSER [Matas ‘02]
- Salient Regions [Kadir & Brady ‘01]
- Others…

- *Those detectors have become a basic building block for many recent applications in Computer Vision.*

Slide credit: Bastian Leibe

Keypoint Localization

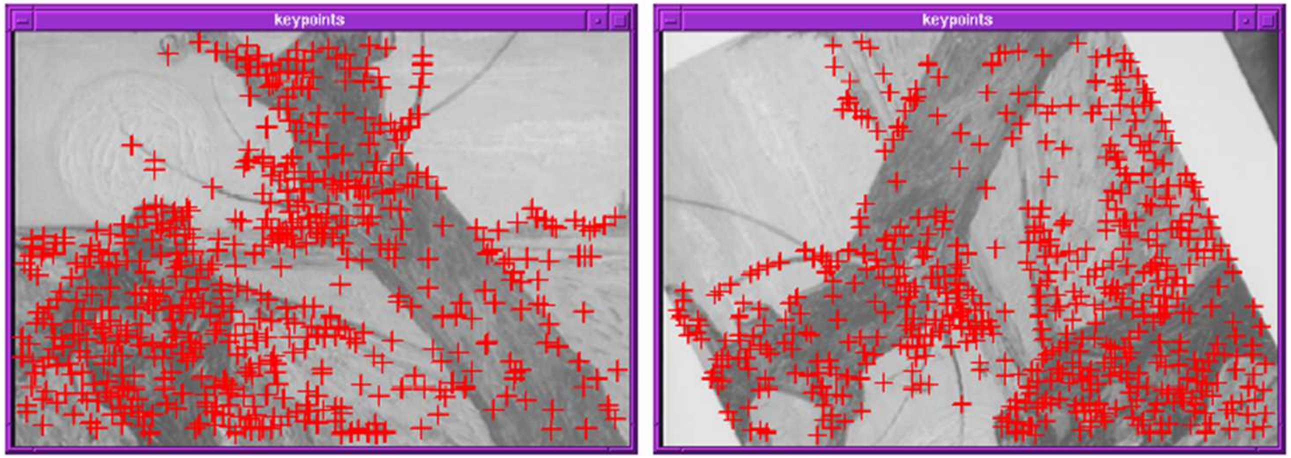


- Goals:
  - Repeatable detection
  - Precise localization
  - Interesting content

$\Rightarrow$ *Look for two-dimensional signal changes*

Slide credit: Bastian Leibe

Finding Corners



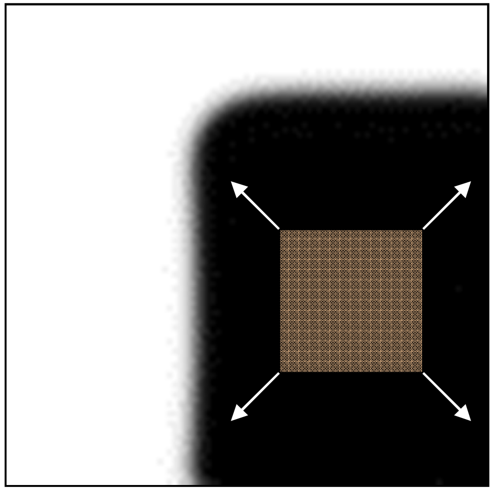
- Key property:
  - In the region around a corner, image gradient has two or more dominant directions
- Corners are *repeatable* and *distinctive*

C.Harris and M.Stephens. "A Combined Corner and Edge Detector.“
Proceedings of the 4th Alvey Vision Conference, 1988.

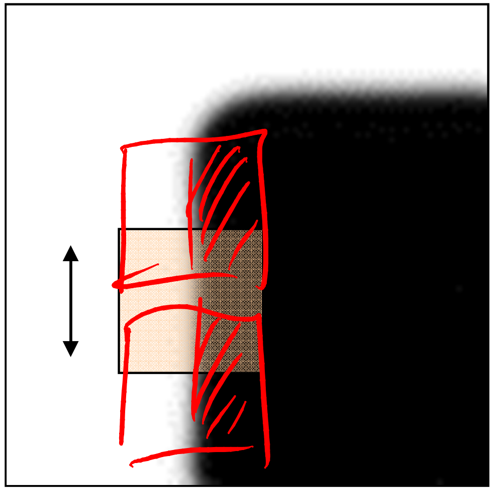
Slide credit: Svetlana Lazebnik

Corners as Distinctive Interest Points

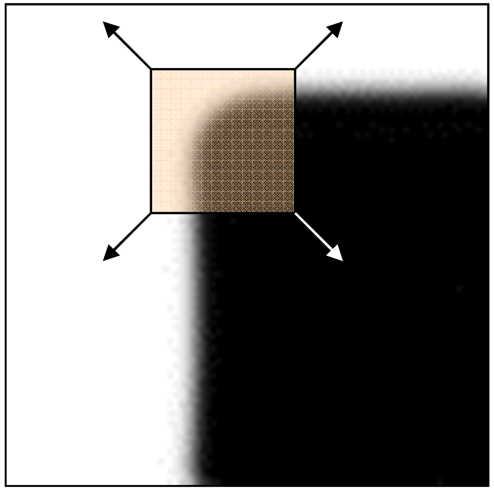
- Design criteria
  - We should easily recognize the point by looking through a small window (*locality*)
  - Shifting the window in *any direction* should give *a large change* in intensity (*good localization*)



"flat" region:
no change in all directions



"edge":
no change along the edge direction



"corner":
significant change in all directions

Slide credit: Alyosha Efros

Harris Detector Formulation

- Change of intensity for the shift [u,v]:

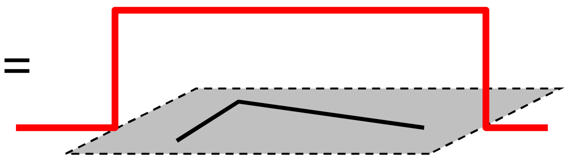
$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

Window function

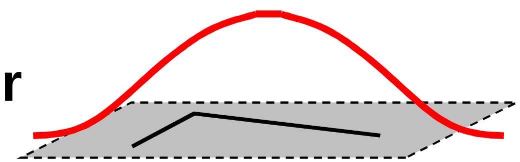
Shifted intensity

Intensity

Window function $w(x,y)$ =



or



1 in window, 0 outside

Gaussian

Slide credit: Rick Szeliski

Harris Detector Formulation

- This measure of change can be approximated by:

$$E(u,v) \approx [u \ \ v] \ M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Gradient with respect to x, times gradient with respect to y

Sum over image region – the area we are checking for corner

$\sum w$

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \ I_y]$$

Slide credit: Rick Szeliski

Harris Detector Formulation

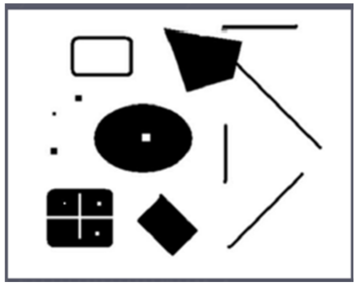
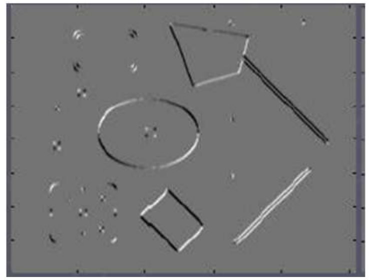


Image I

I_x

I_y



$I_x I_y$

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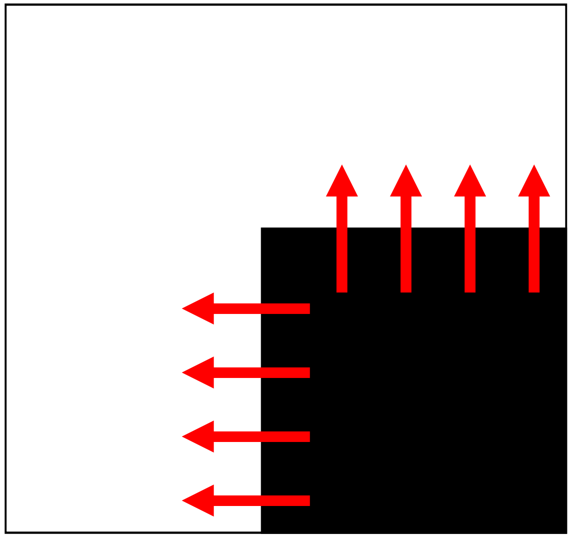
Slide credit: Rick Szeliski

What Does This Matrix Reveal?

- First, let's consider an axis-aligned corner:

$$M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$



Slide credit: David Jacobs

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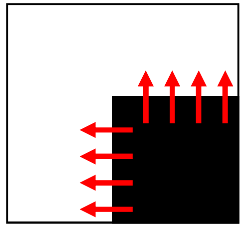


- This means:
  - Dominant gradient directions align with x or y axis
  - If either λ is close to 0, then this is not a corner, so look for locations where both are large.
- What if we have a corner that is not aligned with the image axes?

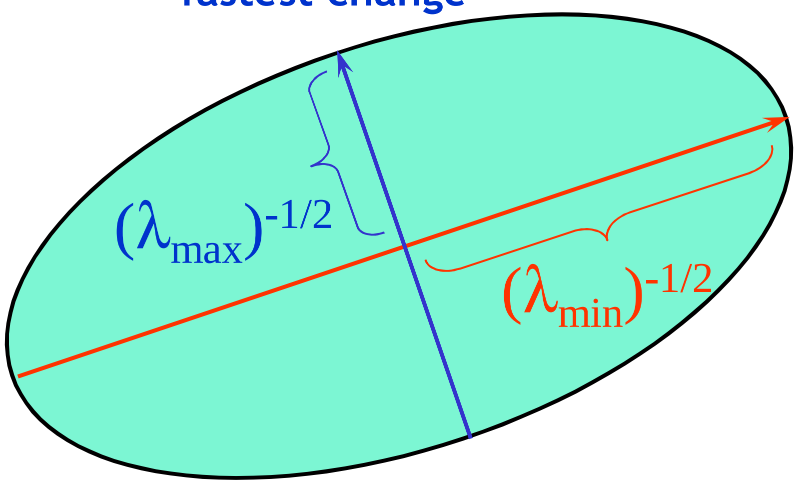
Slide credit: David Jacobs

General Case

- Since M is symmetric, we have $M = R^{-1}\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

  (Eigenvalue decomposition)

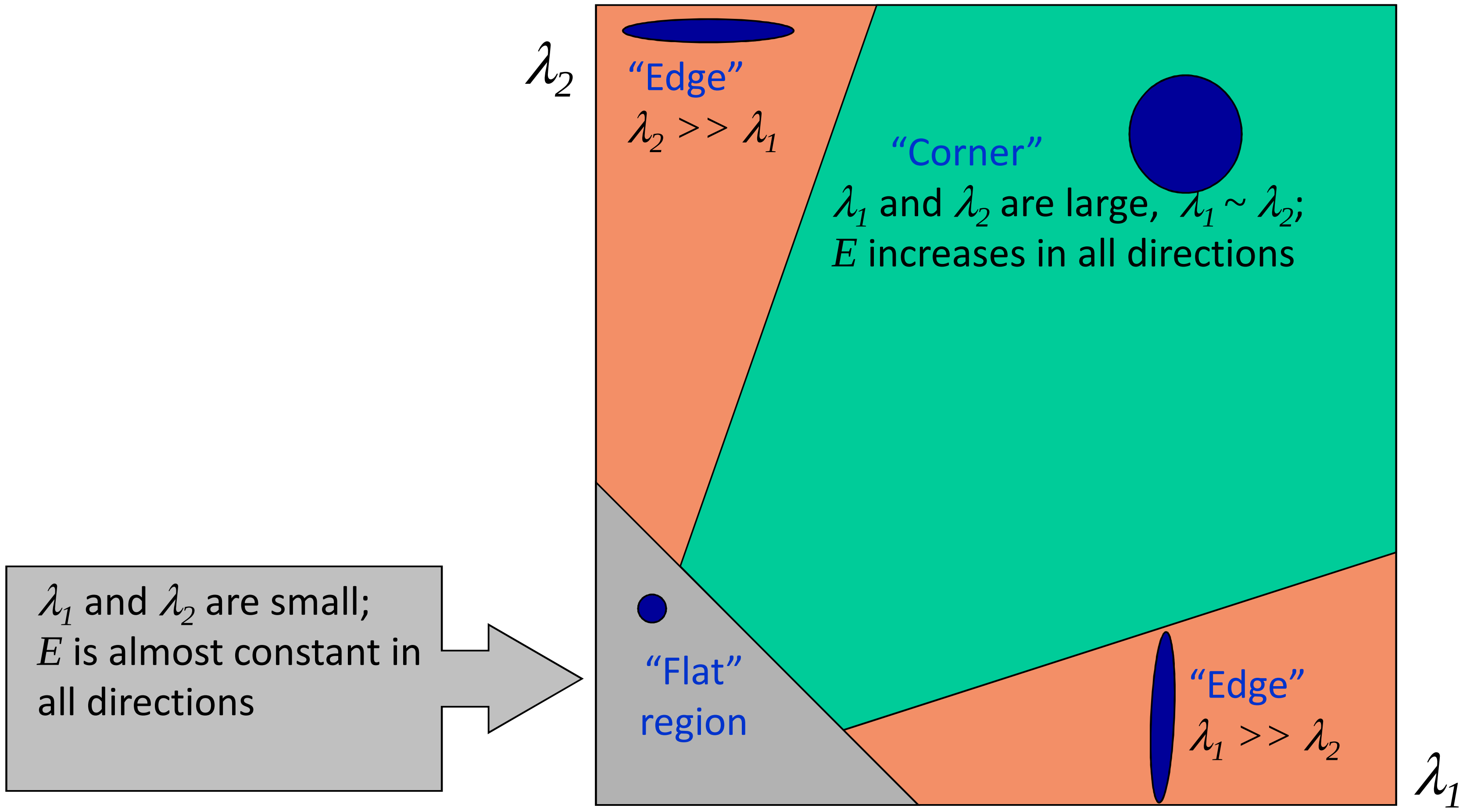
- We can visualize M as an ellipse with axis lengths determined by the eigenvalues and orientation determined by R



adapted from Darya Frolova, Denis Simakov

Interpreting the Eigenvalues

- Classification of image points using eigenvalues of M:

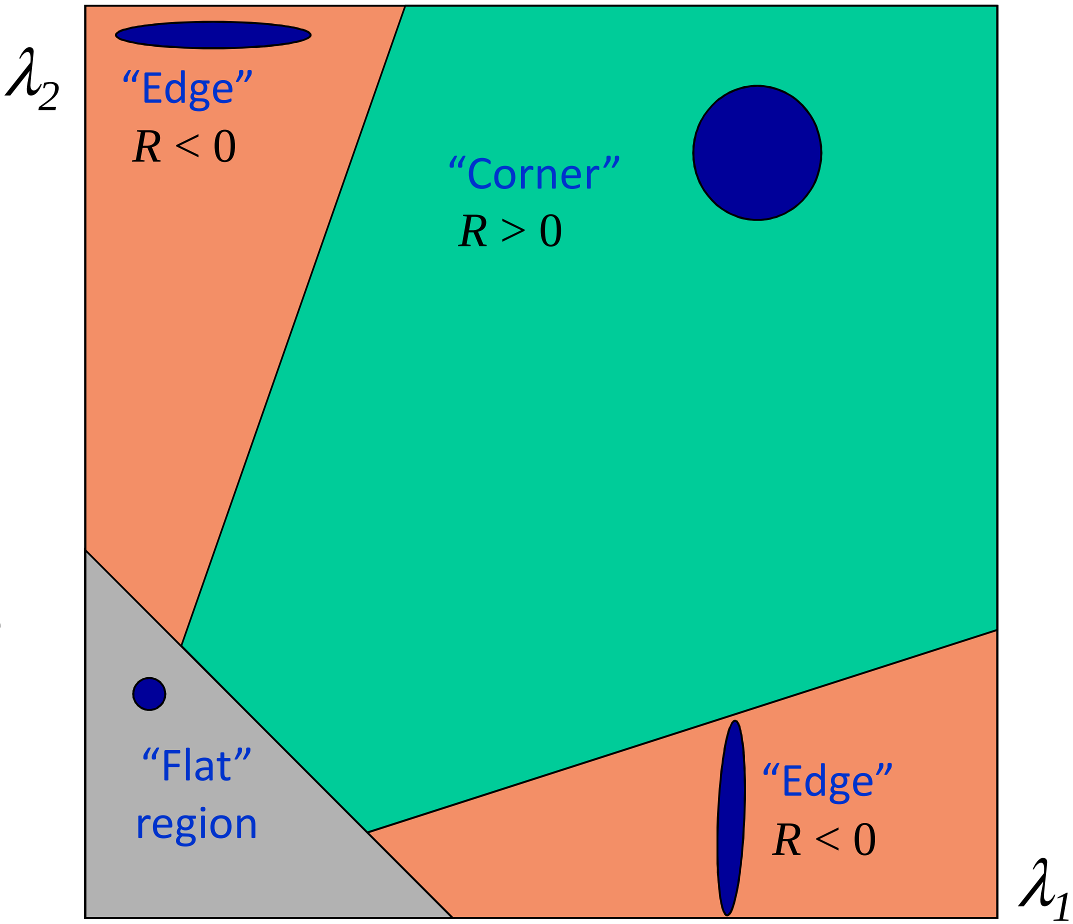


Slide credit: Kristen Grauman

Corner Response Function

$$R = \det(M) - \alpha\,\mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

abuse of notation
Threshold



- Fast approximation
  - Avoid computing the eigenvalues
  - α: constant (0.04 to 0.06)

Slide credit: Kristen Grauman

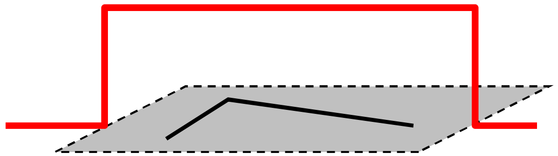
Window Function $w(x,y)$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Option 1: uniform window
  - Sum over square window

