Lecture 9

Fitting and Matching

- Problem formulation
- Least square methods
- RANSAC
- Hough transforms
- Multi-model fitting
- Fitting helps matching!

Reading:

[HZ] Chapter: 4 “Estimation – 2D projective transformation”,
Chapter 11 “Computation of the fundamental matrix F”

[FP] Chapters: 16 “Segmentation and fitting using probabilistic methods”

Some slides of this lectures are courtesy of profs. S. Lazebnik & K. Grauman

Fitting

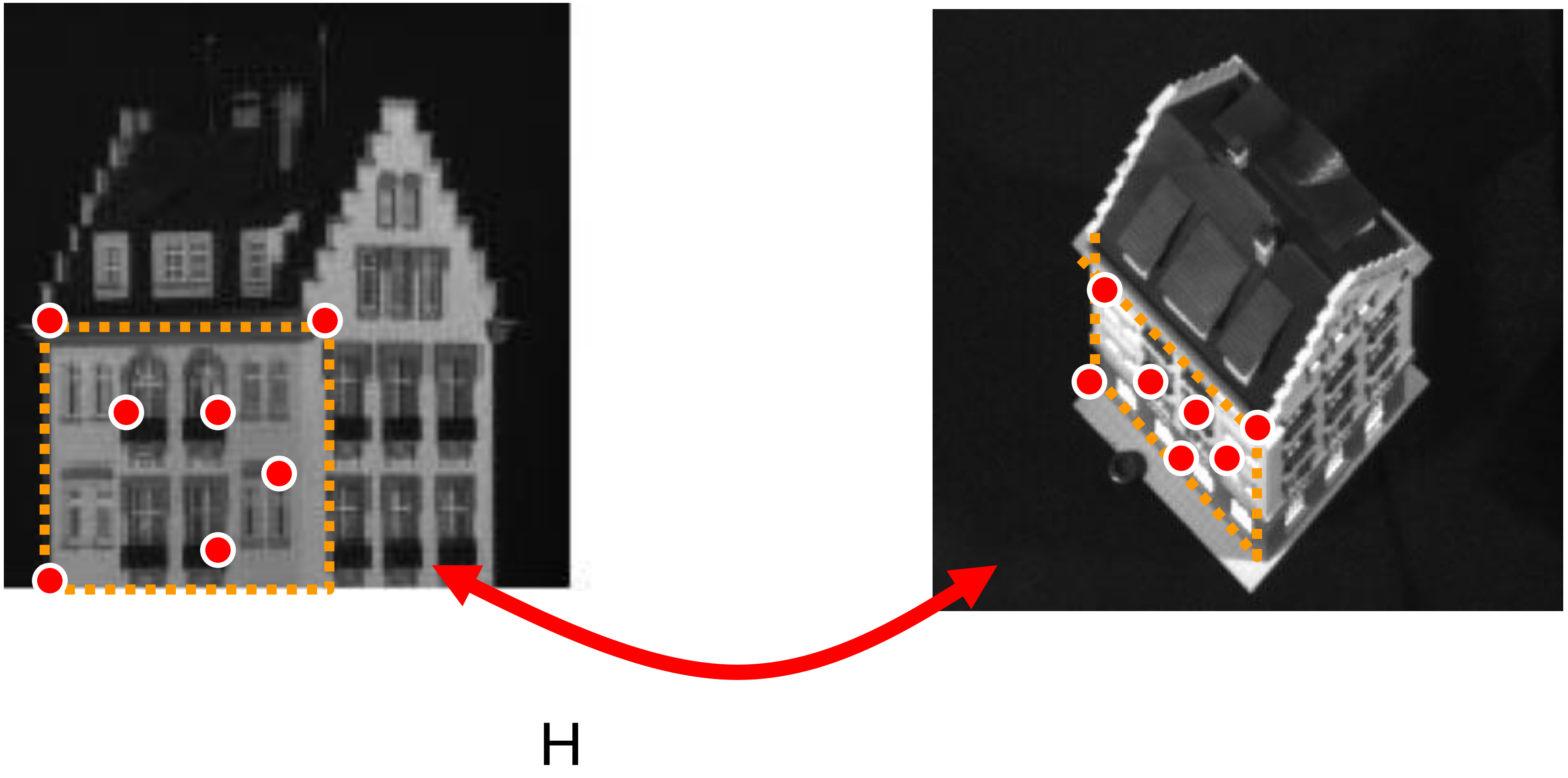
Goals:

- Choose a parametric model to fit a certain quantity from data
- Estimate model parameters

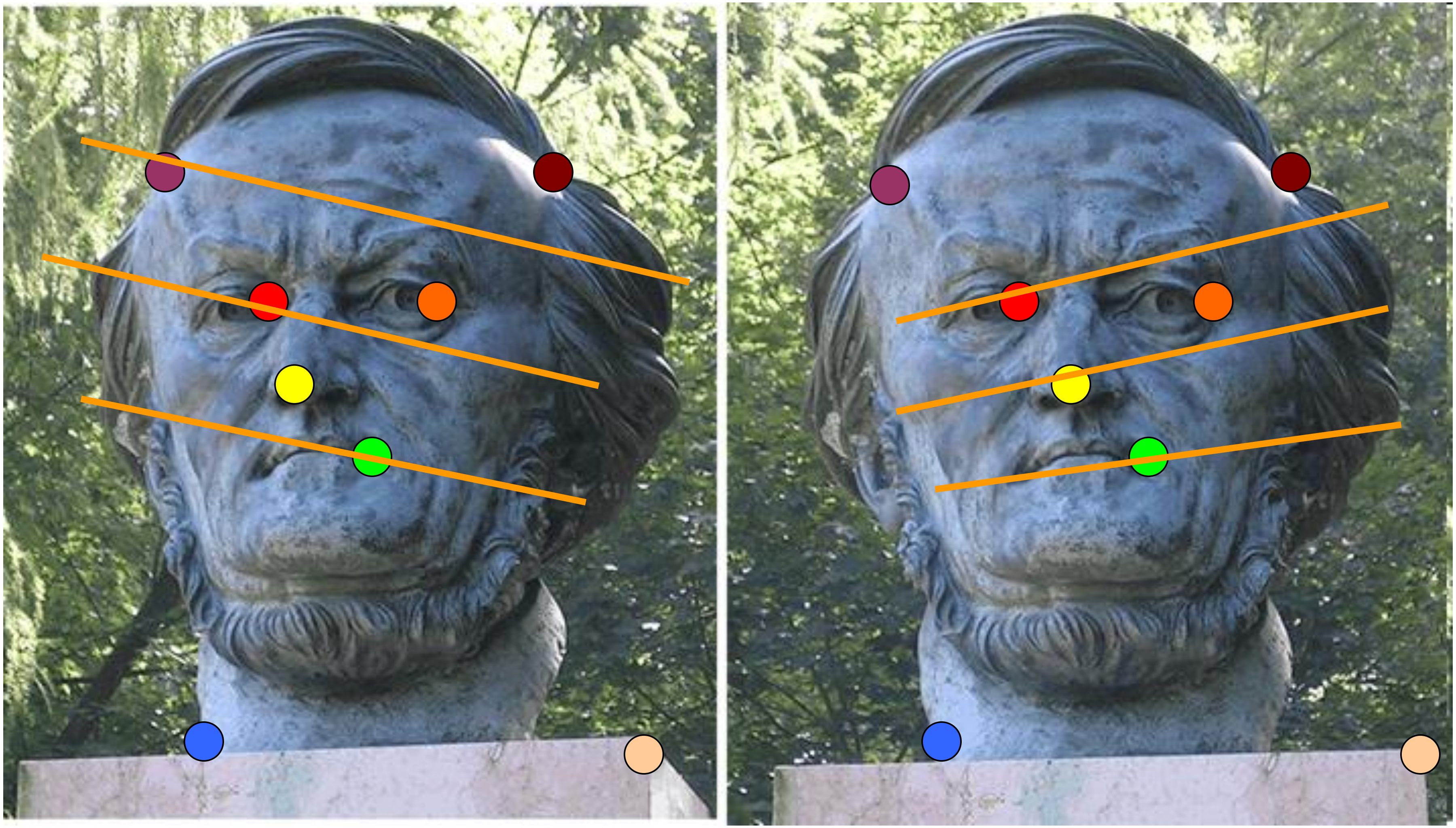
- Lines
- Curves
- Homographic transformation
- Fundamental matrix
- Shape model

Example: fitting lines
(for computing vanishing points)

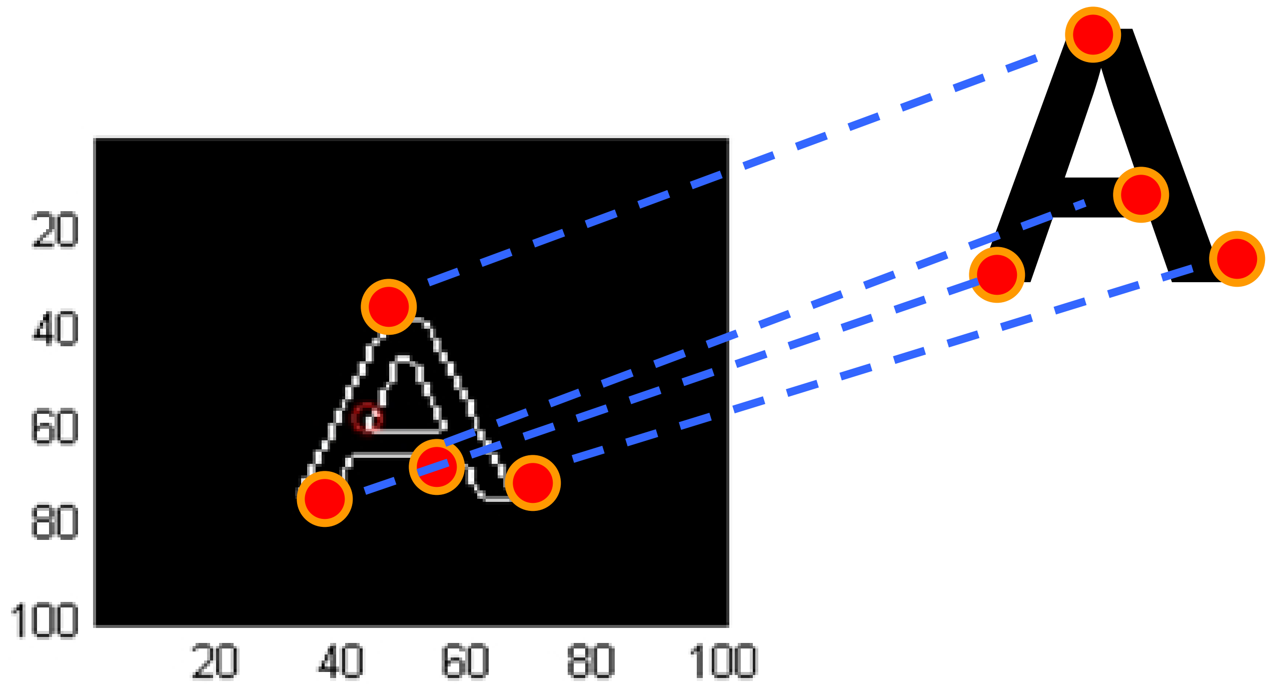
Example: Estimating an homographic transformation