$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

  - Problem: not rotation invariant

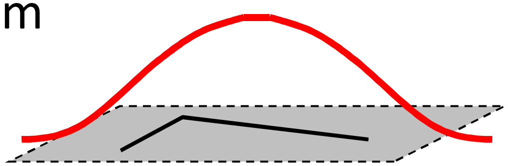


1 in window, 0 outside

- Option 2: Smooth with Gaussian
  - Gaussian already performs weighted sum

$$M = g(\sigma) * \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

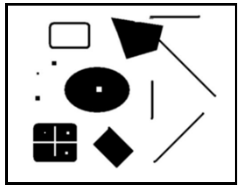
  - Result is rotation invariant



Gaussian

Slide credit: Bastian Leibe

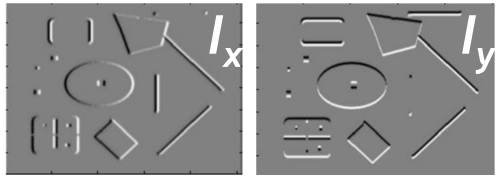
Summary: Harris Detector [Harris88]



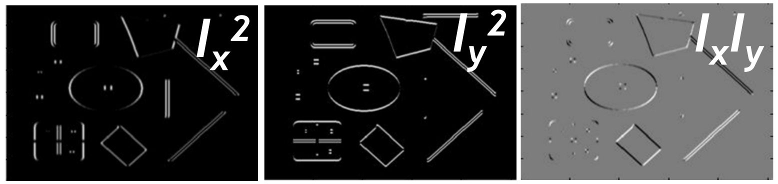
- Compute second moment matrix (autocorrelation matrix)

$$M(\sigma_I,\sigma_D)=g(\sigma_I)*\begin{bmatrix} I_x^2(\sigma_D) & I_xI_y(\sigma_D) \\ I_xI_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

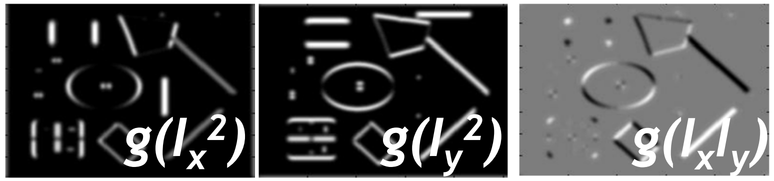
1. Image derivatives



2. Square of derivatives



3. Gaussian filter $g(\sigma_I)$



4. Cornerness function – two strong eigenvalues

$$R=\det[M(\sigma_I,\sigma_D)]-\alpha[\mathrm{trace}(M(\sigma_I,\sigma_D))]^2$$
$$=g(I_x^2)g(I_y^2)-[g(I_xI_y)]^2-\alpha[g(I_x^2)+g(I_y^2)]^2$$

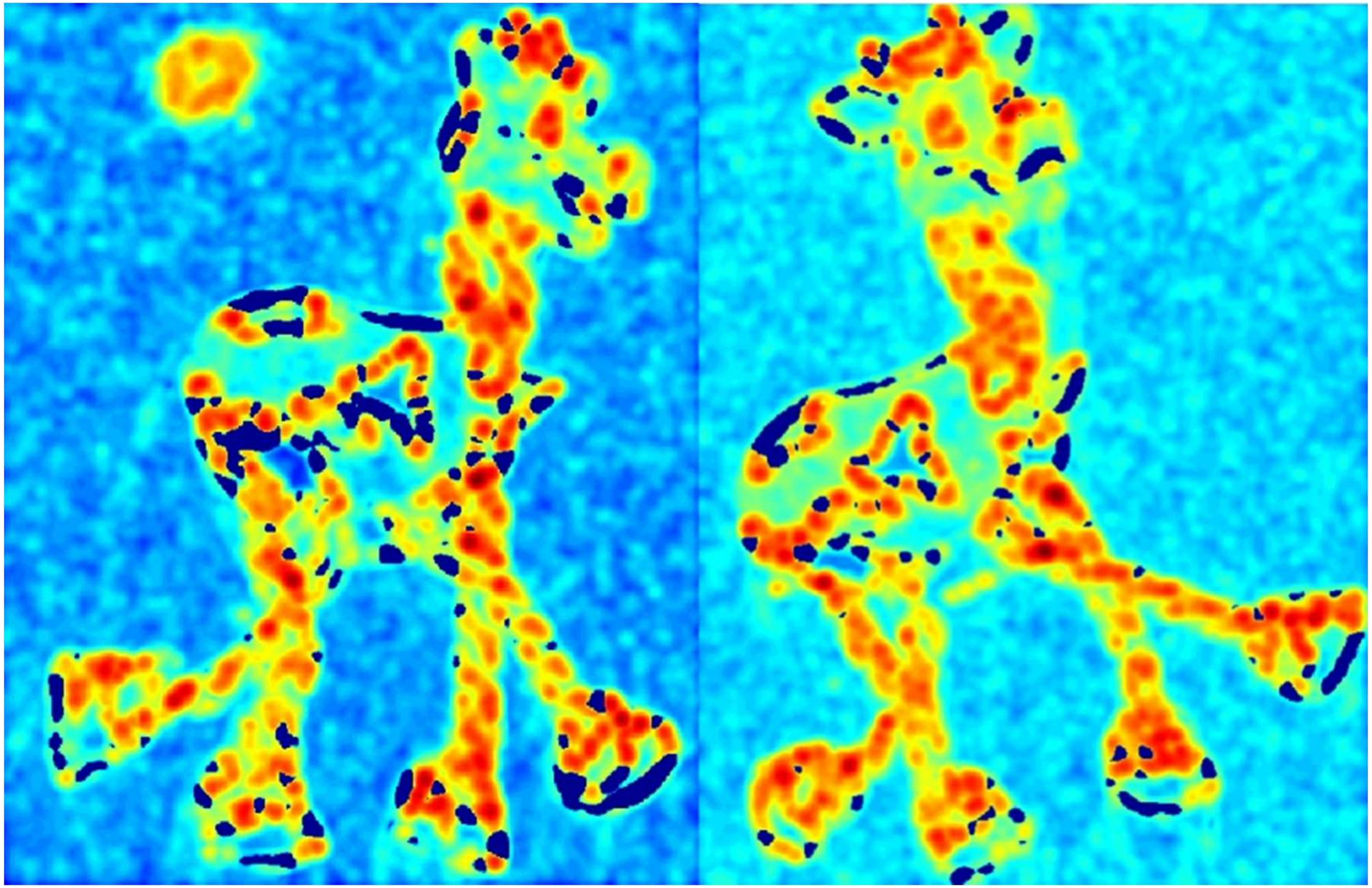
5. Perform non-maximum suppression

Slide credit: Krystian Mikolajczyk

Harris Detector: Workflow

Slide adapted from Darya Frolova, Denis Simakov

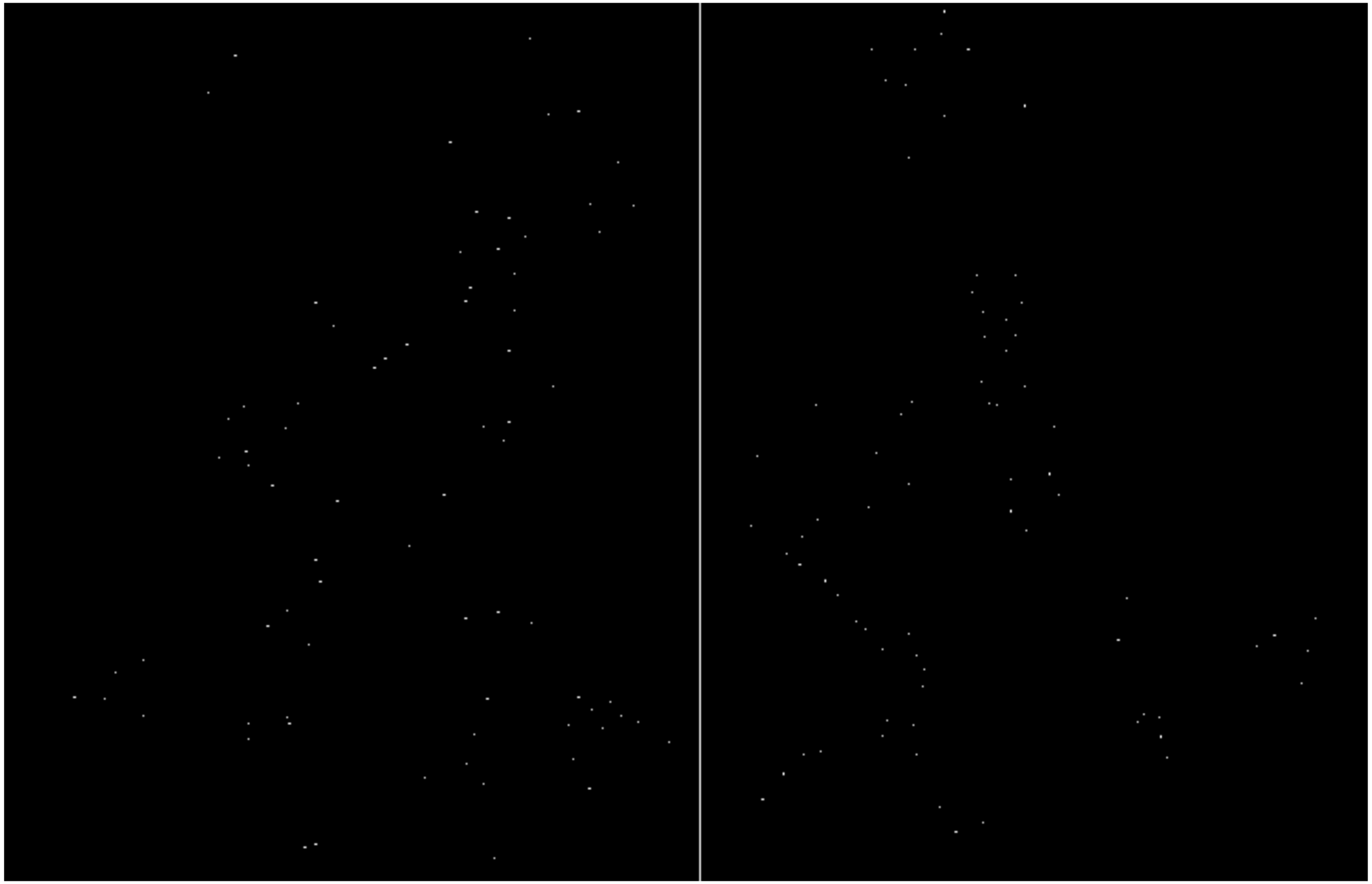
Harris Detector: Workflow
- computer corner responses R



Slide adapted from Darya Frolova, Denis Simakov

Harris Detector: Workflow

- Take only the local maxima of R, where R>threshold

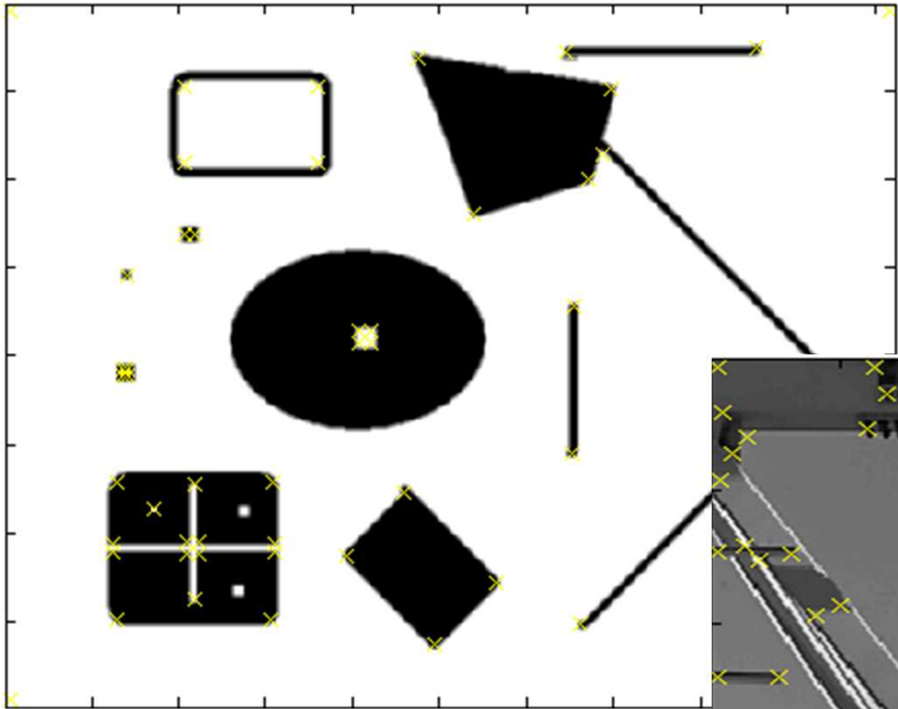


Slide adapted from Darya Frolova, Denis Simakov

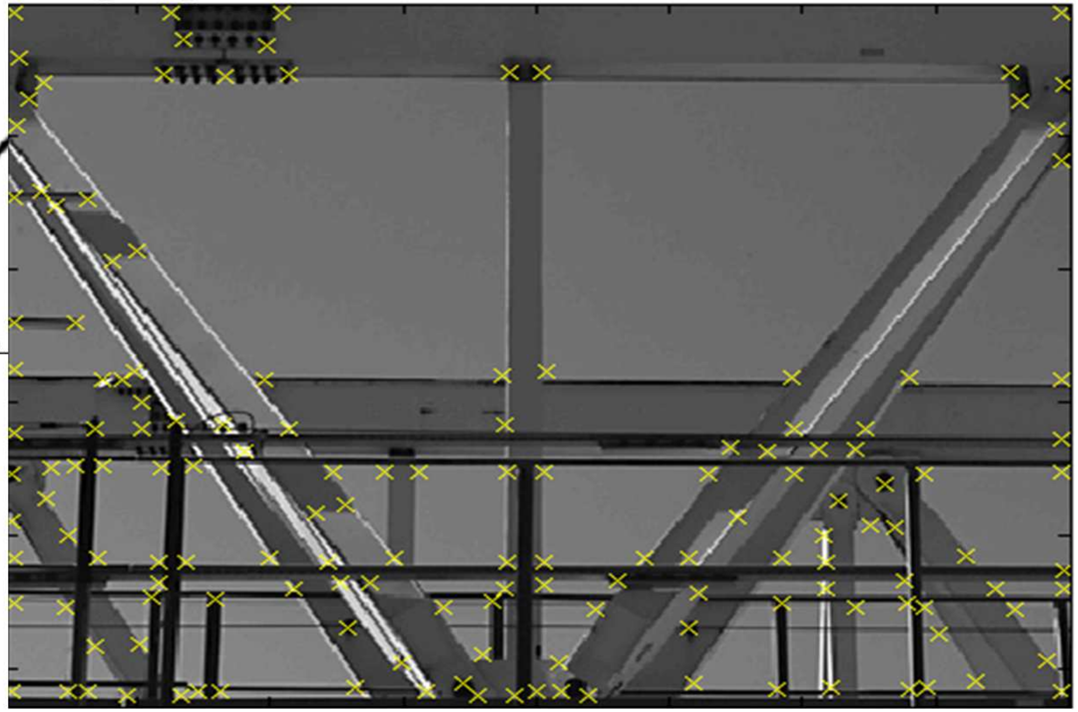
Harris Detector: Workflow
- Resulting Harris points

Slide adapted from Darya Frolova, Denis Simakov

Harris Detector – Responses [Harris88]



Effect: **A very precise corner detector.**



Slide credit: Krystian Mikolajczyk

Harris Detector – Responses [Harris88]

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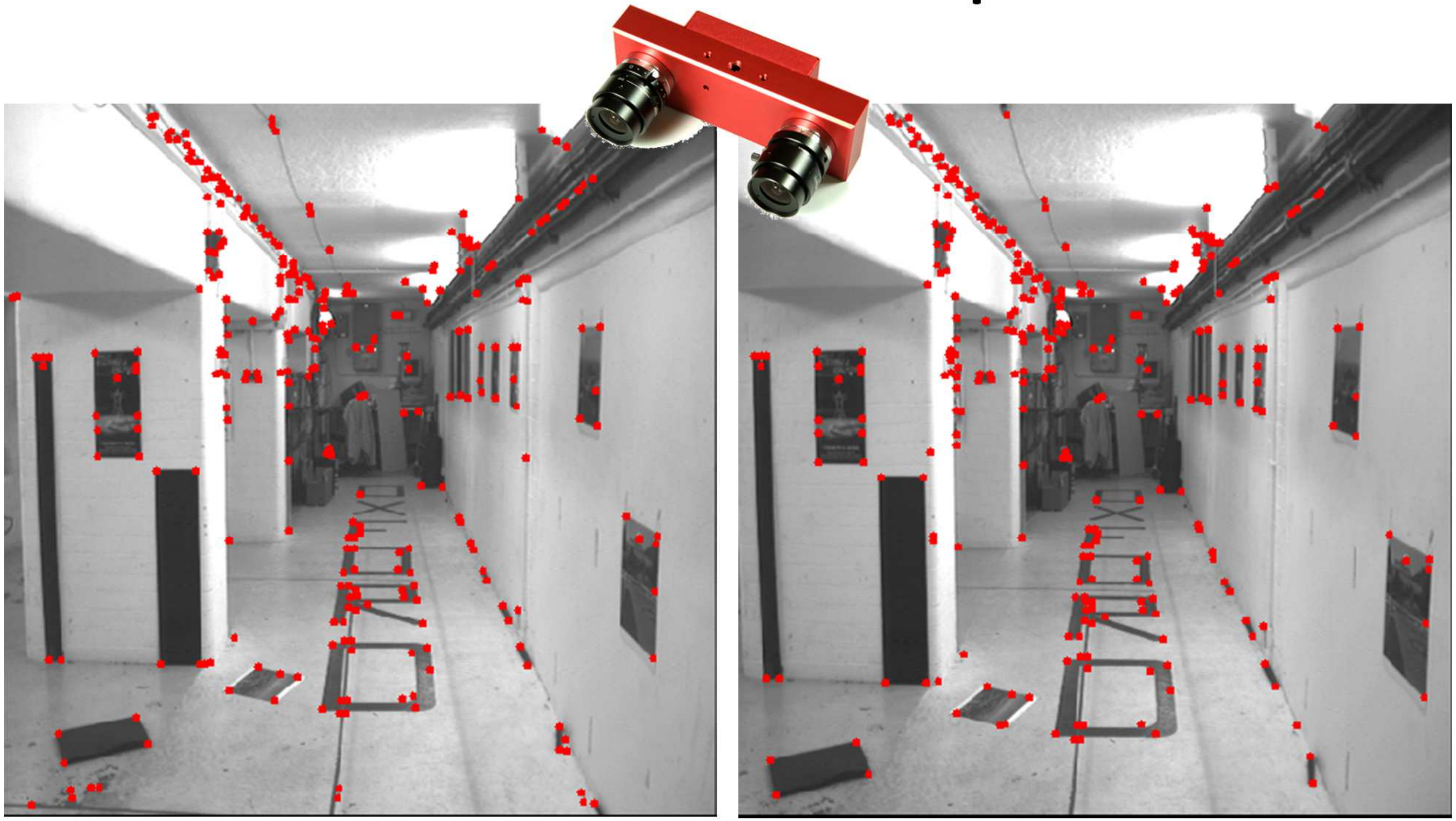


- Results are well suited for finding stereo correspondences

Slide credit: Kristen Grauman

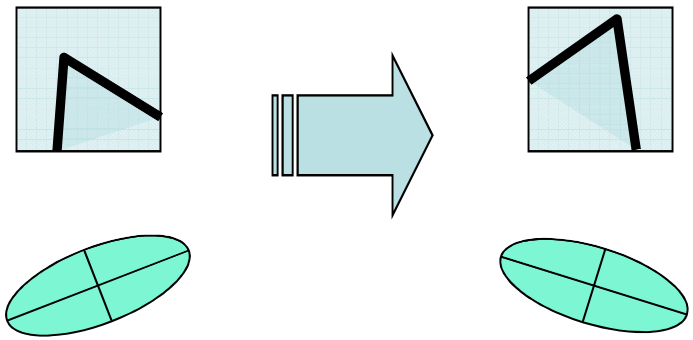
Harris Detector: Properties

- Translation invariance?

Slide credit: Kristen Grauman

Harris Detector: Properties

- Translation invariance
- Rotation invariance?



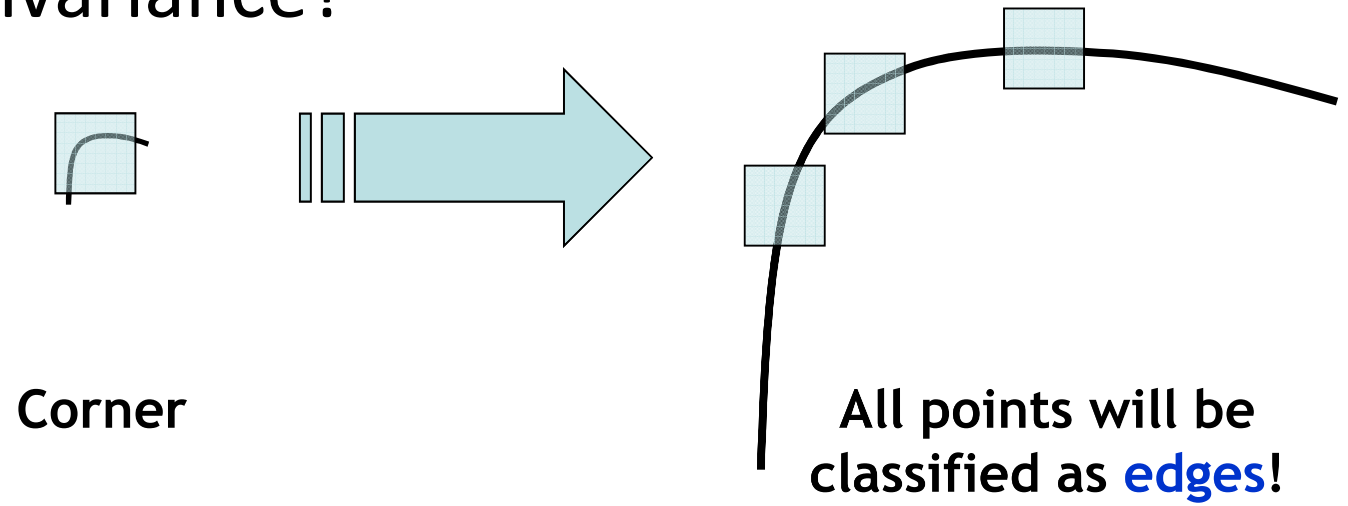
Ellipse rotates but its shape (i.e. eigenvalues) remains the same

***Corner response R* is invariant to image rotation**

Slide credit: Kristen Grauman

Harris Detector: Properties

- Translation invariance
- Rotation invariance
- Scale invariance?



***Not* invariant to image scale!**

Slide credit: Kristen Grauman

What we will learn today?

- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector
- Scale invariant region selection
  - Automatic scale selection
  - Laplacian-of-Gaussian detector
  - Difference-of-Gaussian detector (Problem Set 3 (Q2))
  - Combinations
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From Points to Regions…

- The Harris and Hessian operators define interest points.
  - Precise localization
  - High repeatability

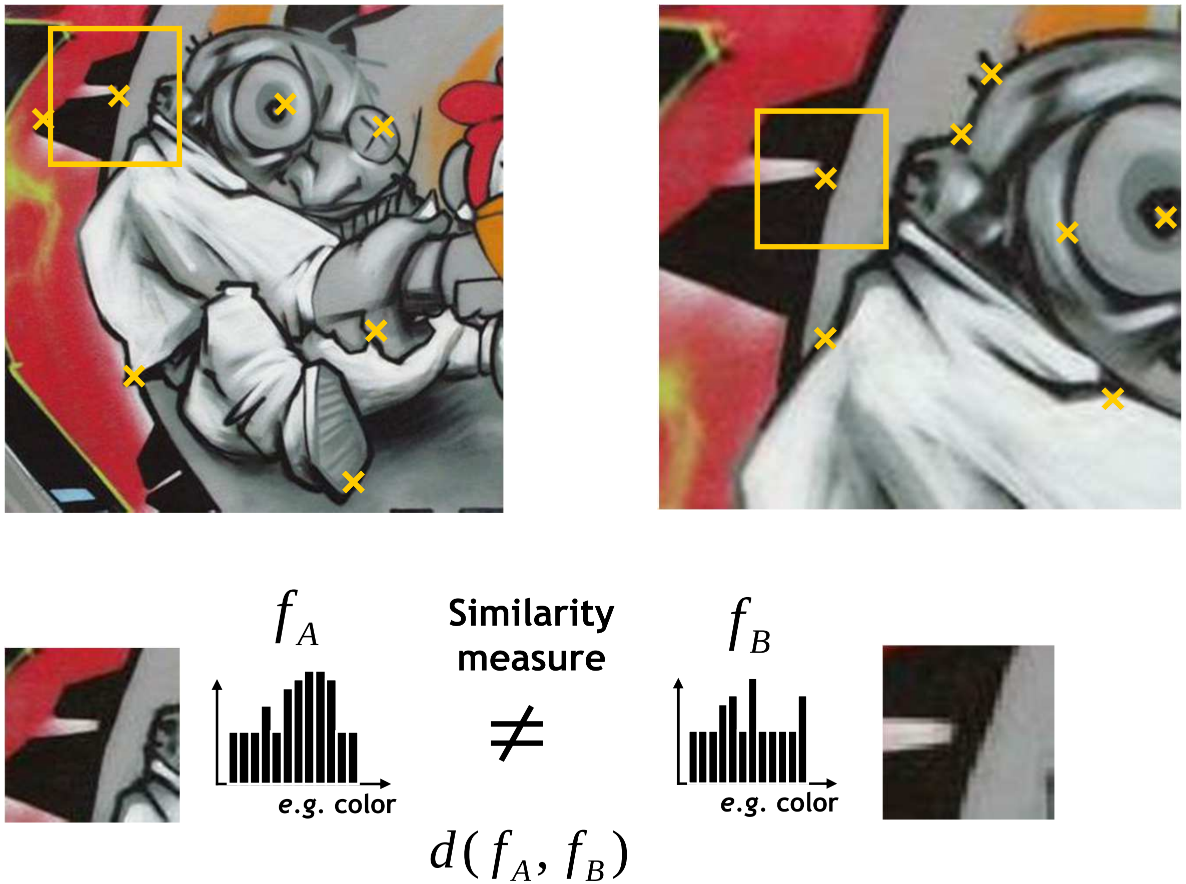
- In order to compare those points, we need to compute a descriptor over a region.
  - How can we define such a region in a scale invariant manner?

- *I.e. how can we detect scale invariant interest regions?*

Source: Bastian Leibe

Naïve Approach: Exhaustive Search

- Multi-scale procedure
  - Compare descriptors while varying the patch size



Slide credit: Krystian Mikolajczyk

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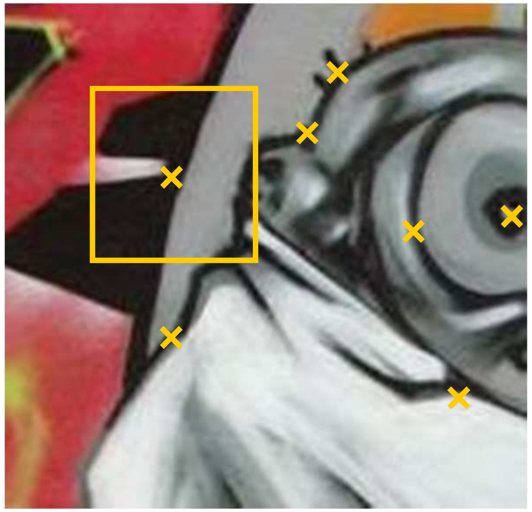
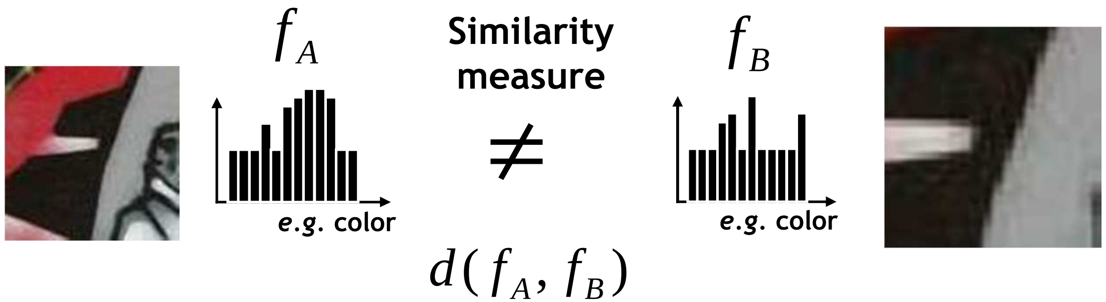
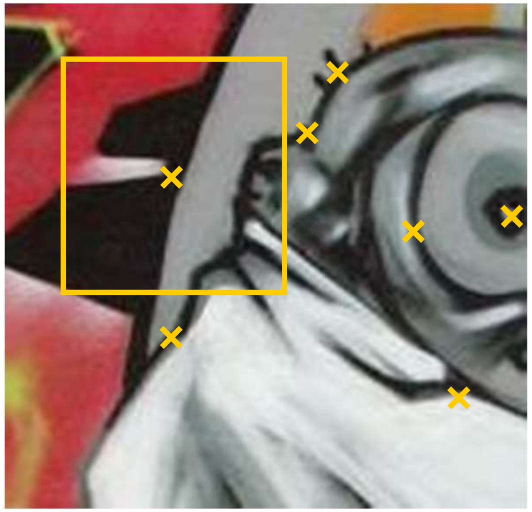


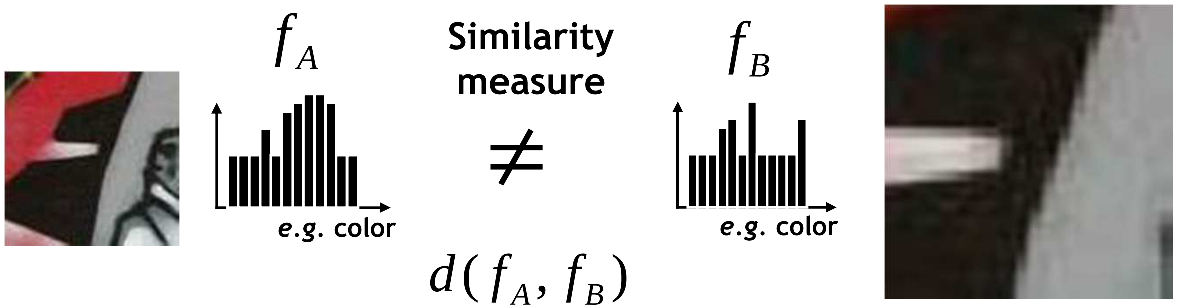


Slide credit: Krystian Mikolajczyk

Naïve Approach: Exhaustive Search

- Multi-scale procedure
  - Compare descriptors while varying the patch size

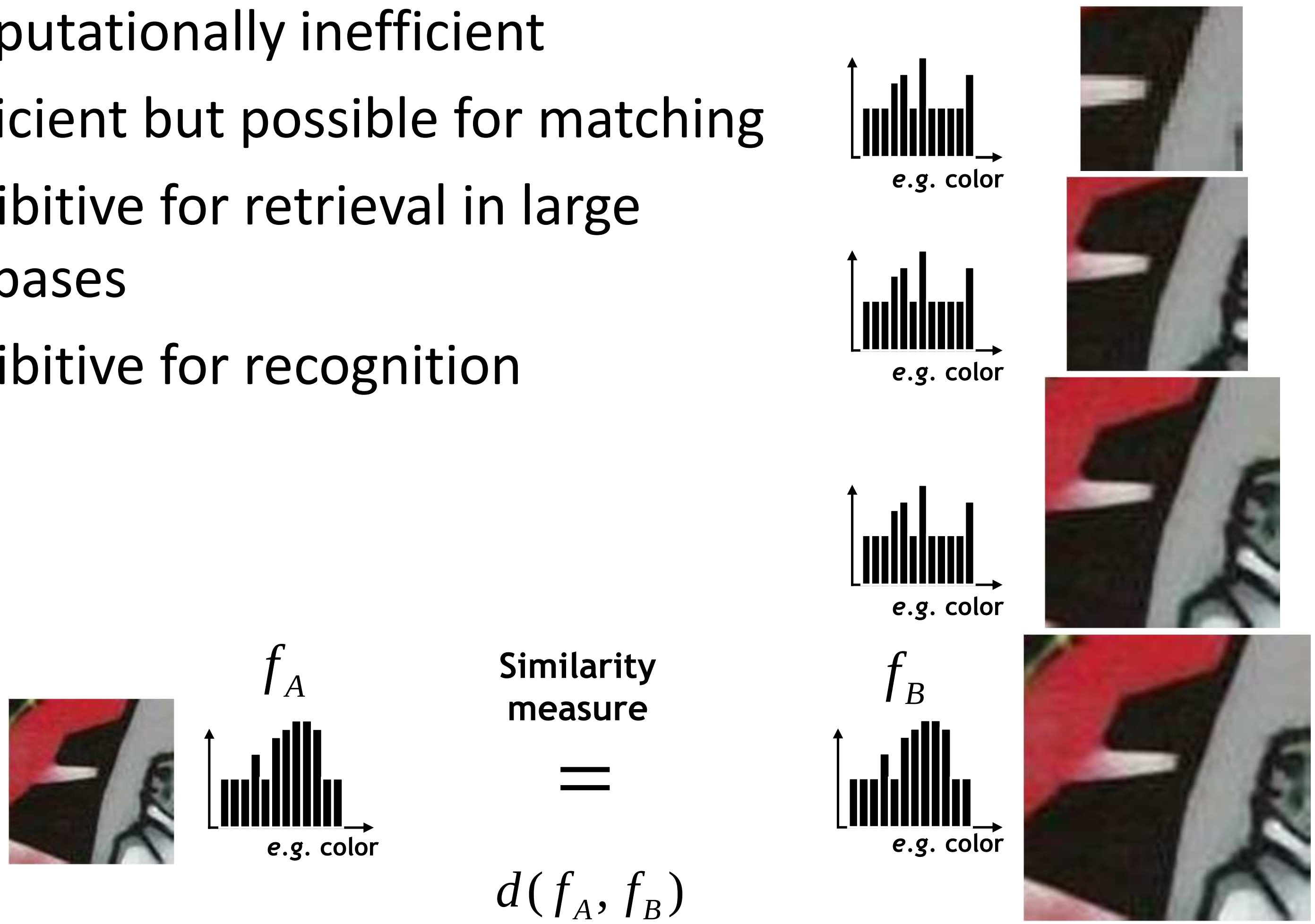




Slide credit: Krystian Mikolajczyk

Naïve Approach: Exhaustive Search

- Comparing descriptors while varying the patch size
  - Computationally inefficient
  - Inefficient but possible for matching
  - Prohibitive for retrieval in large databases
  - Prohibitive for recognition



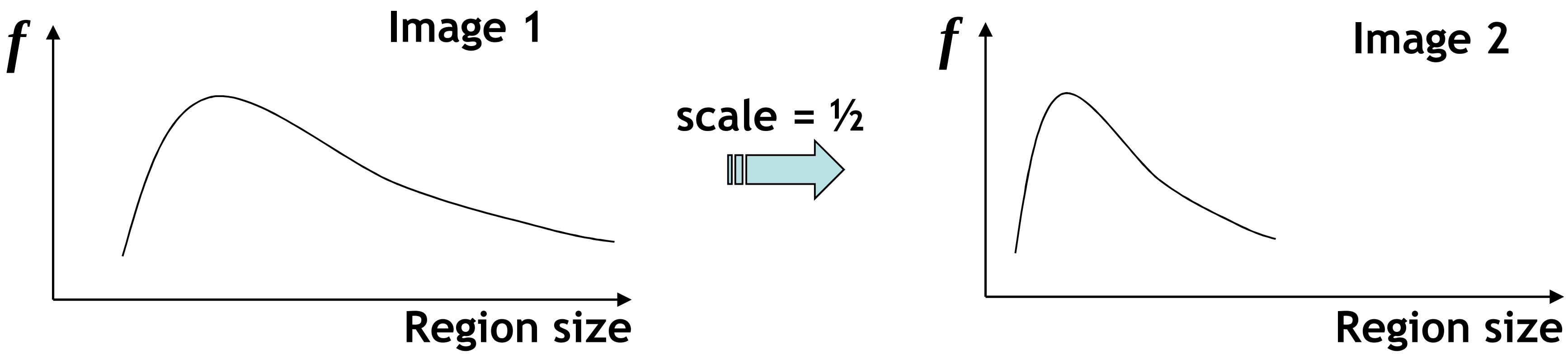
Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

- Solution:
  - Design a function on the region, which is "scale invariant" (*the same for corresponding regions, even if they are at different scales*)

    Example: average intensity. For corresponding regions (even of different sizes) it will be the same.