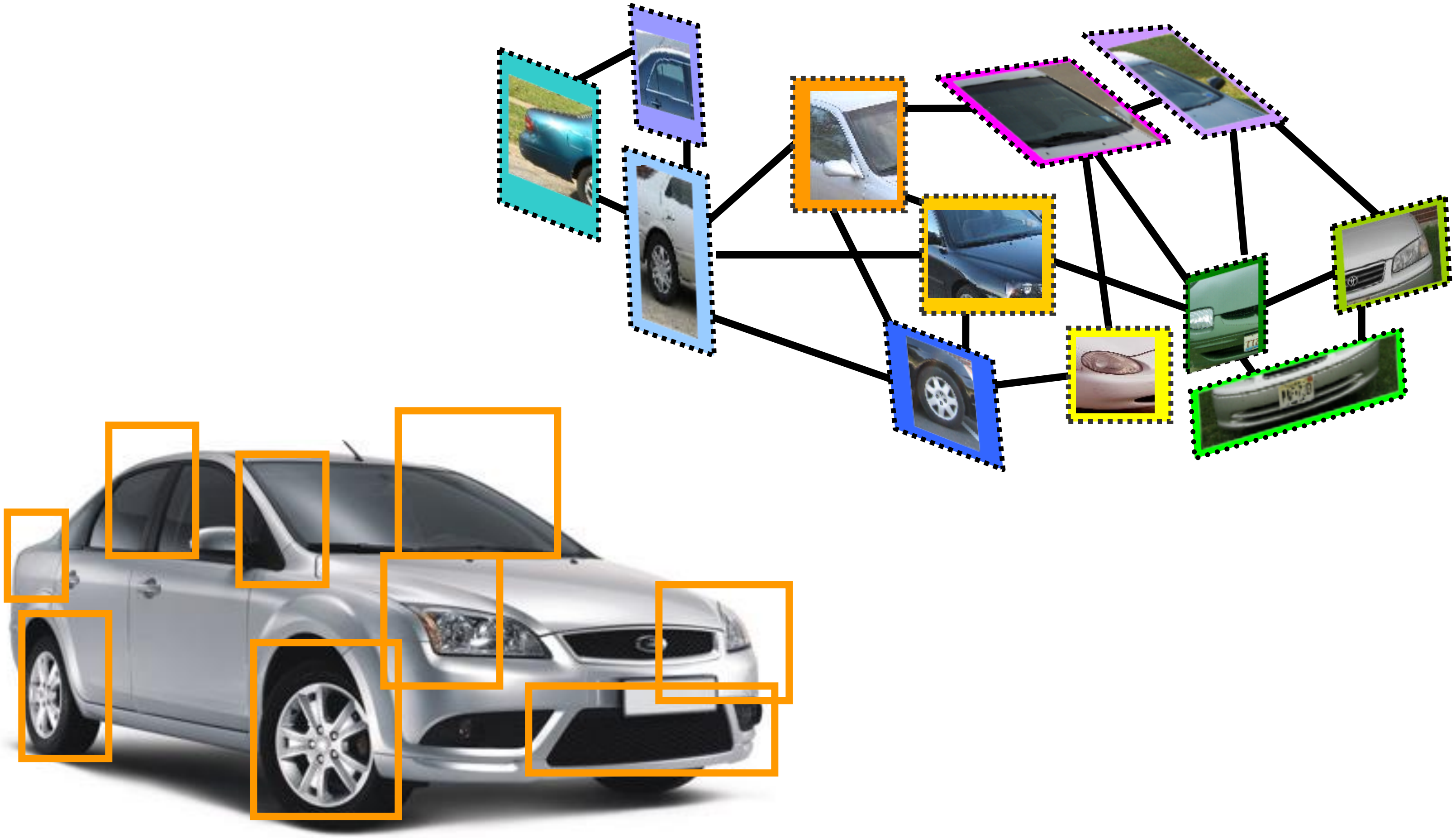
Example: Estimating F



Example: fitting a 2D shape template



Example: fitting a 3D object model



Fitting, matching and recognition are interconnected problems

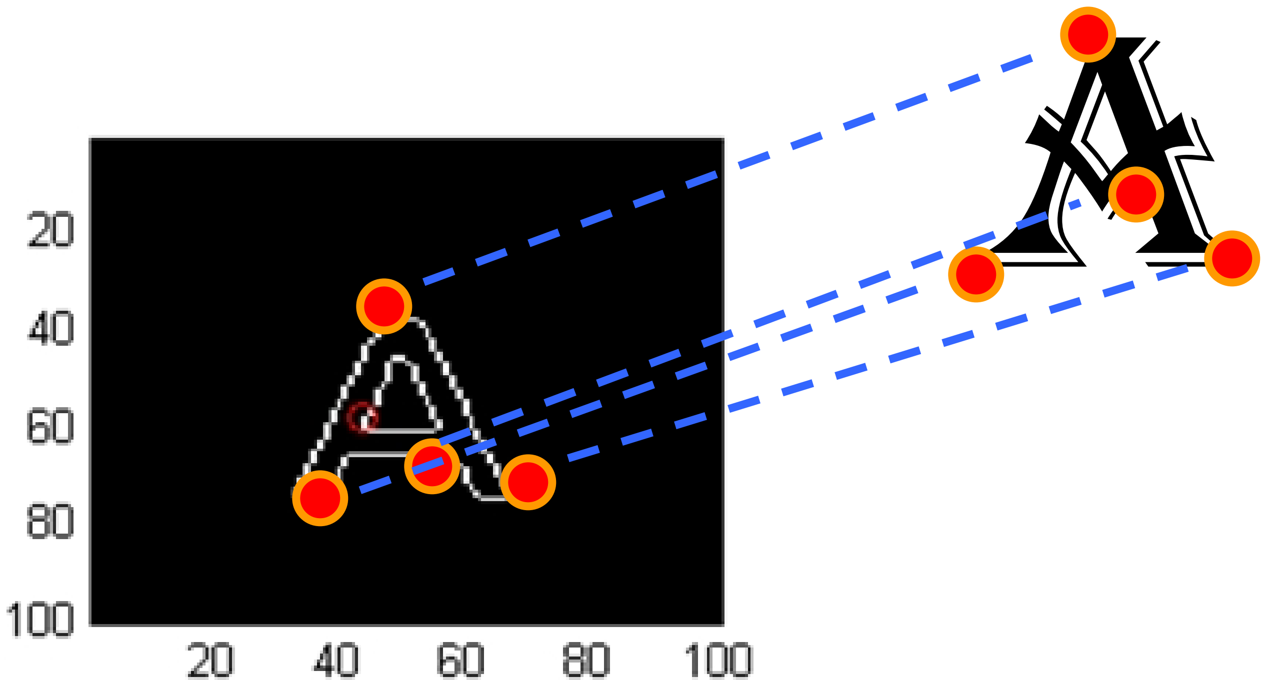
Fitting

Critical issues:

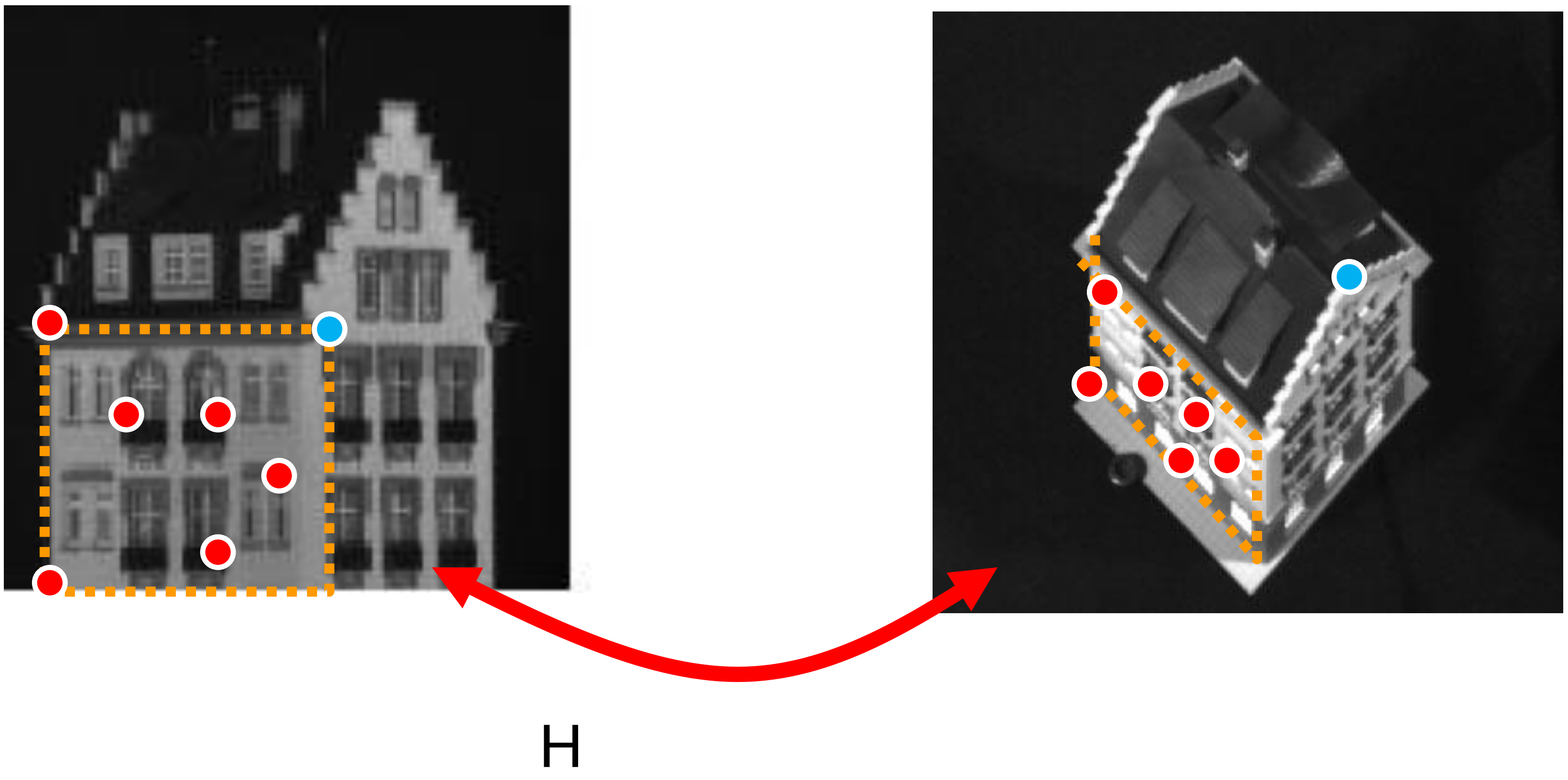
- noisy data
- outliers
- missing data

Critical issues: noisy data

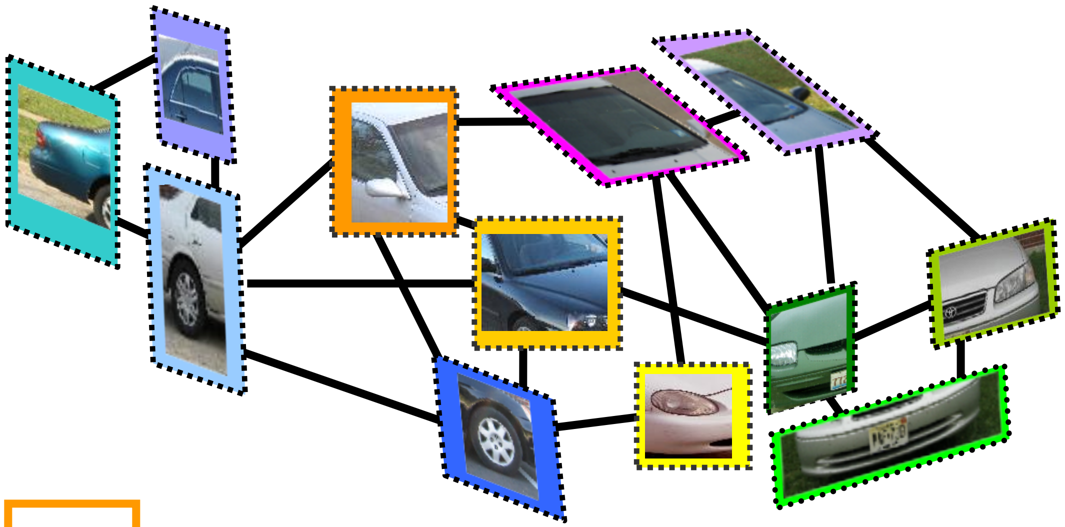
Critical issues: noisy data (intra-class variability)

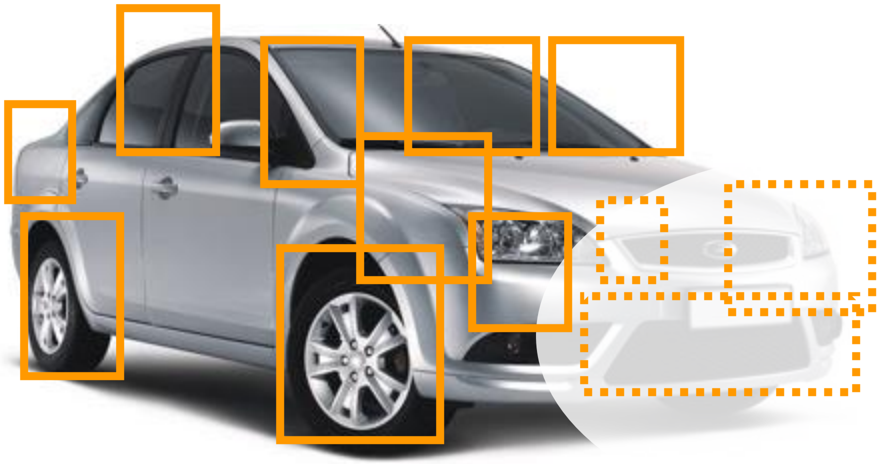


Critical issues: outliers



Critical issues: missing data (occlusions)





Fitting

Goal: Choose a parametric model to fit a certain quantity from data

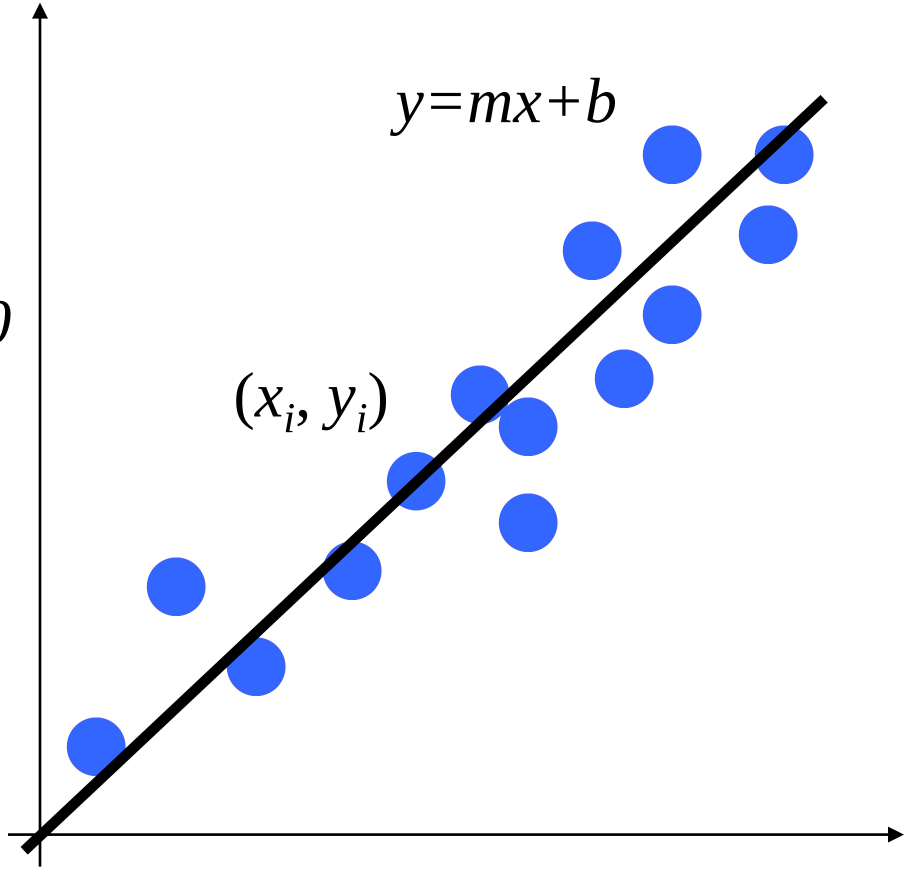
Techniques:

- Least square methods
- RANSAC
- Hough transform
- EM (Expectation Maximization) [not covered]

Least squares methods
- fitting a line -

- Data: $(x_1, y_1), \ldots, (x_n, y_n)$
- Line equation: $y_i - m x_i - b = 0$
- Find (m, b) to minimize

$$\boxed{E = \sum_{i=1}^{n} (y_i - m x_i - b)^2}$$



Least squares methods

- fitting a line -

$$\boxed{E = \sum_{i=1}^{n} (y_i - m x_i - b)^2}$$

$$\mathrm{E} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \left(\mathrm{y_i} - \begin{bmatrix} \mathrm{x_i} & 1 \end{bmatrix} \begin{bmatrix} \mathrm{m} \\ \mathrm{b} \end{bmatrix} \right)^2 = \left\| \begin{bmatrix} \mathrm{y_1} \\ \vdots \\ \mathrm{y_n} \end{bmatrix} - \begin{bmatrix} \mathrm{x_1} & 1 \\ \vdots & \vdots \\ \mathrm{x_n} & 1 \end{bmatrix} \begin{bmatrix} \mathrm{m} \\ \mathrm{b} \end{bmatrix} \right\|^2 = \left\| \mathrm{Y} - \mathrm{XB} \right\|^2$$

$$= (\mathrm{Y} - \mathrm{XB})^{\mathrm{T}} (\mathrm{Y} - \mathrm{XB}) = \mathrm{Y}^{\mathrm{T}}\mathrm{Y} - 2(\mathrm{XB})^{\mathrm{T}}\mathrm{Y} + (\mathrm{XB})^{\mathrm{T}}(\mathrm{XB})$$