  - For a point in one image, we can consider it as a function of region size (patch width)

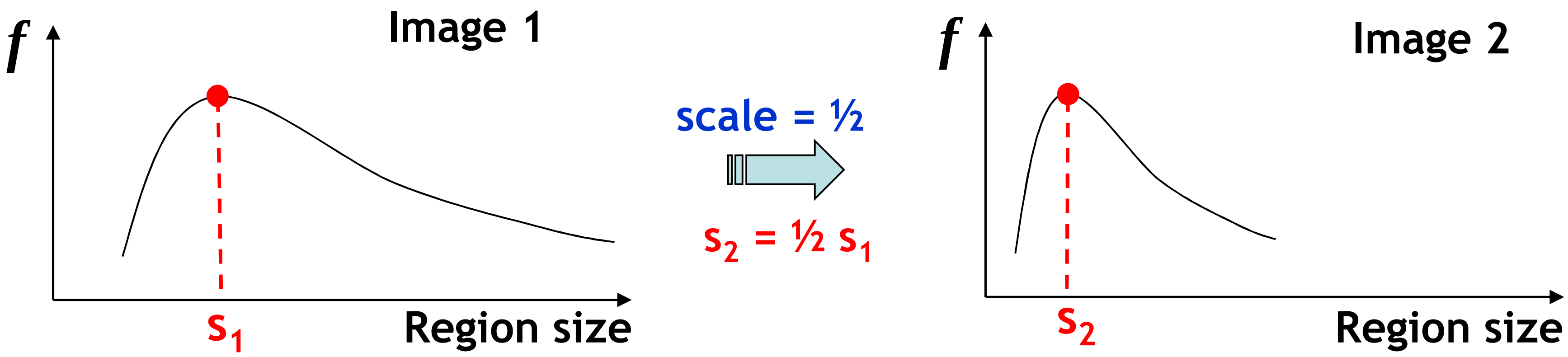


Slide credit: Kristen Grauman

Automatic Scale Selection

- Common approach:
  - Take a local maximum of this function.
  - Observation: region size for which the maximum is achieved should be *invariant* to image scale.

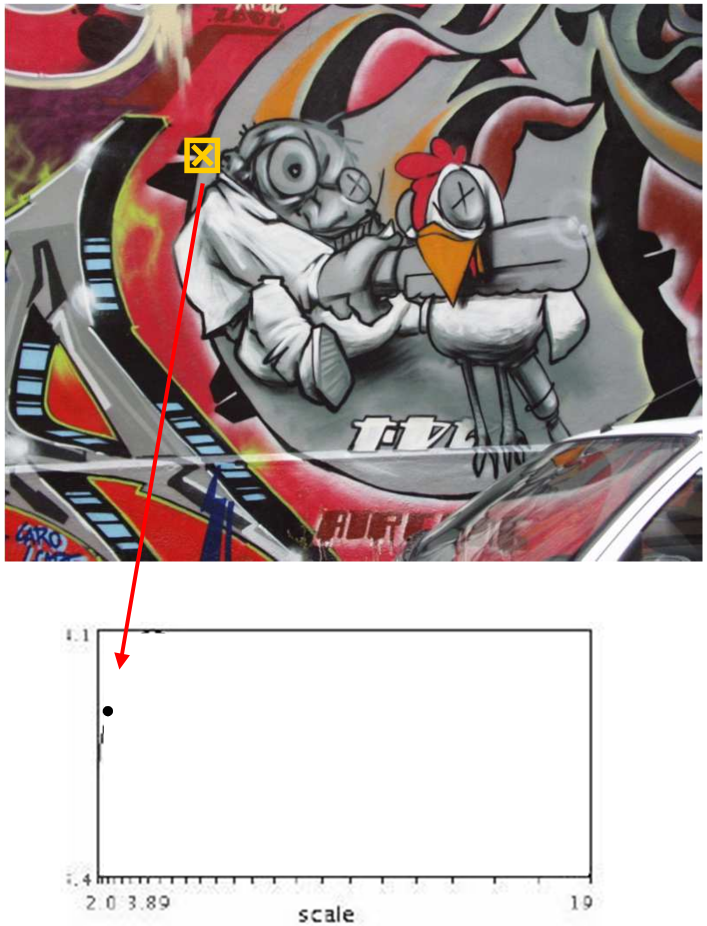
Important: this scale invariant region size is found in each image independently!



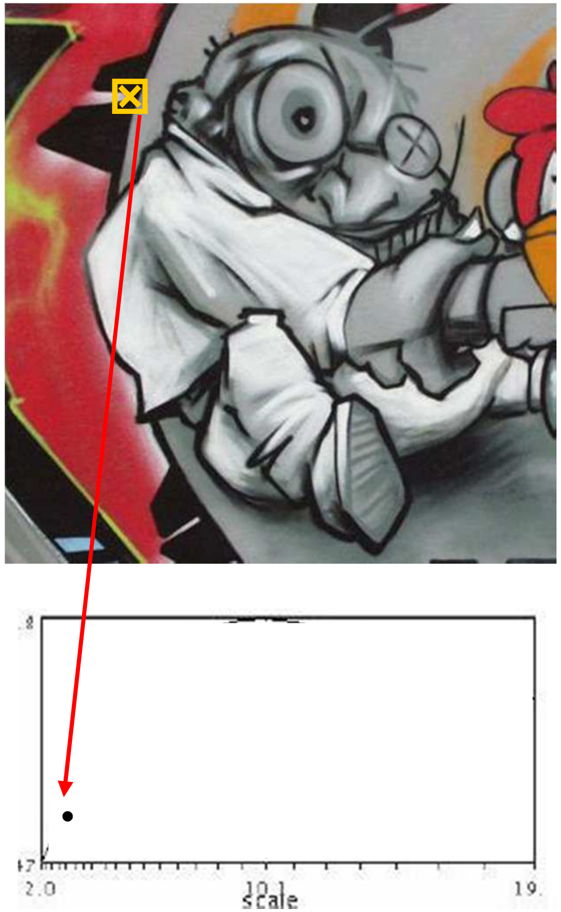
Slide credit: Kristen Grauman

Automatic Scale Selection

- Function responses for increasing scale (scale signature)



$f(I_{i_1 \ldots i_m}(x, \sigma))$



$f(I_{i_1 \ldots i_m}(x', \sigma))$

Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

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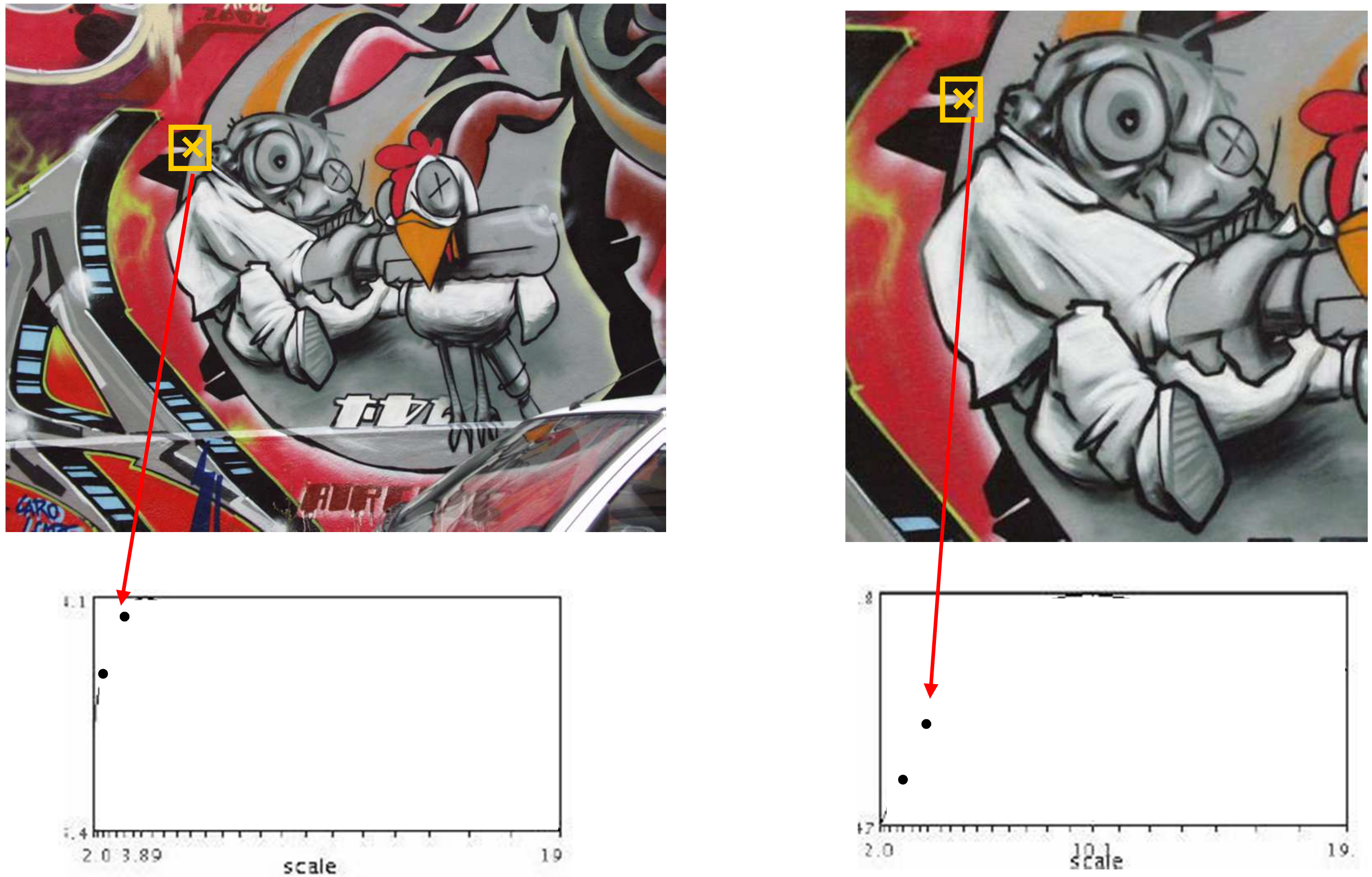
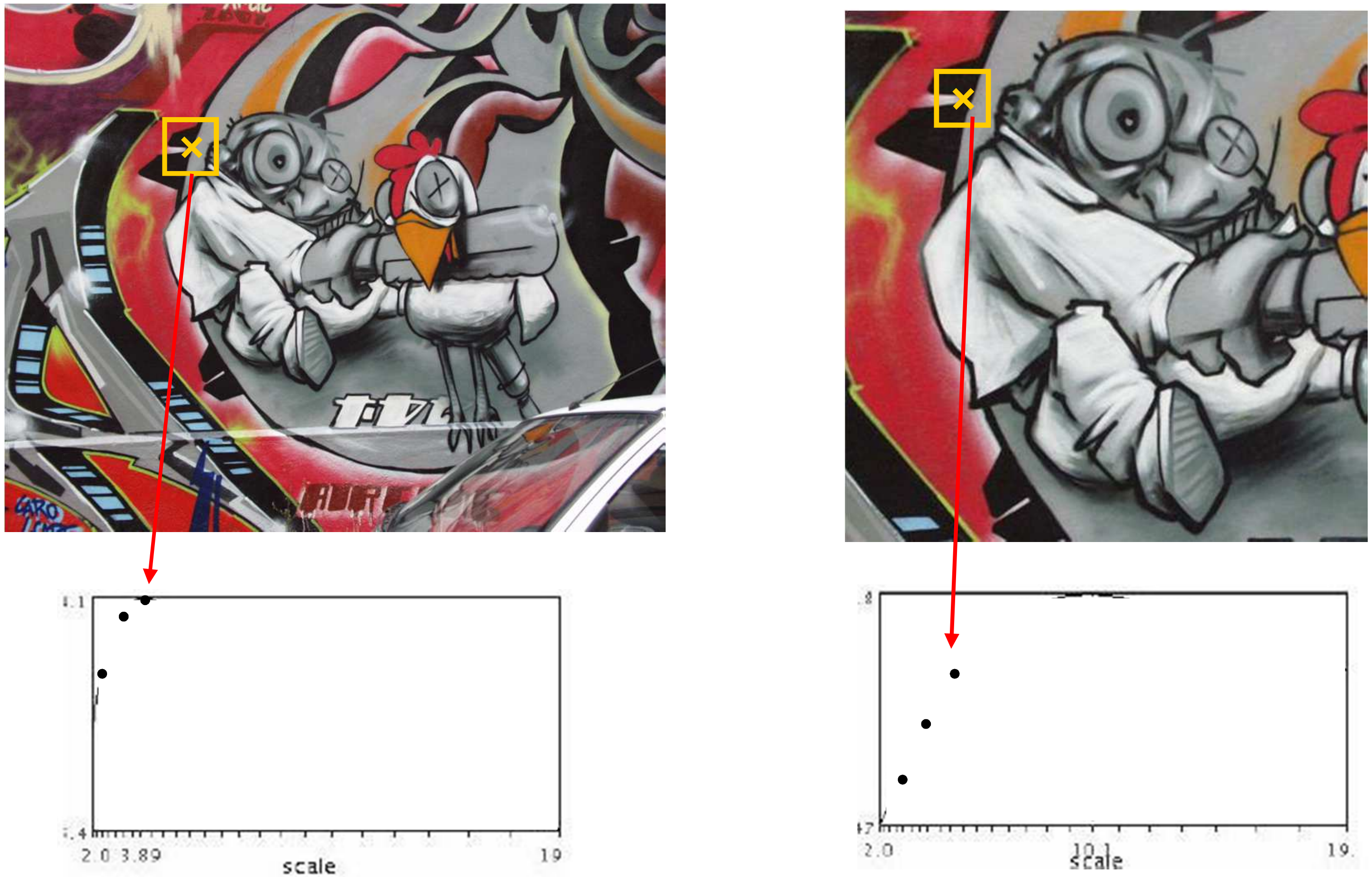


$f(I_{i_1 \ldots i_m}(x, \sigma))$ $\qquad$ $f(I_{i_1 \ldots i_m}(x', \sigma))$

Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

- Function responses for increasing scale (scale signature)

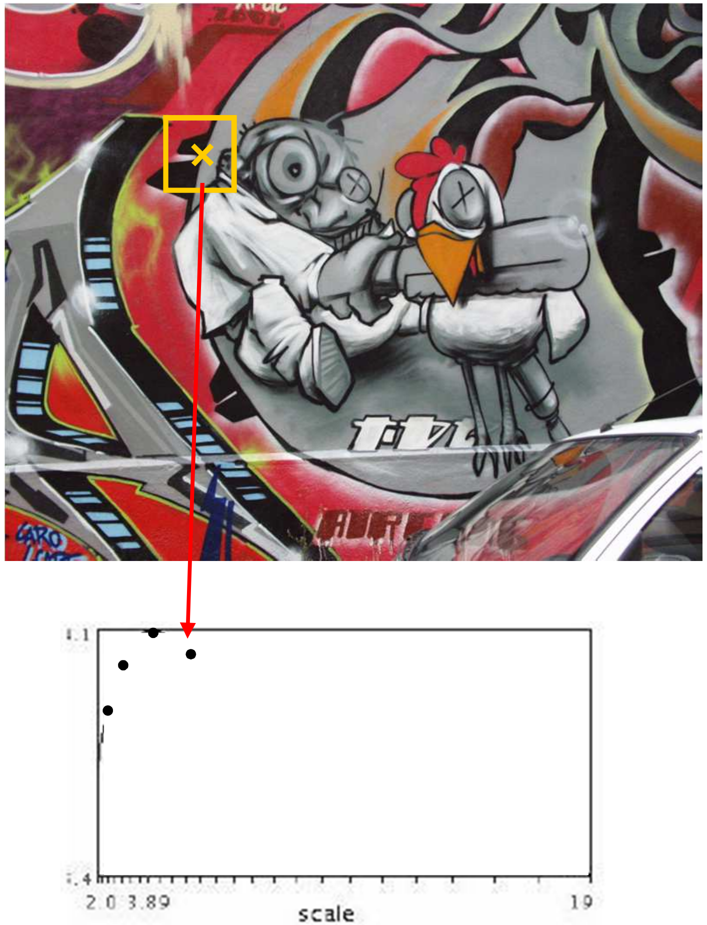


$f(I_{i_1 \ldots i_m}(x, \sigma))$ $\quad$ $f(I_{i_1 \ldots i_m}(x', \sigma))$

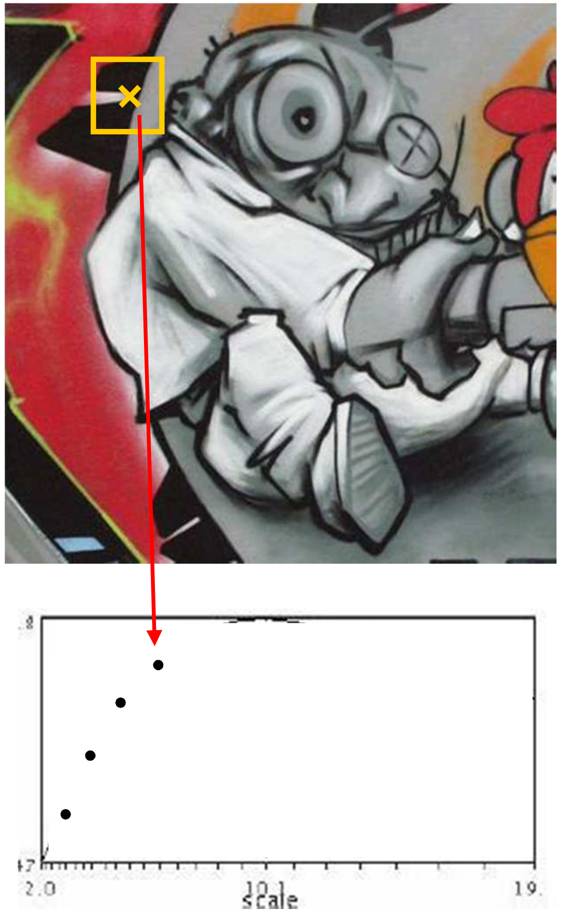
Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

- Function responses for increasing scale (scale signature)



$f(I_{i_1 \ldots i_m}(x, \sigma))$

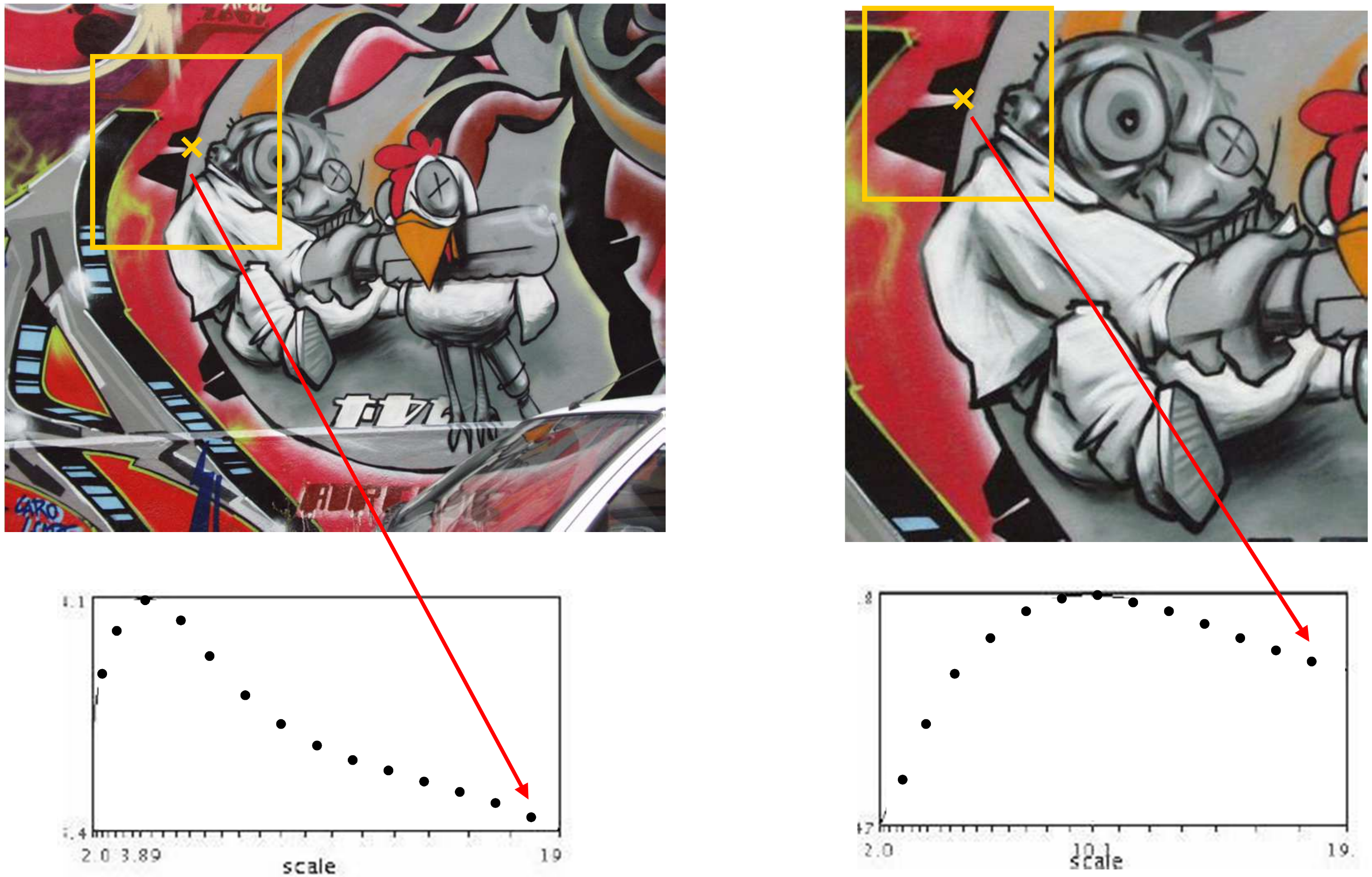


$f(I_{i_1 \ldots i_m}(x', \sigma))$

Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

- Function responses for increasing scale (scale signature)

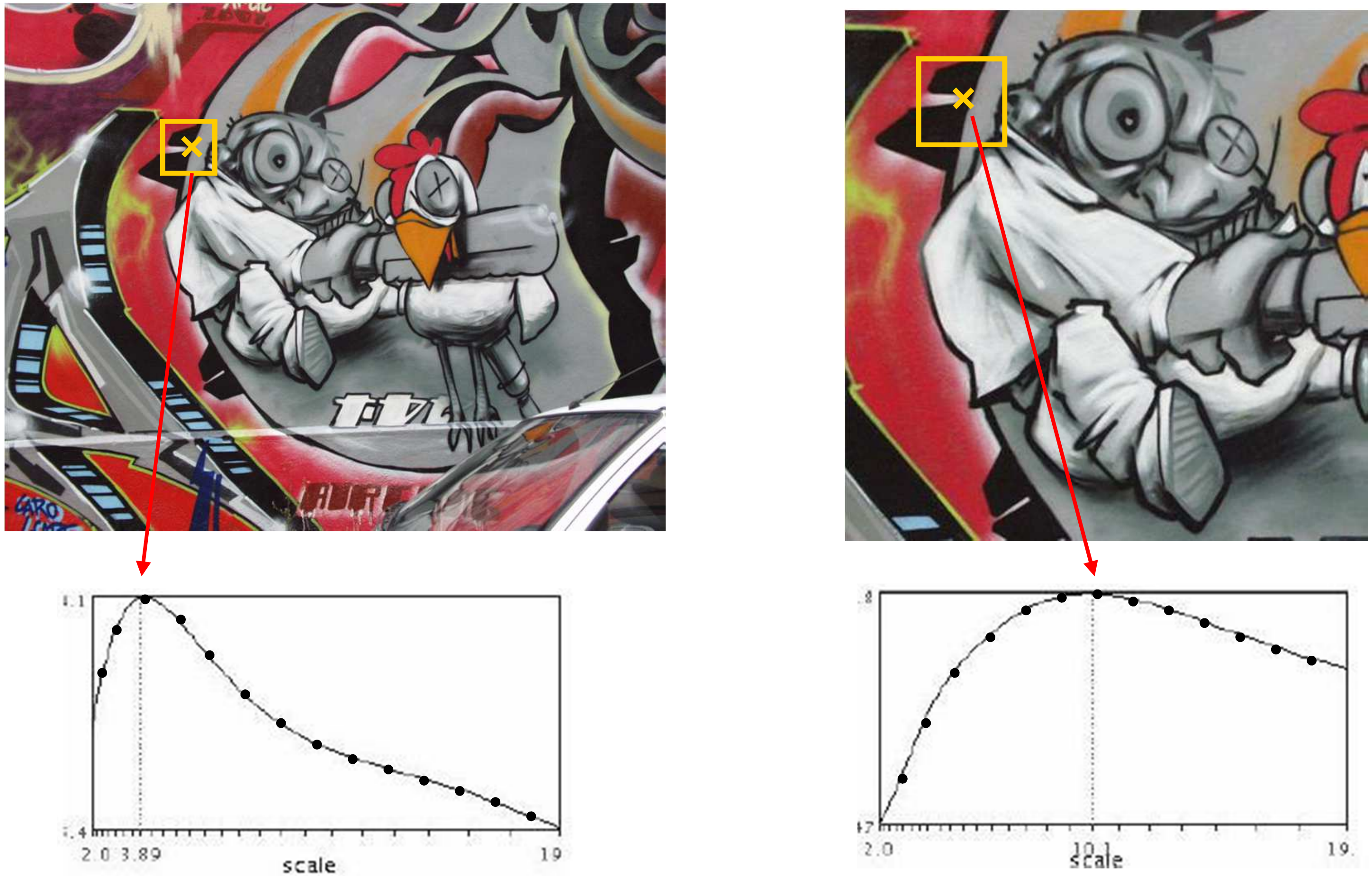


$f(I_{i_1 \ldots i_m}(x, \sigma))$ $f(I_{i_1 \ldots i_m}(x', \sigma))$

Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

- Function responses for increasing scale (scale signature)

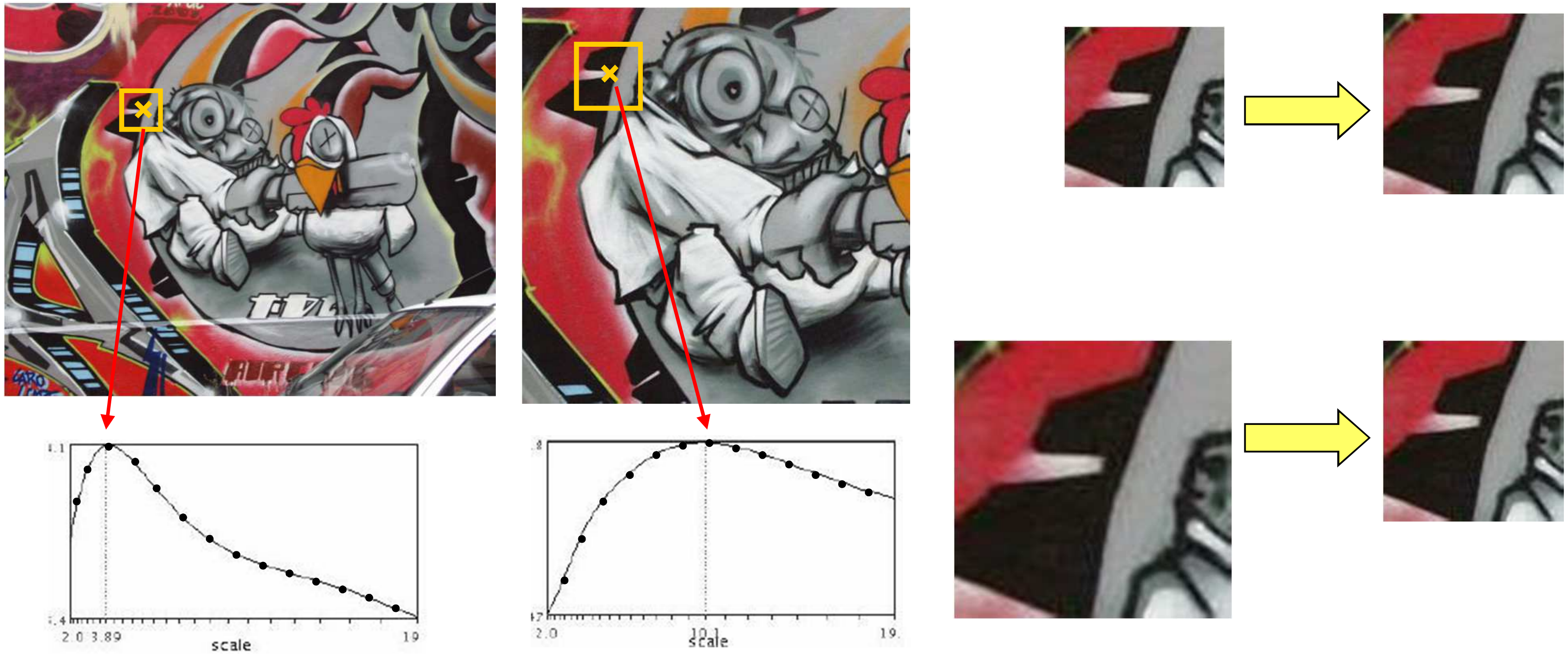


$f(I_{i_1 \ldots i_m}(x, \sigma))$ $f(I_{i_1 \ldots i_m}(x', \sigma'))$

Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

- Normalize: Rescale to fixed size



$f(I_{i_1 \ldots i_m}(x,\sigma))$ $f(I_{i_1 \ldots i_m}(x',\sigma'))$

Slide credit: Tinne Tuytelaars

What Is A Useful Signature Function?

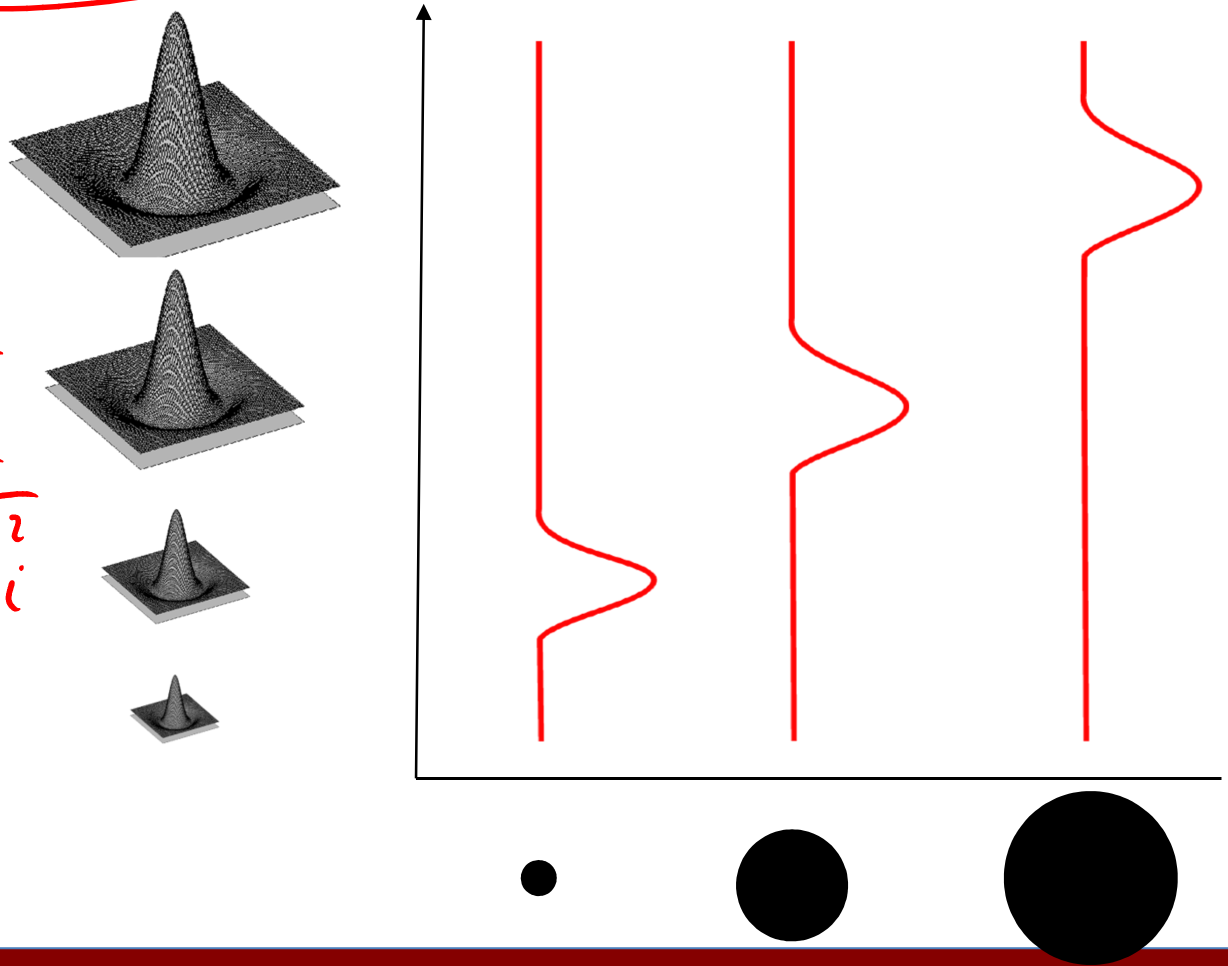
$f(I(x, y, \text{neighborhood}))$

- Laplacian-of-Gaussian = "blob" detector

$\Delta f = \nabla^2 f$

$= \nabla \cdot \nabla f$

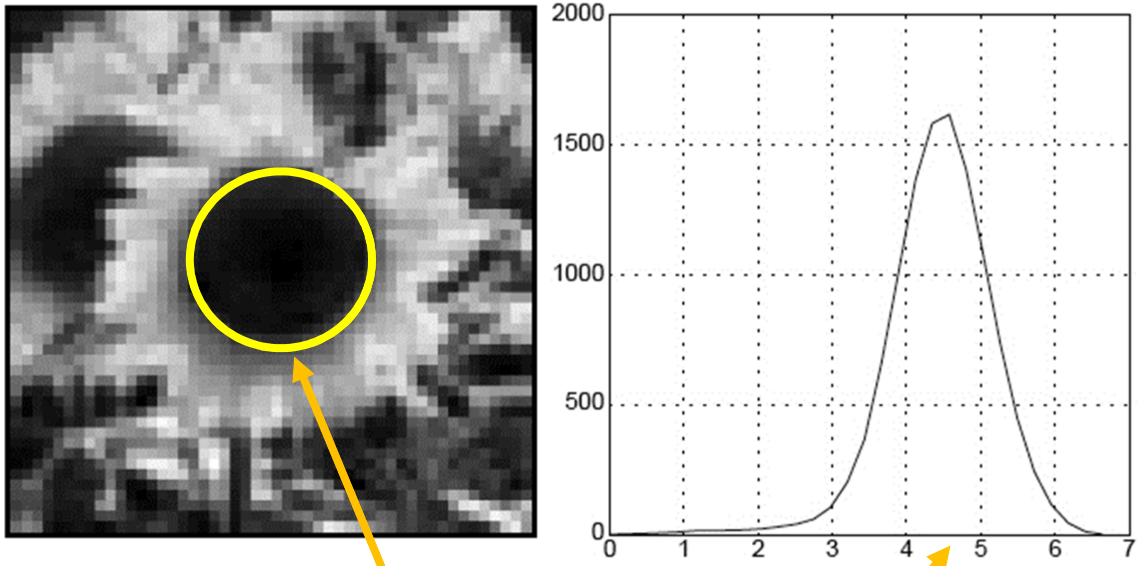
$= \sum_{i=1}^{n} \frac{\partial^2 f}{\partial x_i^2}$



Slide credit: Bastian Leibe

Characteristic Scale

- We define the *characteristic scale* as the scale that produces peak of Laplacian response



Characteristic scale

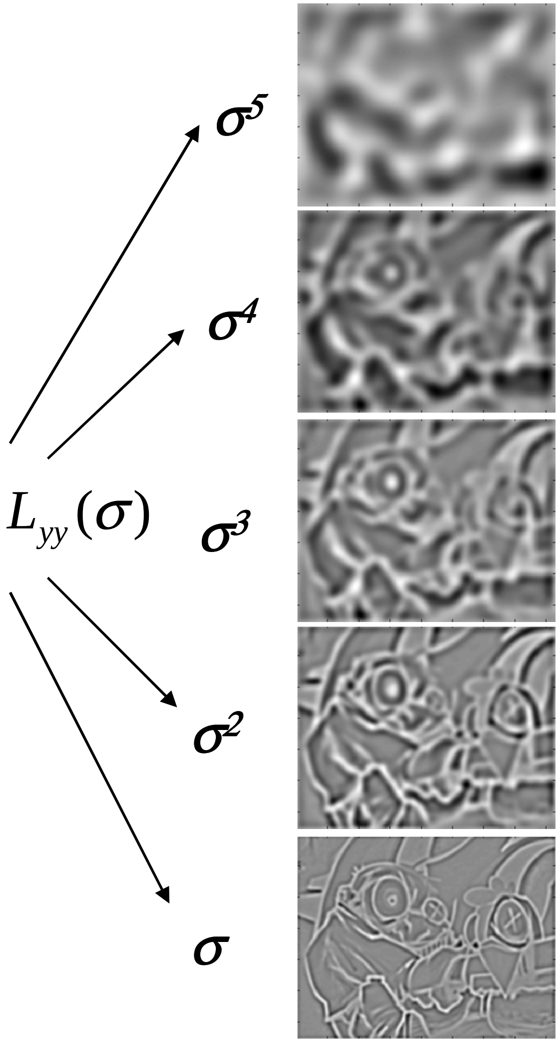
T. Lindeberg (1998). ["Feature detection with automatic scale selection."]() *International Journal of Computer Vision* 30 (2): pp 77--116.