Find B=[m, b]$^{\mathrm{T}}$ that minimizes E

$$\frac{dE}{dB} = -2X^TY + 2X^TXB = 0$$

$$\mathrm{X}^{\mathrm{T}}\mathrm{XB} = \mathrm{X}^{\mathrm{T}}\mathrm{Y}$$

Normal equation

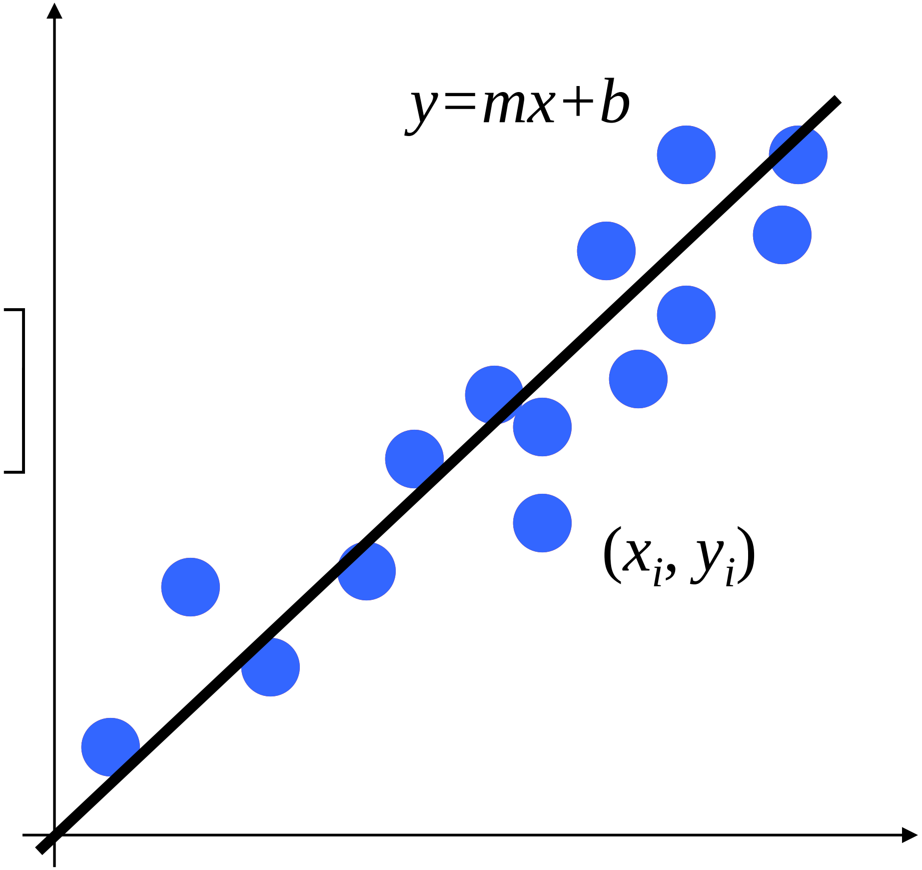
$$\boxed{\mathrm{B} = \left(\mathrm{X}^{\mathrm{T}}\mathrm{X}\right)^{-1}\mathrm{X}^{\mathrm{T}}\mathrm{Y}}$$

Least squares methods

- fitting a line -

$$E = \sum_{i=1}^{n} (y_i - mx_i - b)^2$$

$$\boxed{B = \left(X^T X\right)^{-1} X^T Y} \qquad B = \begin{bmatrix} m \\ b \end{bmatrix}$$



Limitations

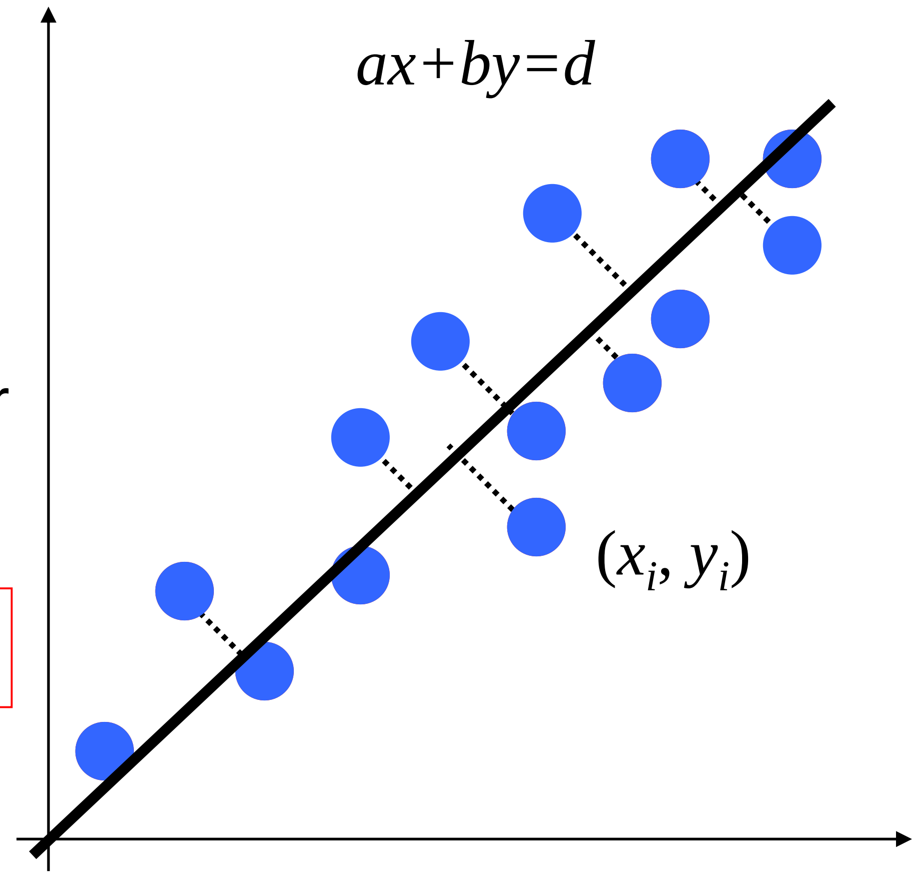
- Fails completely for vertical lines

Least squares methods

- fitting a line -

- Distance between point (x_n, y_n) and line $ax+by=d$

- Find (a, b, d) to minimize the sum of squared perpendicular distances



$$E = \sum_{i=1}^{n} (a x_i + b y_i - d)^2$$

$$\boxed{U}\boxed{N} = 0$$

data model parameters

Least squares methods

- fitting a line -

$$\mathrm{A\ h} = 0$$

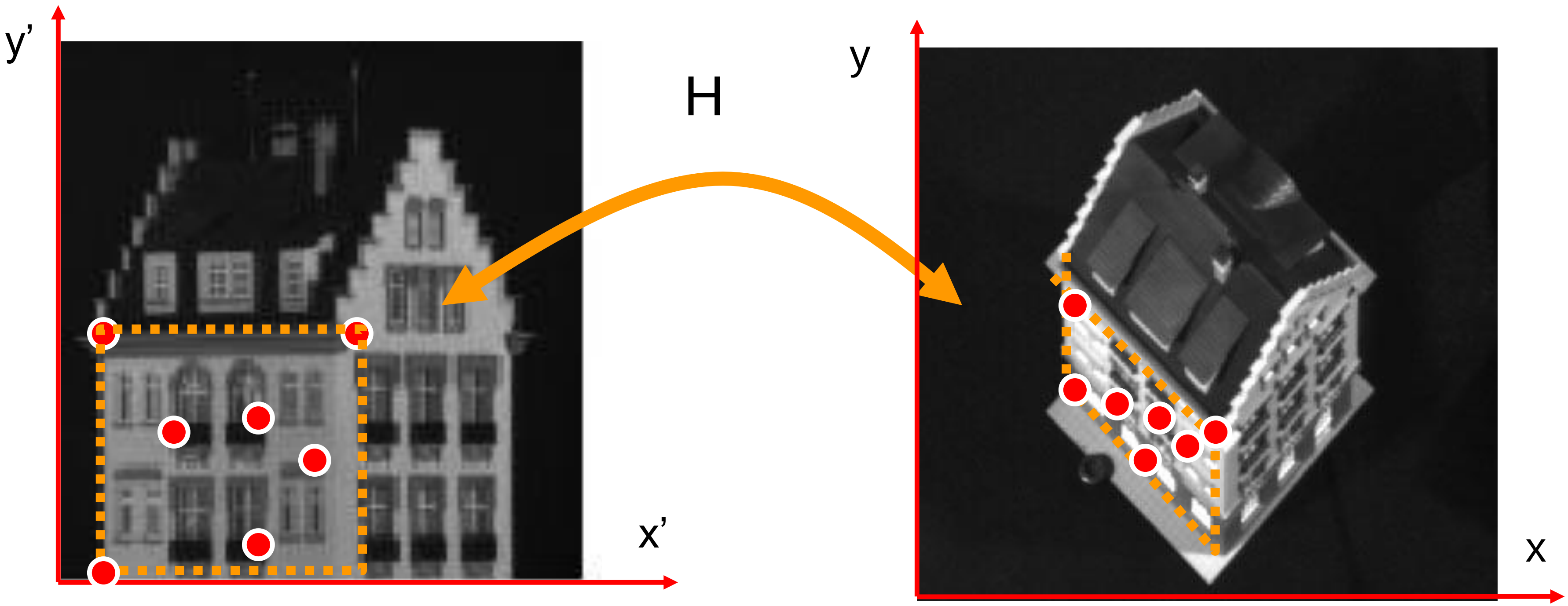
$$\text{Minimize} \| \mathrm{A\ h} \| \quad \text{subject to} \quad \| \mathrm{h} \| = 1$$

$$A = UDV^T$$

$$\mathrm{h} = \text{last column of } \mathrm{V}$$

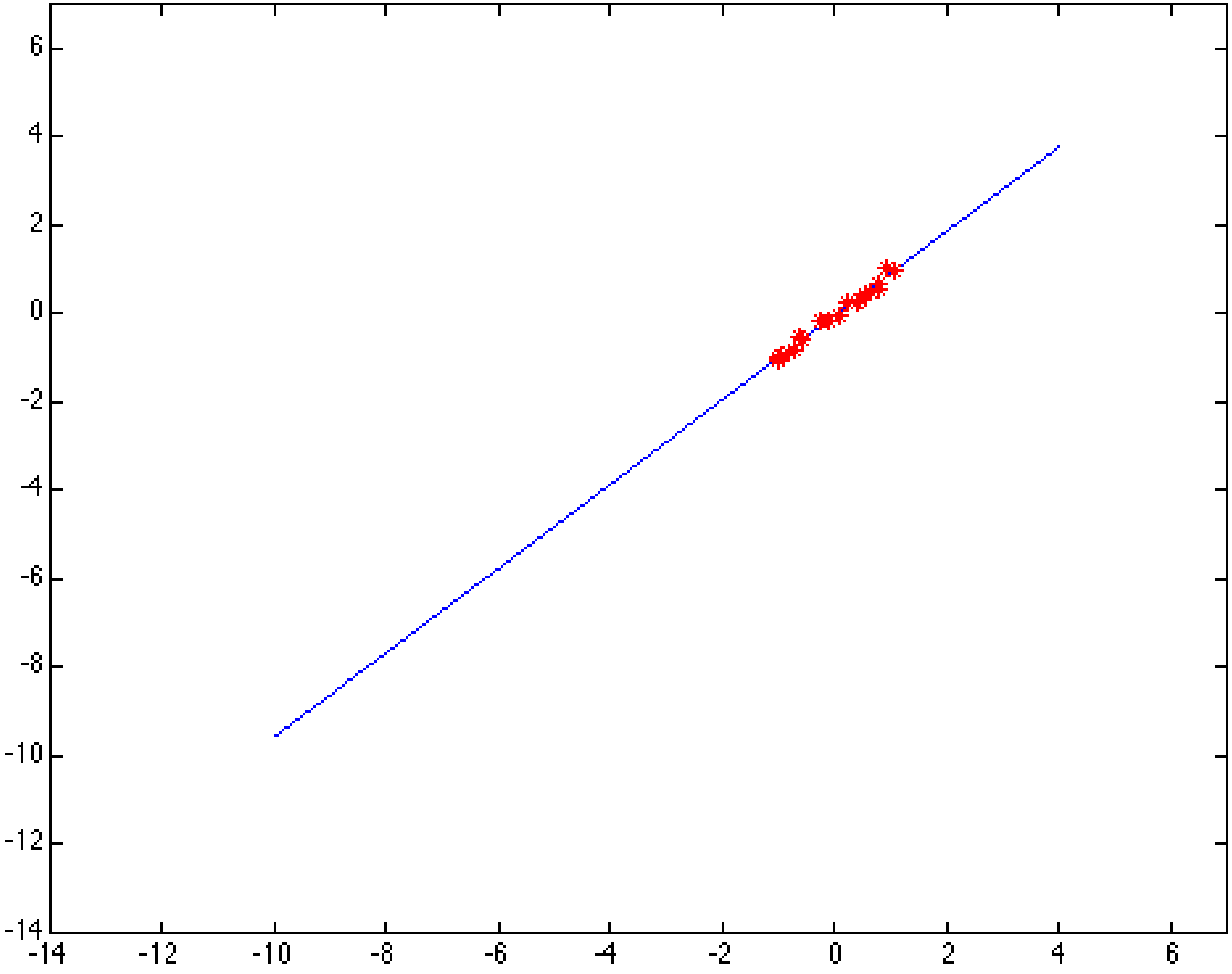
Least squares methods

- fitting an homography -

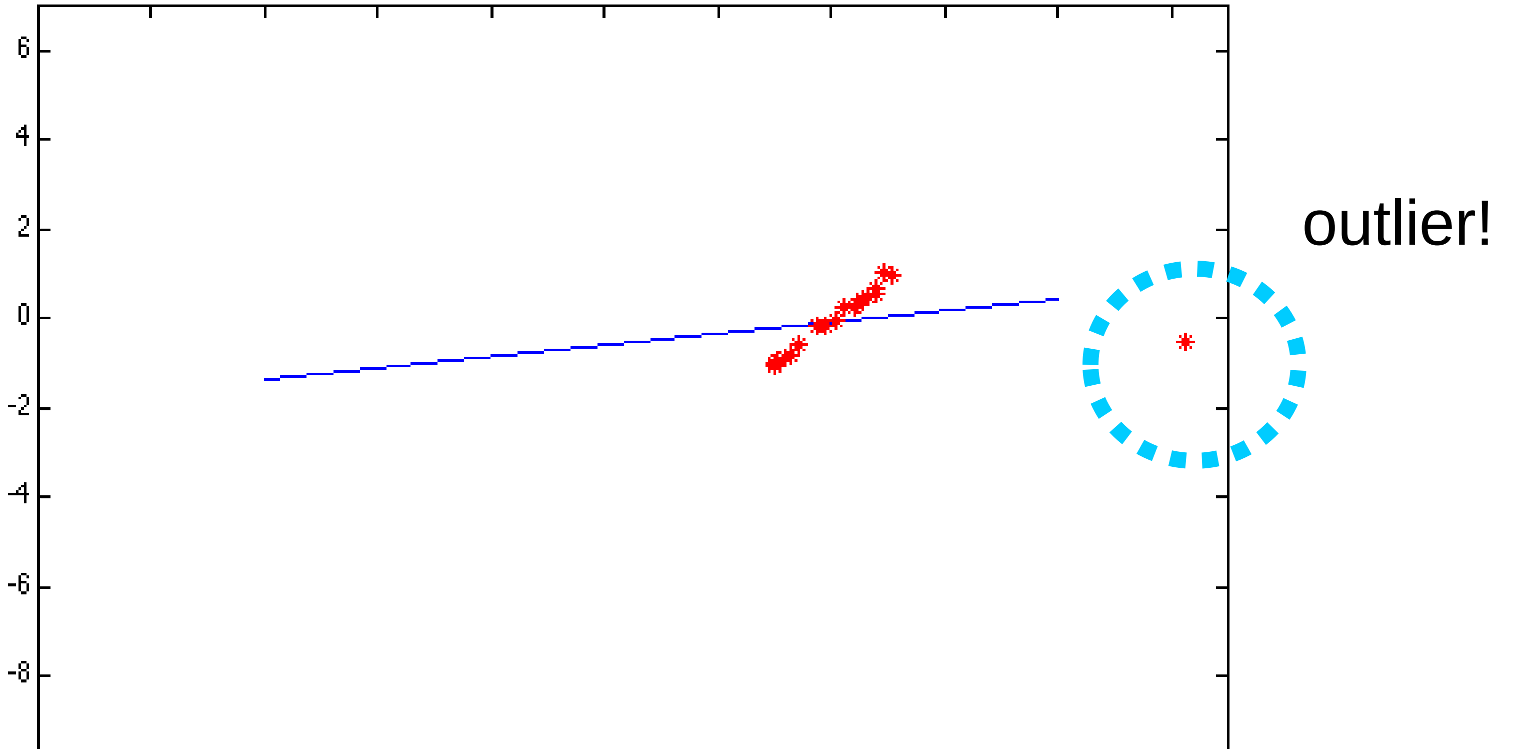


$$\underbrace{U}_{\text{data}} \underbrace{N}_{\text{model parameters}} = 0$$

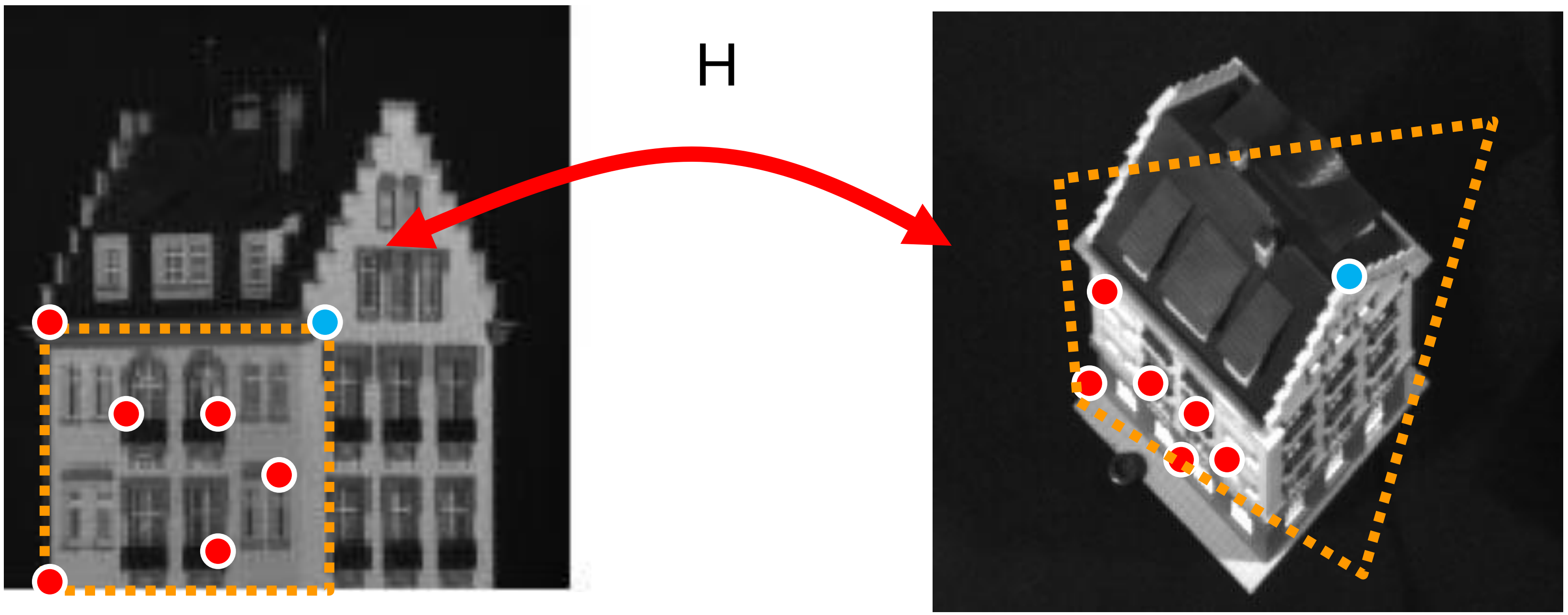
Least squares: Robustness to noise



Least squares: Robustness to noise



Critical issues: outliers



CONCLUSION: Least square is not robust w.r.t. outliers

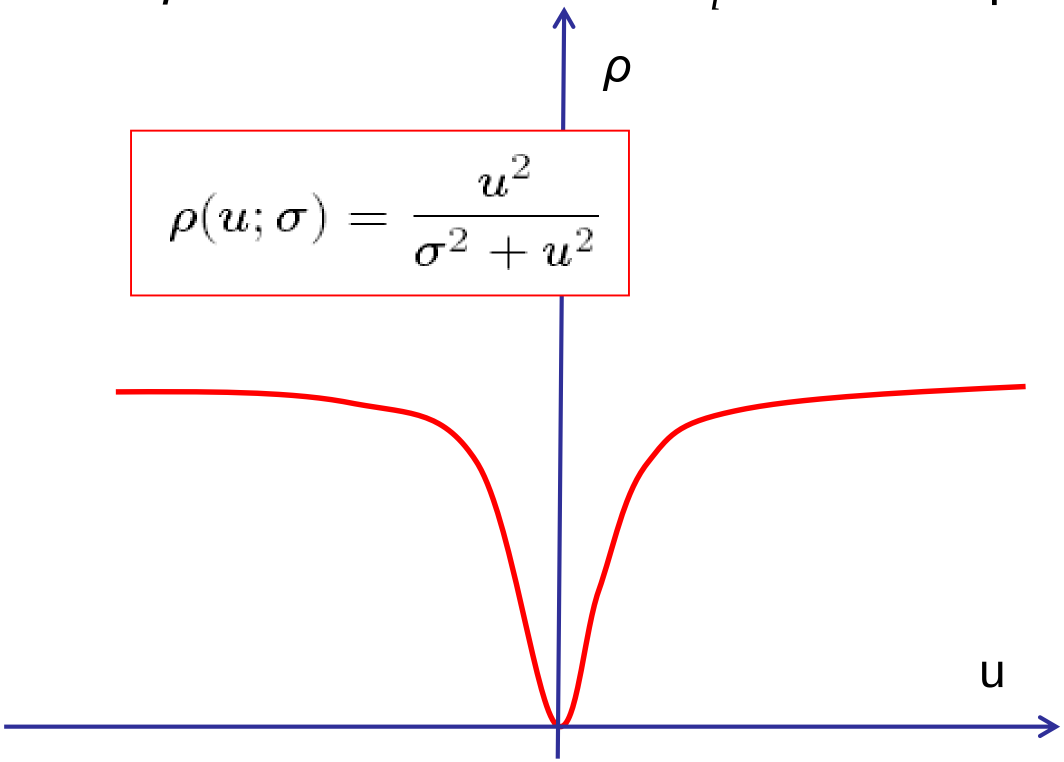
Least squares: Robust estimators

Instead of minimizing $E = \sum_{i=1}^{n} (ax_i + by_i - d)^2$