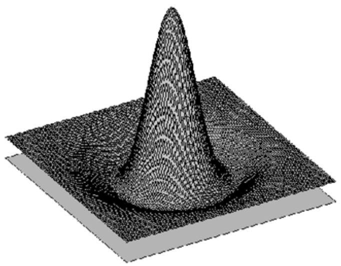
Slide credit: Svetlana Lazebnik

Laplacian-of-Gaussian (LoG)

- Interest points:
  - Local maxima in scale space of Laplacian-of-Gaussian

$$L_{xx}(\sigma) + L_{yy}(\sigma)$$





Slide adapted from Krystian Mikolajczyk

Laplacian-of-Gaussian (LoG)

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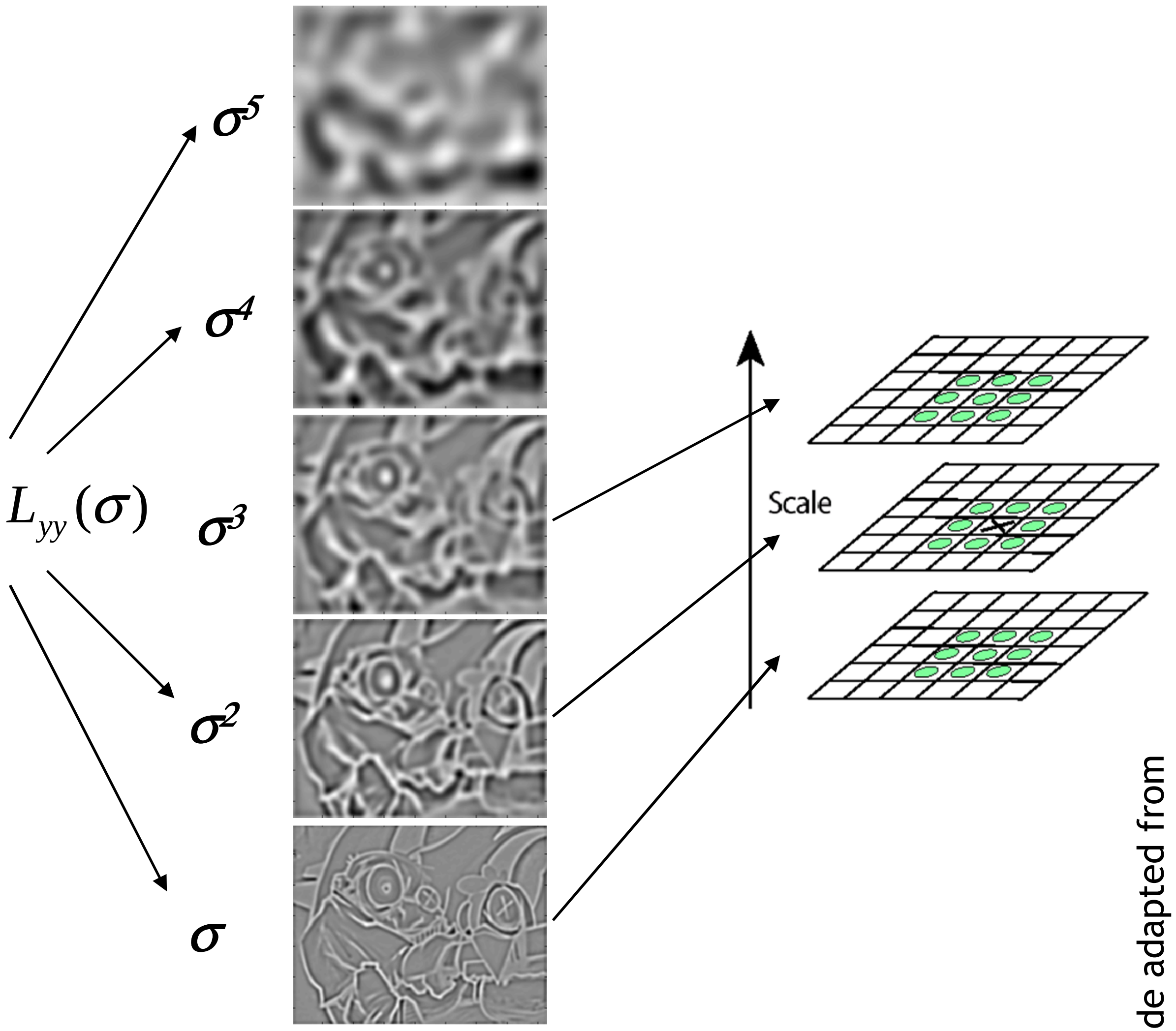

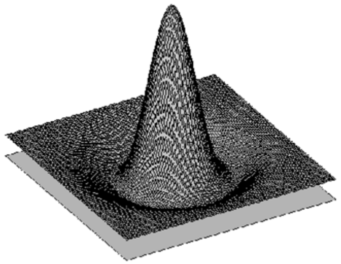

Slide adapted from

Laplacian-of-Gaussian (LoG)

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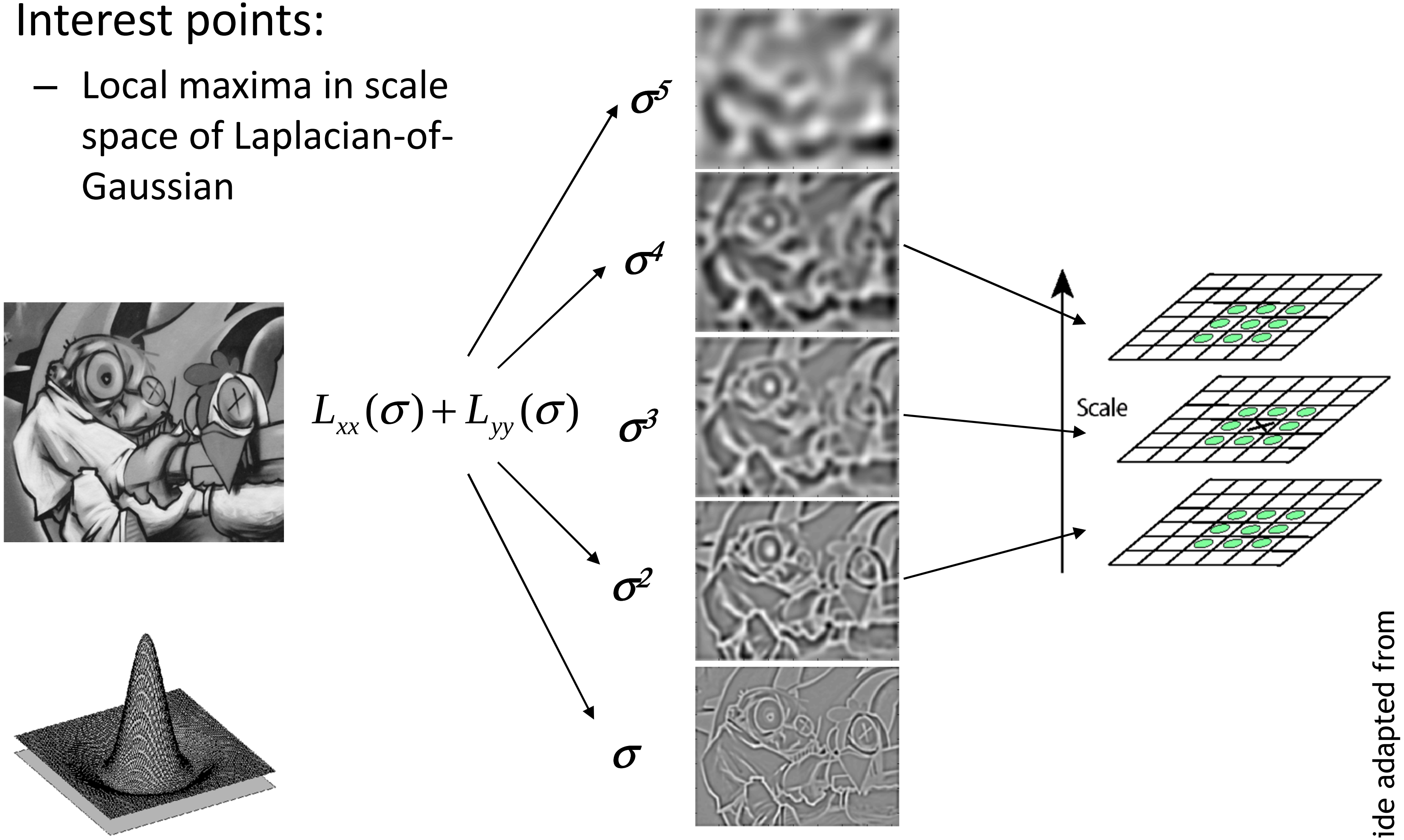
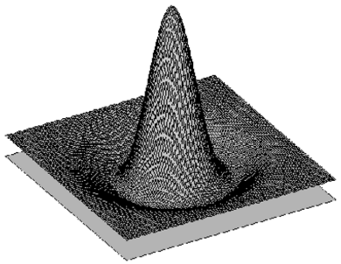
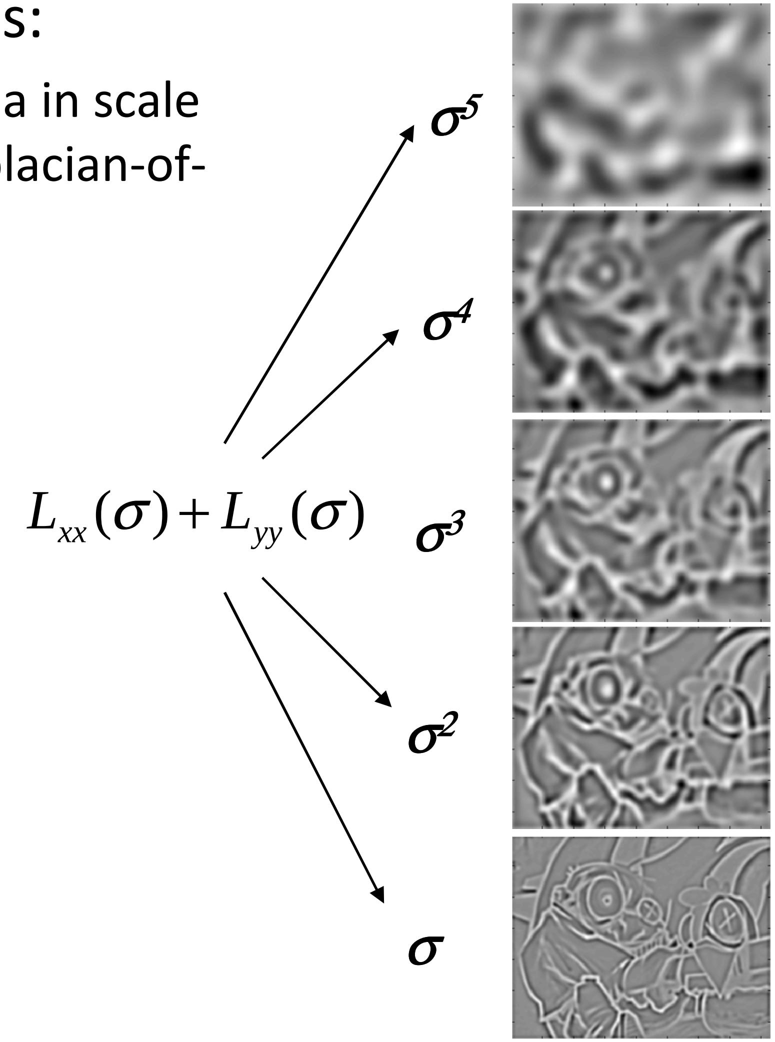


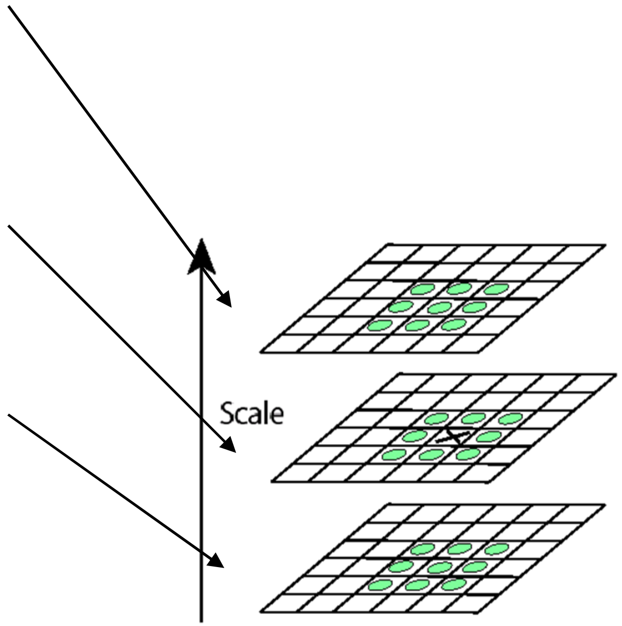
Slide adapted from

Laplacian-of-Gaussian (LoG)

- Interest points:
  - Local maxima in scale space of Laplacian-of-Gaussian







$\Rightarrow$ **List of *(x, y, σ)***

Slide adapted from

LoG Detector: Workflow

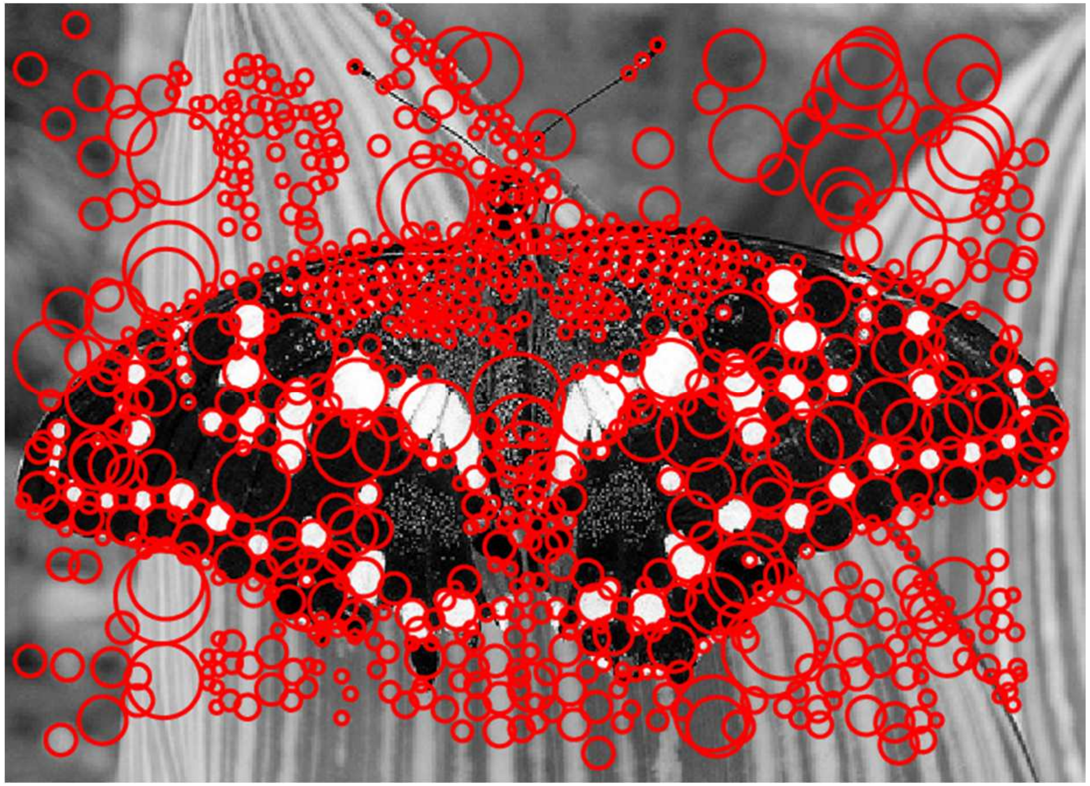
Slide credit: Svetlana Lazebnik

LoG Detector: Workflow

sigma = 11.9912

Slide credit: Svetlana Lazebnik

LoG Detector: Workflow



Slide credit: Svetlana Lazebnik

Technical Detail

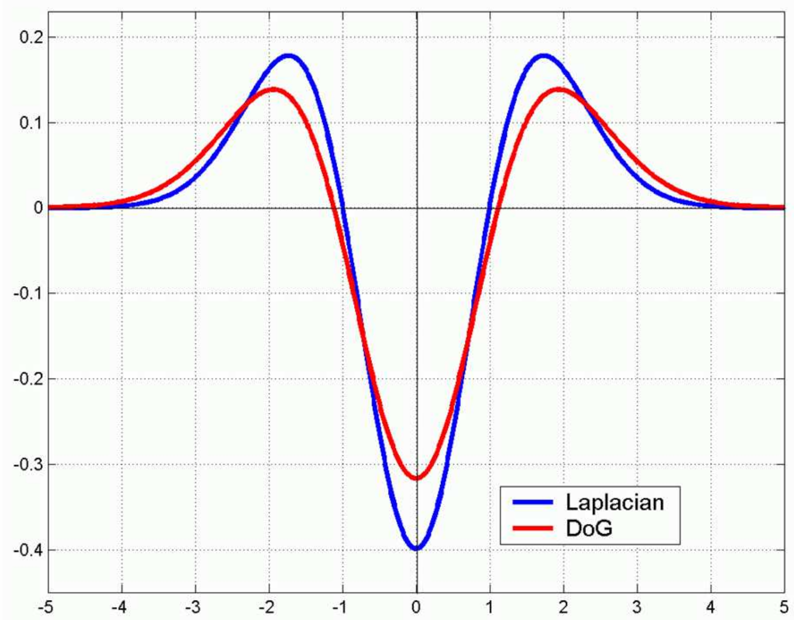
- We can efficiently approximate the Laplacian with a difference of Gaussians:

$$L = \sigma^2 \left(G_{xx}(x,y,\sigma) + G_{yy}(x,y,\sigma) \right)$$

(Laplacian)

$$DoG = G(x,y,k\sigma) - G(x,y,\sigma)$$

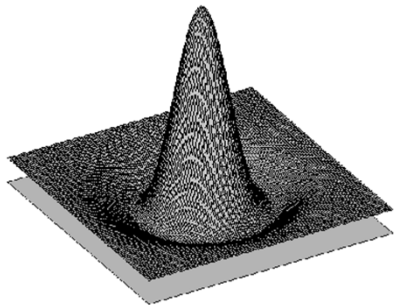
(Difference of Gaussians)



Slide credit: Bastian Leibe

Difference-of-Gaussian (DoG)

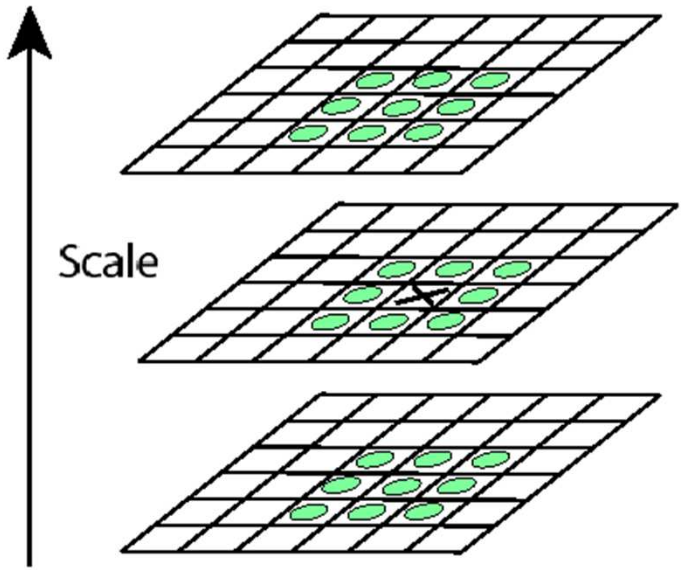
- Difference of Gaussians as approximation of the LoG
  - This is used e.g. in Lowe's SIFT pipeline for feature detection.
- Advantages
  - No need to compute 2nd derivatives
  - Gaussians are computed anyway, e.g. in a Gaussian pyramid.