We minimize $\boxed{E = \sum_i \rho(u_i ; \sigma)}$ $\quad u_i = ax_i + by_i - d$

- u_i = error (residual) of ith point w.r.t. model parameters β = (a,b,d)
- ρ = robust function of u_i with scale parameter σ

$$\boxed{\rho(u;\sigma) = \frac{u^2}{\sigma^2 + u^2}}$$



The robust function ρ

- Favors a configuration with small residuals
- Penalizes large residuals

Least squares: Robust estimators

Instead of minimizing $E = \sum_{i=1}^{n} (a x_i + b y_i - d)^2$

We minimize $\boxed{E = \sum_i \rho(u_i ; \sigma)}$ $\quad u_i = a x_i + b y_i - d$

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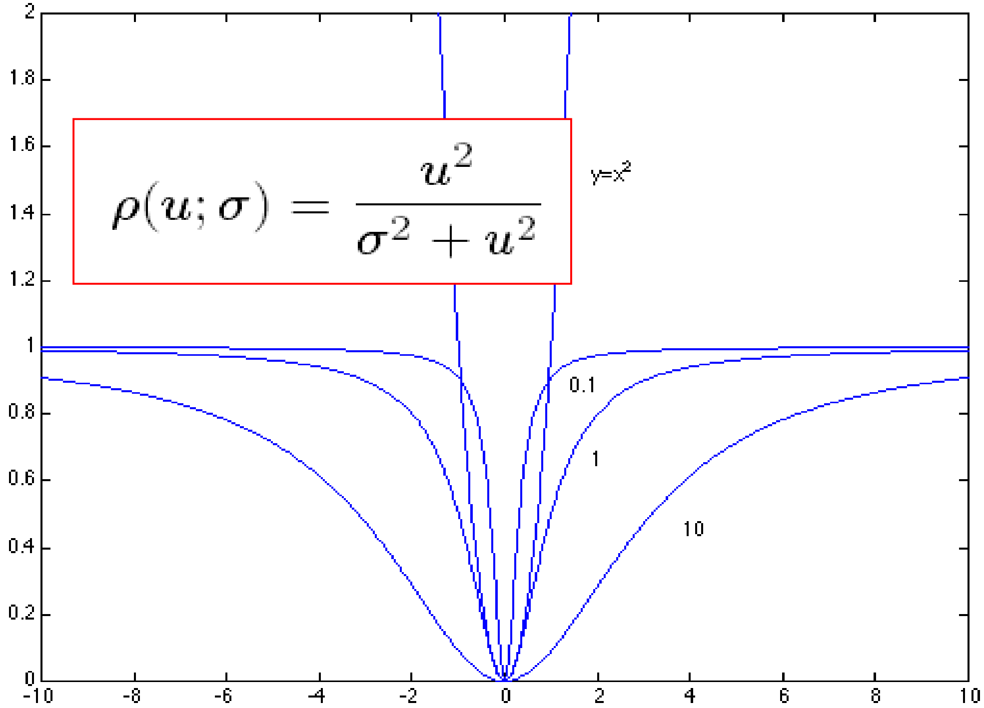


$$\rho(u;\sigma) = \frac{u^2}{\sigma^2 + u^2}$$

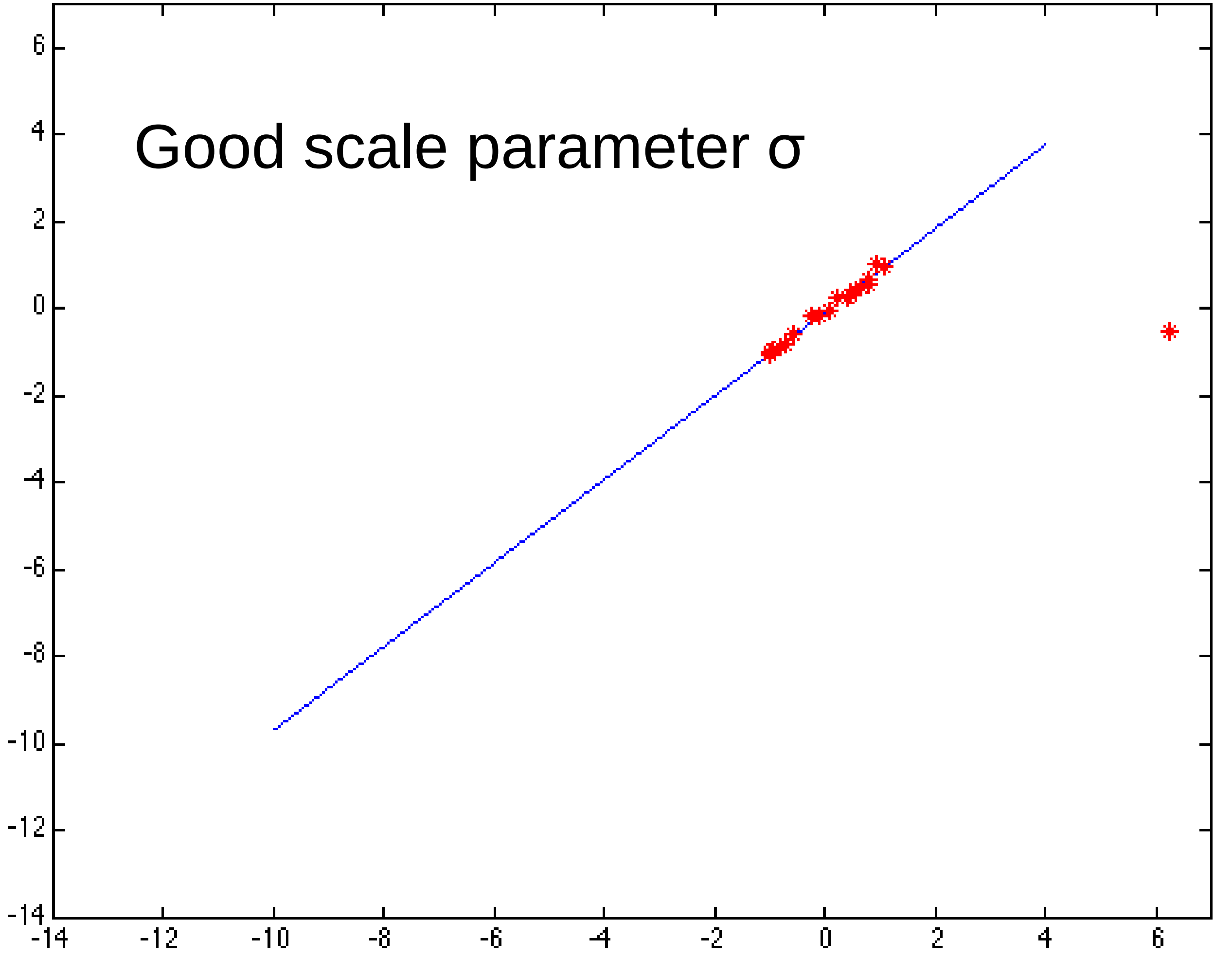
The robust function ρ

- Favors a configuration with small residuals
- Penalizes large residuals

•Small sigma → highly penalize large residuals

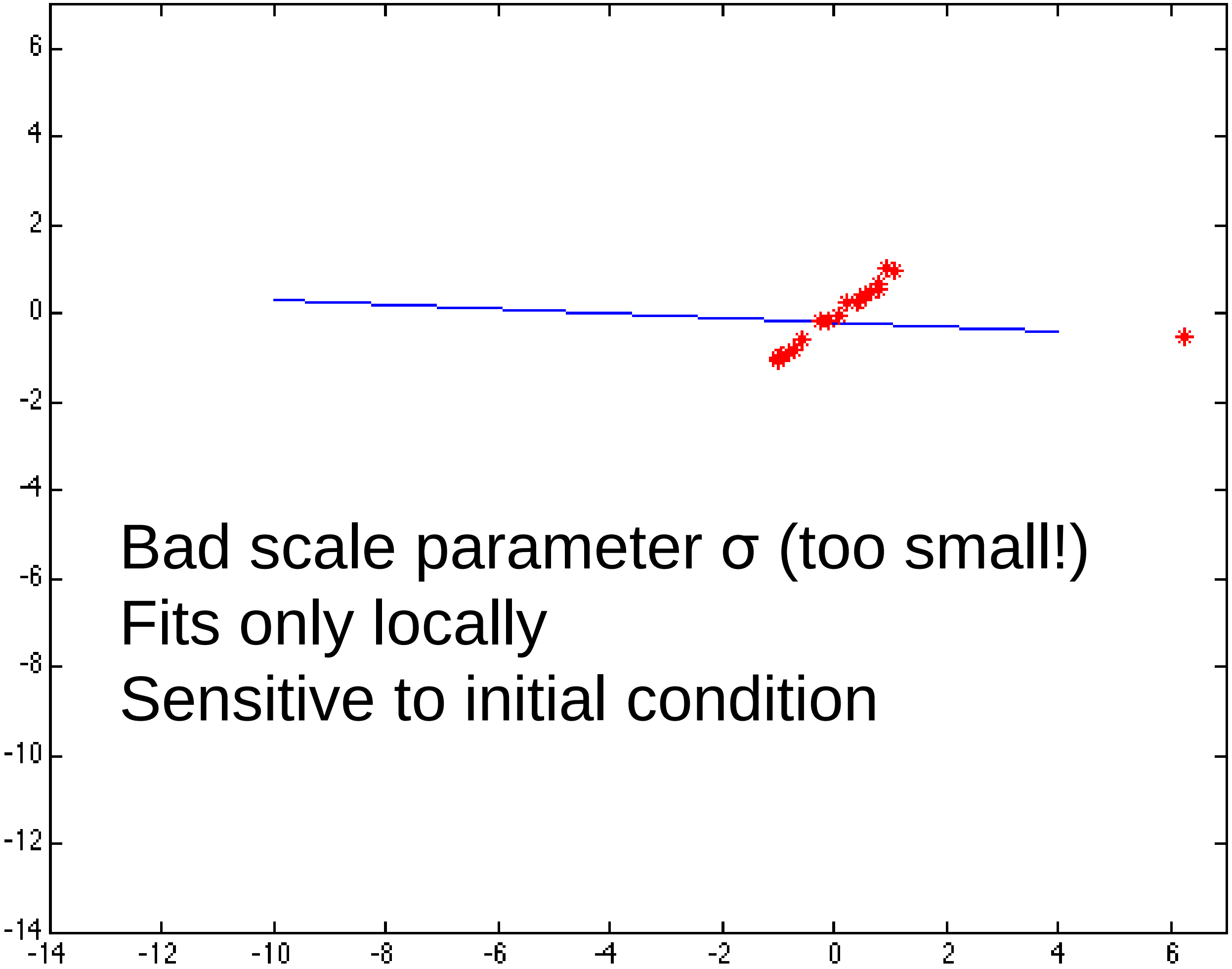
•Large sigma → mildly penalize large residual (like LSQR)

Least squares: Robust estimators



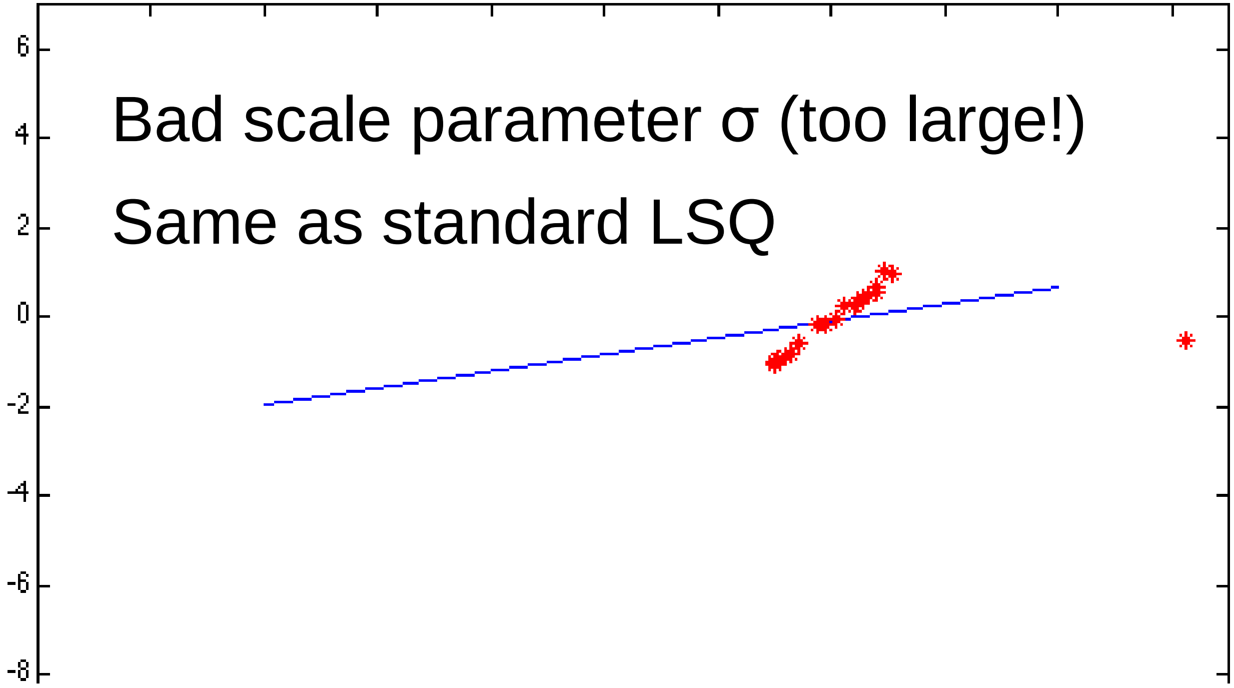
The effect of the outlier is eliminated

Least squares: Robust estimators



Bad scale parameter σ (too small!)
Fits only locally
Sensitive to initial condition

Least squares: Robust estimators



•CONCLUSION: Robust estimator useful if prior info about the distribution of points is known

•Robust fitting is a nonlinear optimization problem (iterative solution)

•Least squares solution provides good initial condition

Fitting

Goal: Choose a parametric model to fit a certain quantity from data

Techniques:

- Least square methods
- RANSAC
- Hough transform

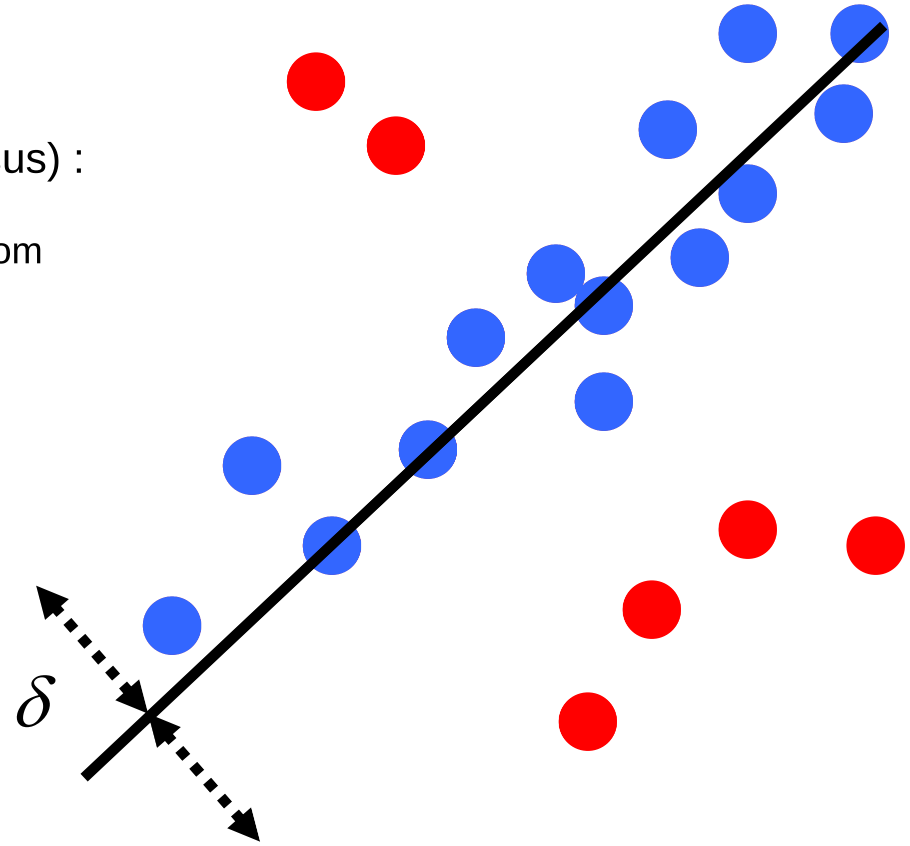
Basic philosophy
(voting scheme)

- Data elements are used to vote for one (or multiple) models
- Robust to outliers and missing data
- Assumption1: Noise features will not vote consistently for any single model ("few" outliers)
- Assumption2: there are enough features to agree on a good model ("few" missing data)

RANSAC

(RANdom SAmple Consensus) :
Learning technique to estimate parameters of a model by random sampling of observed data

Fischler & Bolles in ‘81.