Slide credit: Bastian Leibe

Key point localization with DoG

- Detect maxima of difference-of-Gaussian (DoG) in scale space
- Then reject points with low contrast (threshold)
- Eliminate edge responses

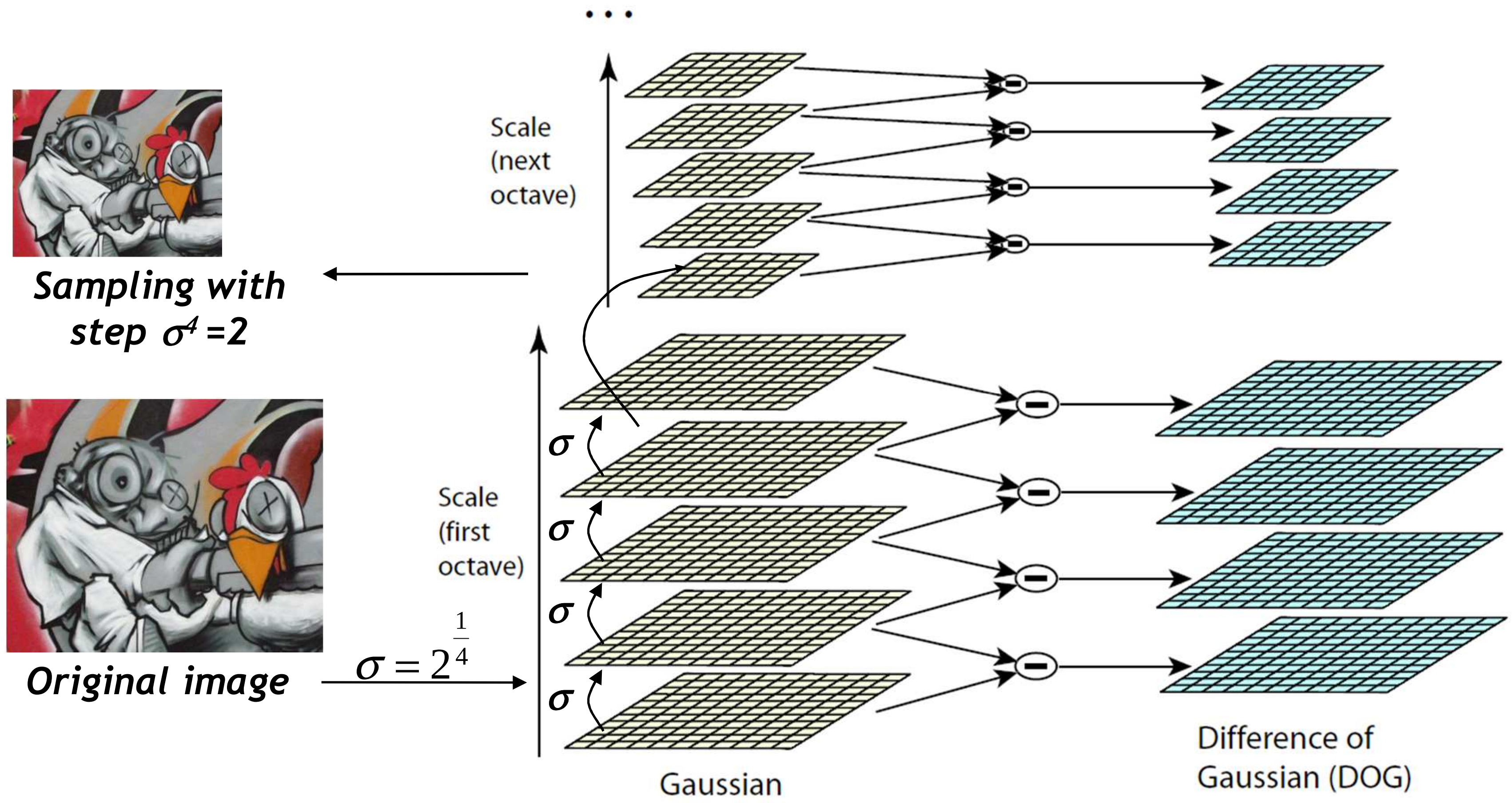


Candidate keypoints: list of (x,y,σ)

Slide credit: David Lowe

DoG – Efficient Computation

- Computation in Gaussian scale pyramid

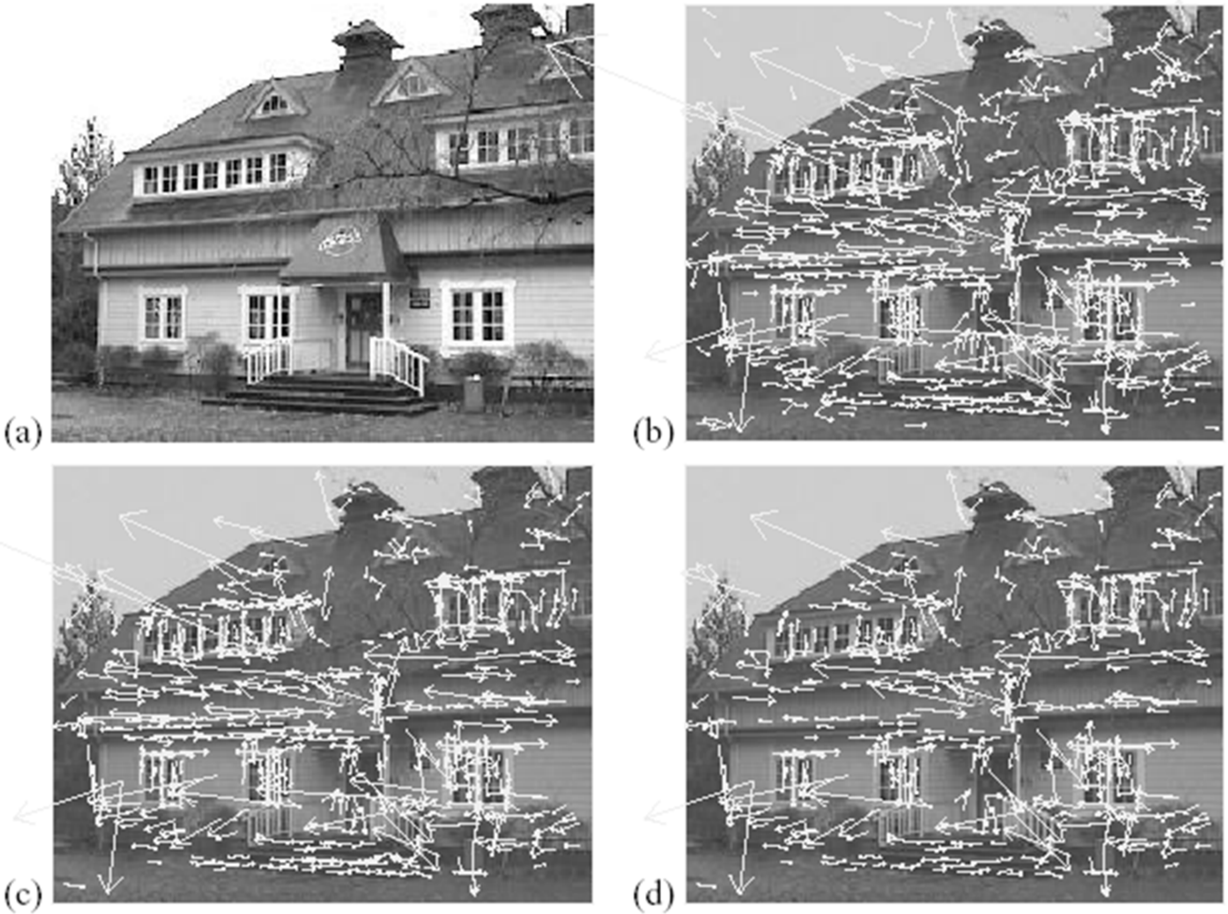


Slide adapted from Krystian Mikolajczyk

Results: Lowe's DoG

Slide credit: Bastian Leibe

Example of Keypoint Detection


(a) (b) (c) (d)

(a) 233x189 image

(b) 832 DoG extrema

(c) 729 left after peak value threshold

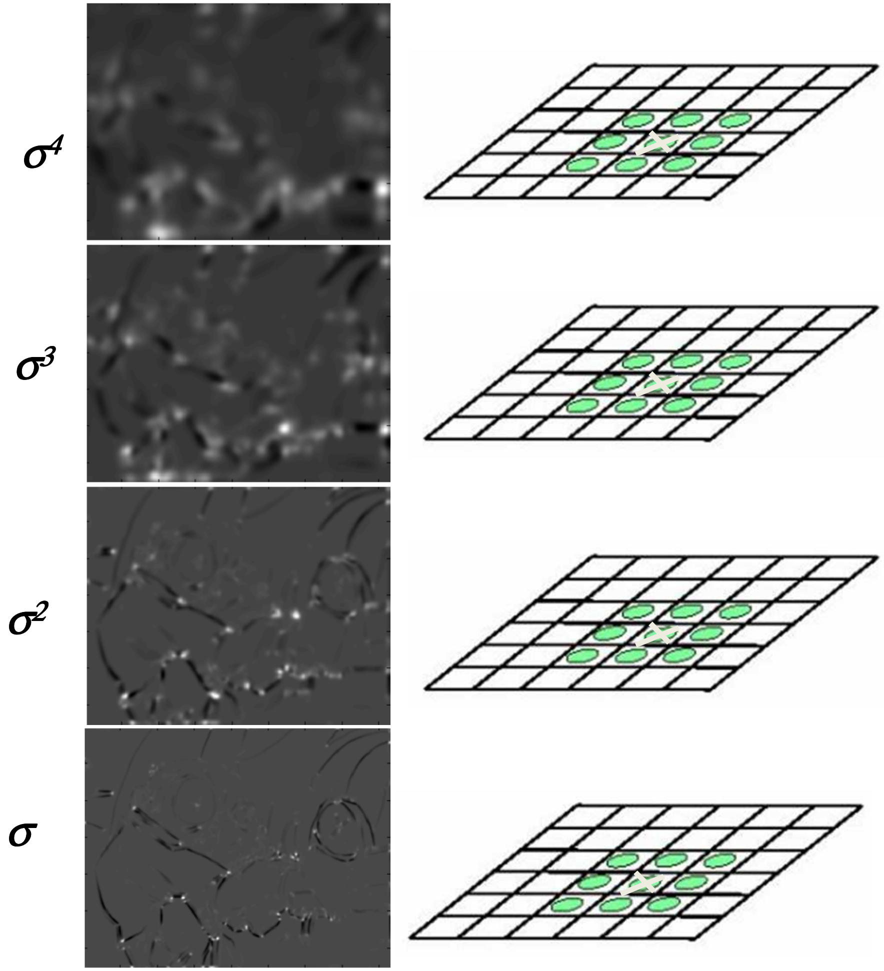
(d) 536 left after testing ratio of principle curvatures (removing edge responses)

Slide credit: David Lowe

Harris-Laplace [Mikolajczyk '01]

1. Initialization: Multiscale Harris corner detection

Slide adapted from Krystian Mikolajczyk

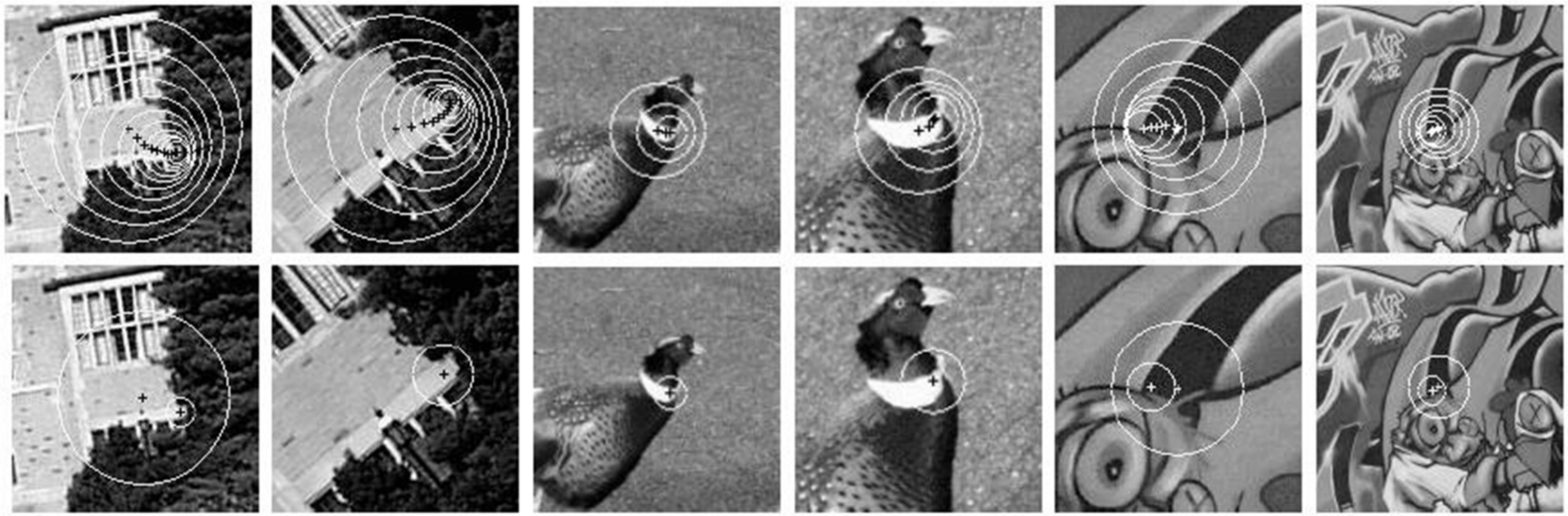


Computing Harris function **Detecting local maxima**

Harris-Laplace [Mikolajczyk '01]

1. Initialization: Multiscale Harris corner detection
2. Scale selection based on Laplacian
   (same procedure with Hessian ⇒ Hessian-Laplace)

Harris points



Harris-Laplace points

Slide adapted from Krystian Mikolajczyk

Summary: Scale Invariant Detection

- Given: Two images of the same scene with a large *scale difference* between them.
- Goal: Find *the same* interest points *independently* in each image.
- Solution: Search for *maxima* of suitable functions in *scale* and in *space* (over the image).

- Two strategies
  - Laplacian-of-Gaussian (LoG)
  - Difference-of-Gaussian (DoG) as a fast approximation
  - *These can be used either on their own, or in combinations with single-scale keypoint detectors (Harris, Hessian).*

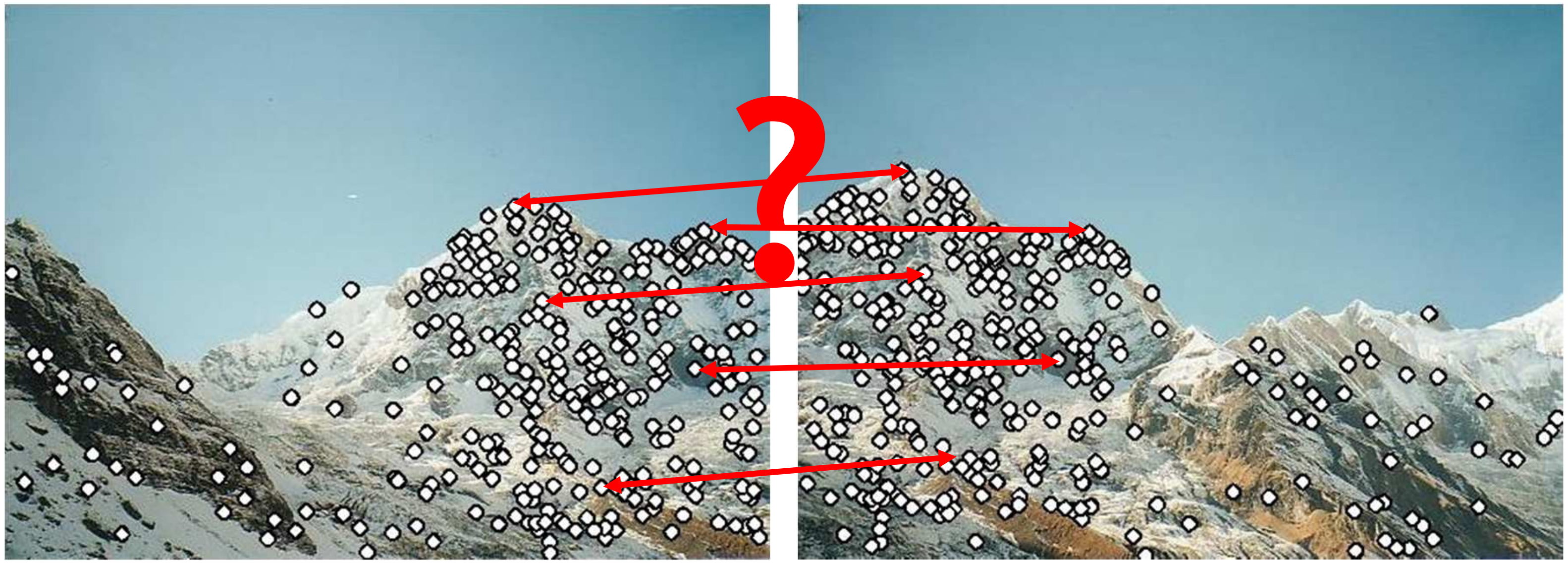
What we will learn today?

- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector
- Scale invariant region selection
  - Automatic scale selection
  - Laplacian-of-Gaussian detector
  - Difference-of-Gaussian detector
  - Combinations
- Local descriptors
  - An intro

Local Descriptors

- We know how to detect points
- Next question:

How to *describe* them for matching?



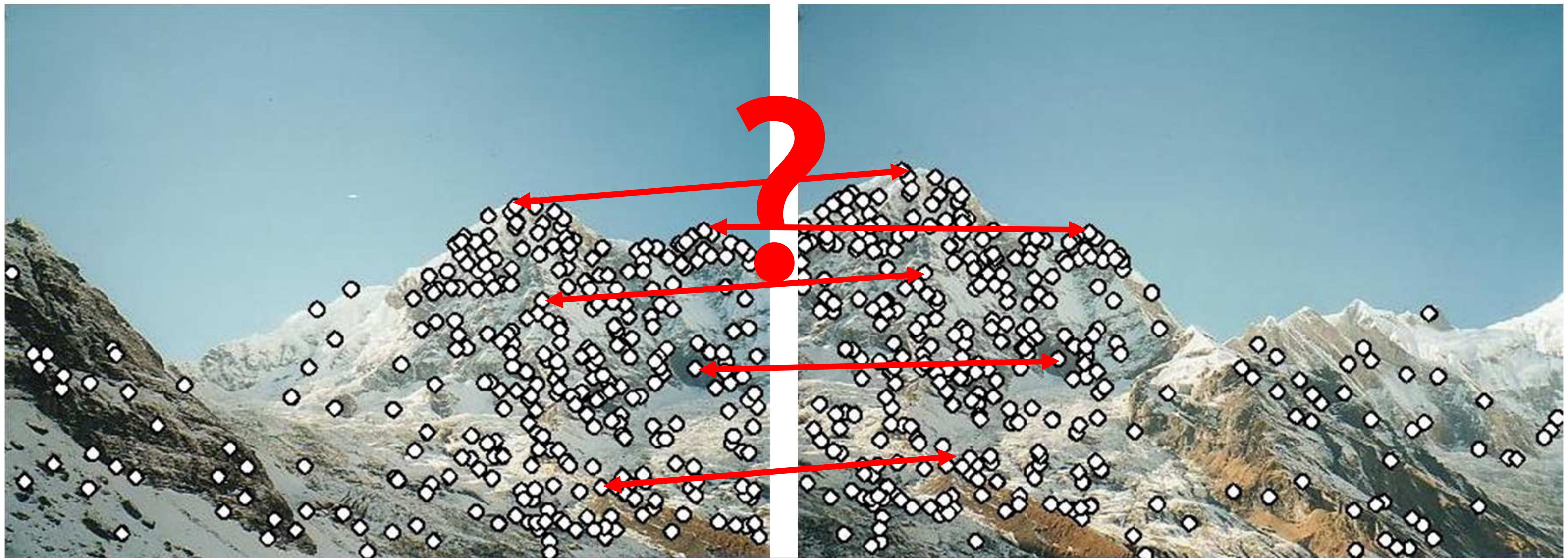
⇒ ***Next lecture…***

Slide credit: Kristen Grauman

Local Descriptors

- We know how to detect points
- Next question:

How to *describe* them for matching?



Point descriptor should be:
1. **Invariant**
2. **Distinctive**

Slide credit: Kristen Grauman

What we have learned today?

- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector
- Scale invariant region selection
  - Automatic scale selection
  - Laplacian-of-Gaussian detector
  - Difference-of-Gaussian detector (Problem Set 3 (Q2))
  - Combinations
- Local descriptors
  - An intro

Supplementary materials

- Hessian detector

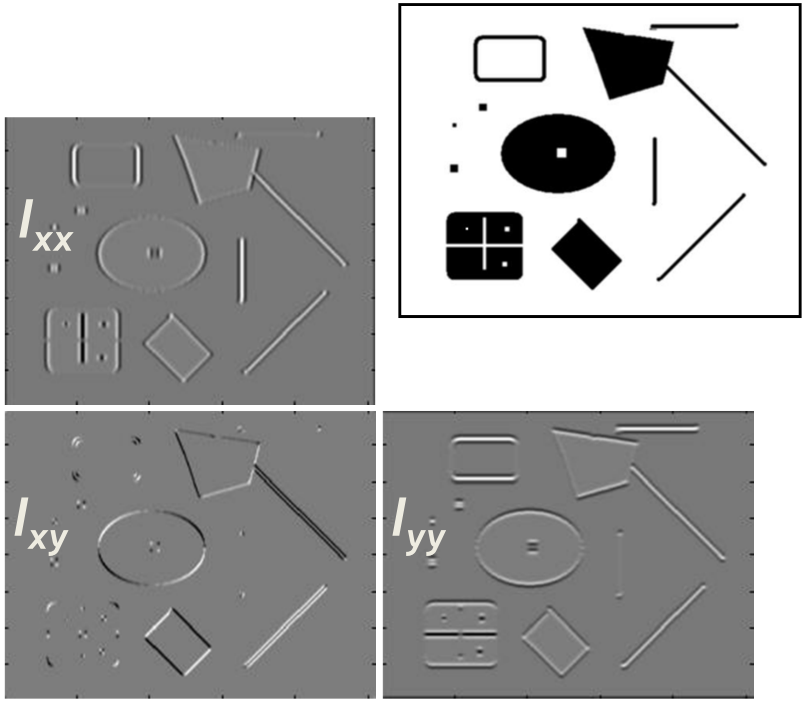
Hessian Detector [Beaudet78]

- Hessian determinant

$$Hessian(I) = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}$$

Note: these are 2nd derivatives!

Intuition: **Search for strong derivatives in two orthogonal directions**



Slide credit: Krystian Mikolajczyk

Hessian Detector [Beaudet78]

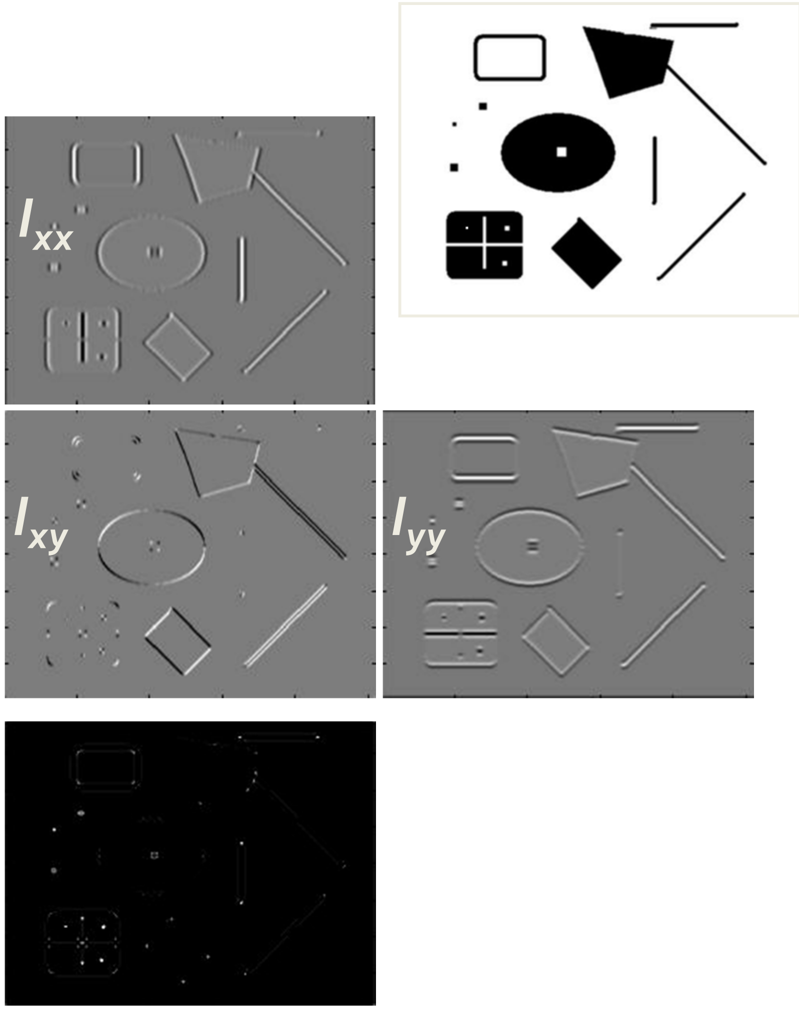
- Hessian determinant

$$Hessian(I) = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}$$

$$\det(Hessian(I)) = I_{xx} I_{yy} - I_{xy}^2$$

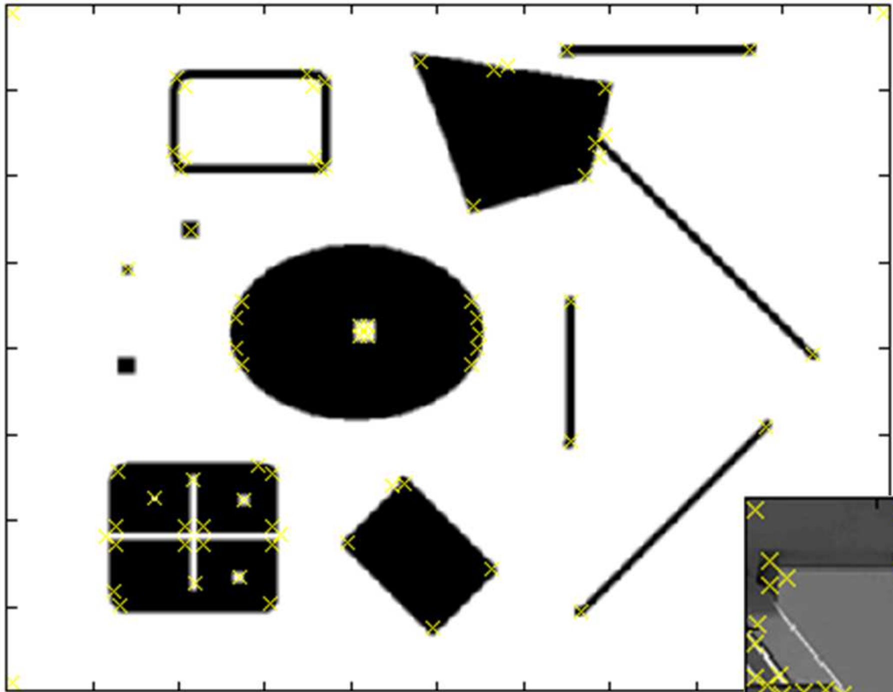
In Matlab:

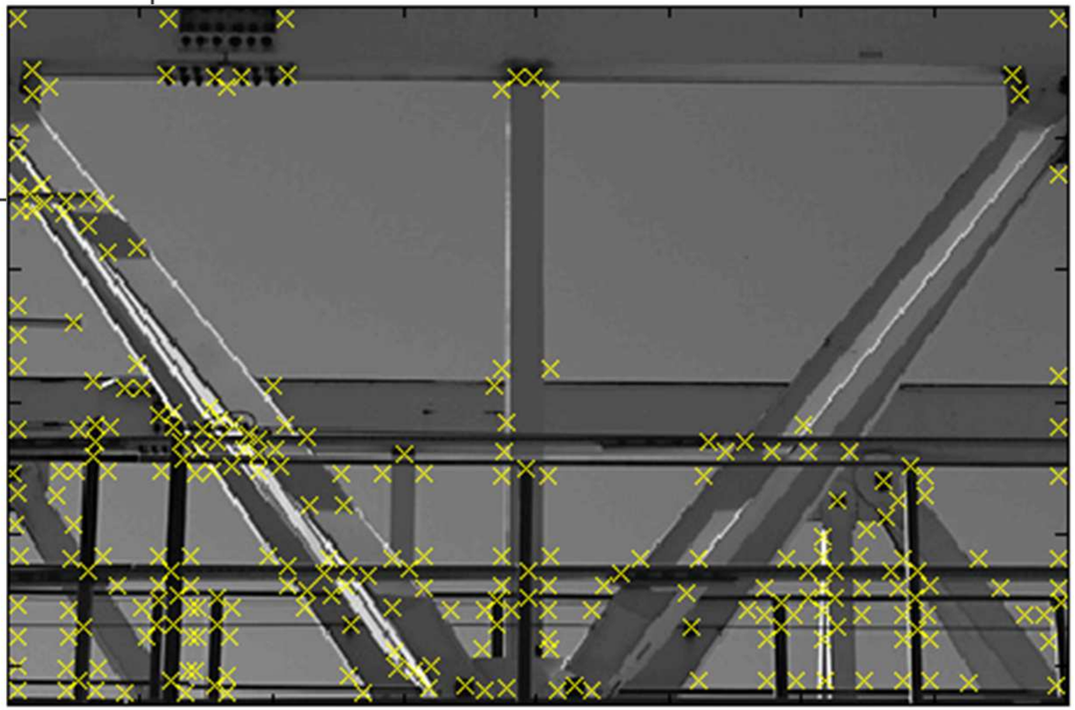
$$I_{xx} .* I_{yy} - (I_{xy})\wedge 2$$



Slide credit: Krystian Mikolajczyk

Hessian Detector – Responses [Beaudet78]

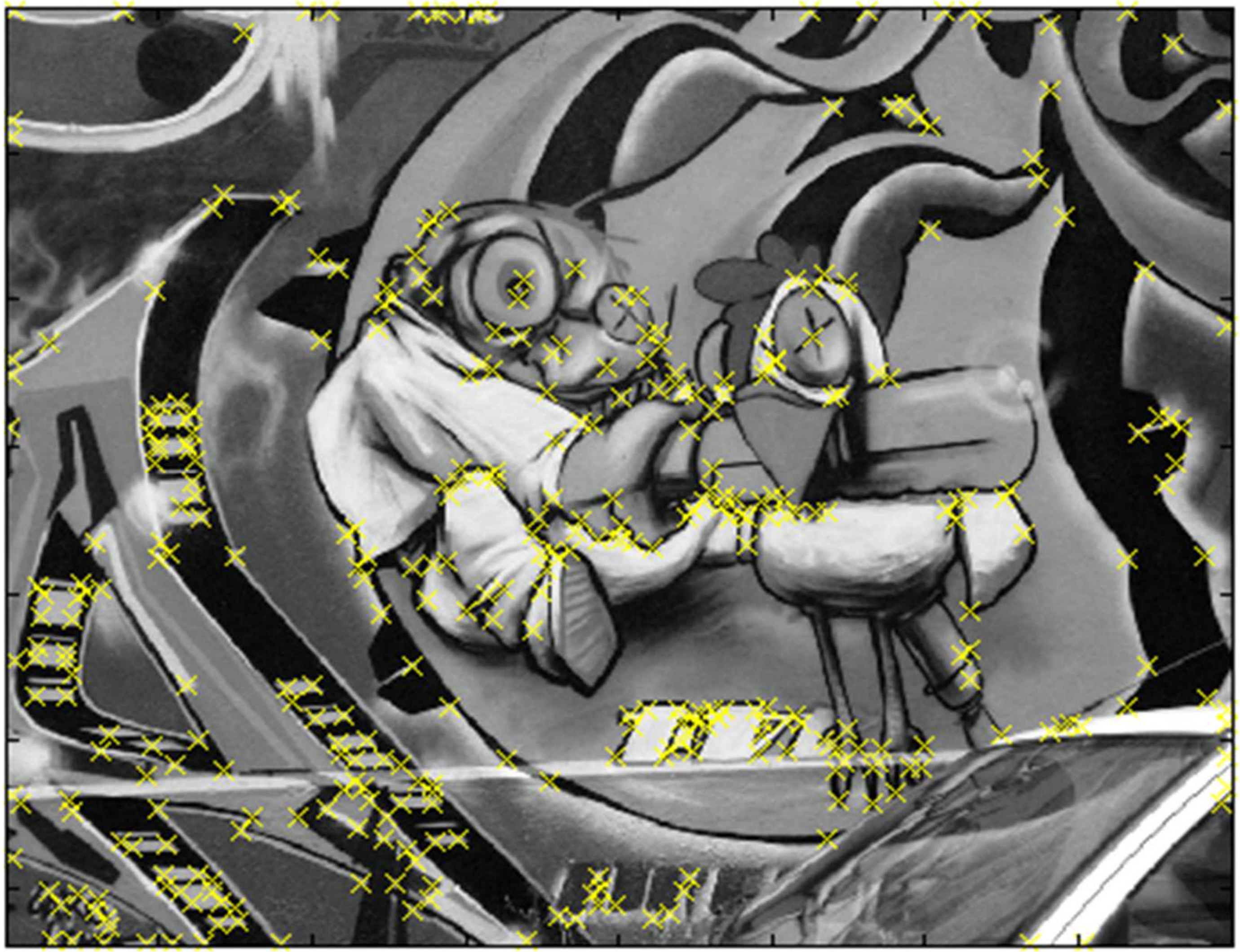




Effect: **Responses mainly on corners and strongly textured areas.**

Slide credit: Krystian Mikolajczyk

Hessian Detector – Responses [Beaudet78]



Slide credit: Krystian Mikolajczyk