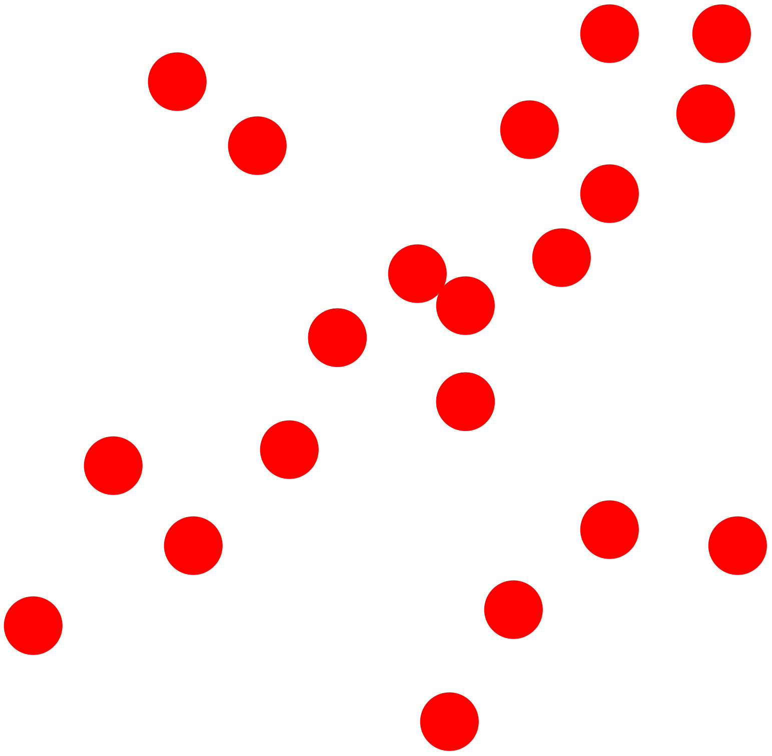
$$\pi : \boldsymbol{I} \rightarrow \{\boldsymbol{P}, \boldsymbol{O}\} \qquad \min_{\pi} |\boldsymbol{O}|$$

Model parameters

such that:

$$f(P, \beta) < \delta, \qquad \forall P \in \mathbf{P} \qquad f(P, \beta) = residual$$

RANSAC



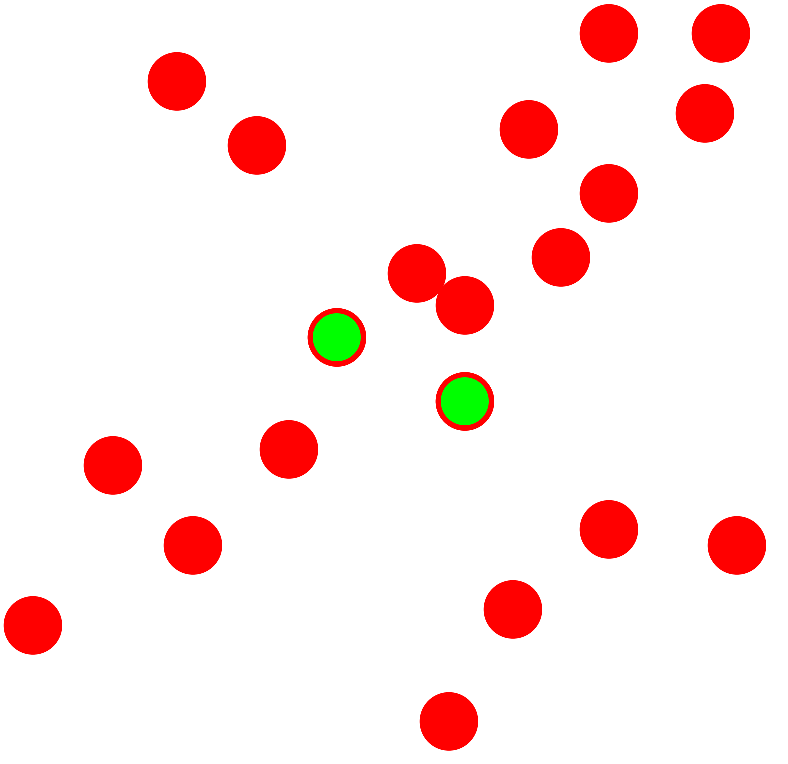
Sample set = set of points in 2D

Algorithm:

1. Select random sample of minimum required size to fit model
2. Compute a putative model from sample set
3. Compute the set of inliers to this model from whole data set

Repeat 1-3 until model with the most inliers over all samples is found

RANSAC



Sample set = set of points in 2D

Algorithm:

1. Select random sample of minimum required size to fit model [?]
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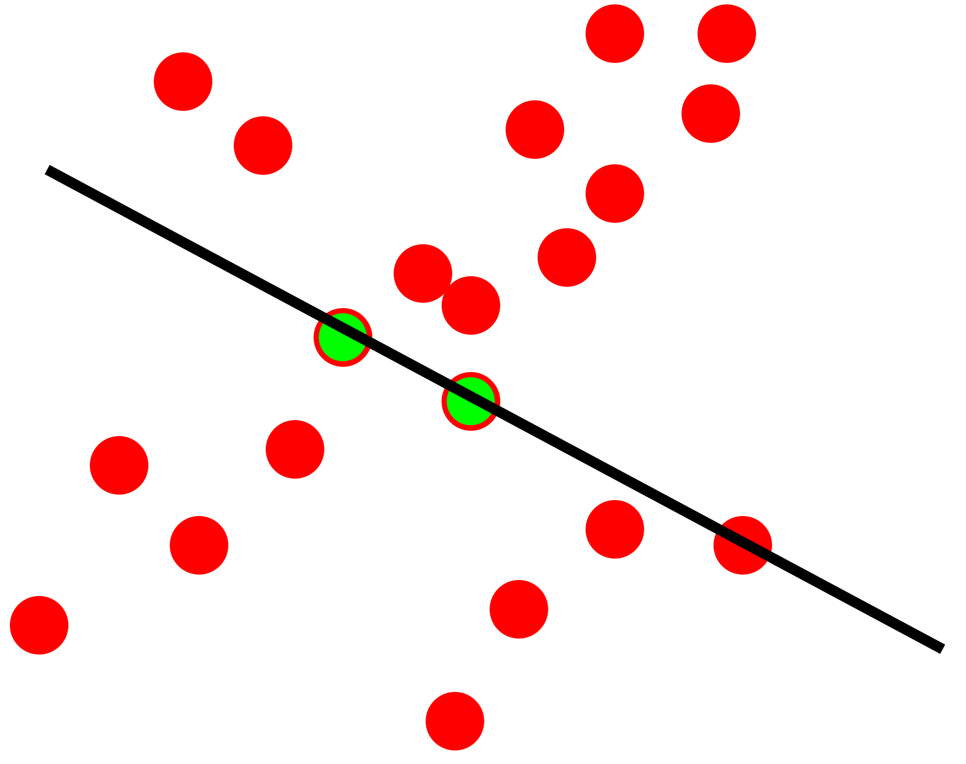
RANSAC



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Algorithm:

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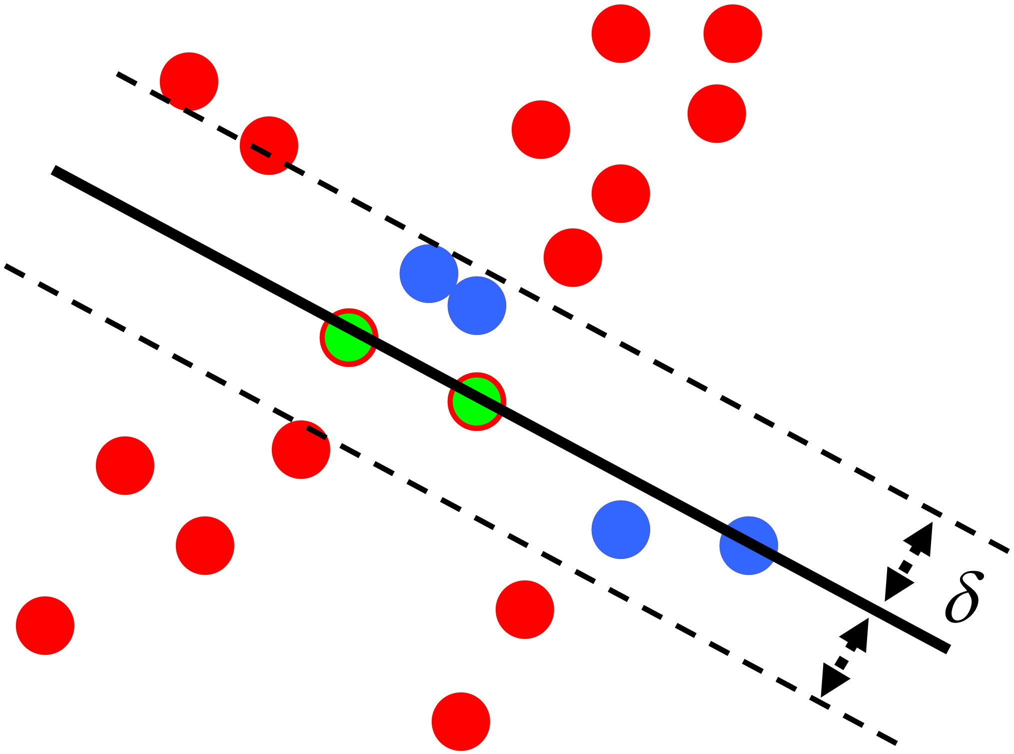
RANSAC



Sample set = set of points in 2D

$$|\boldsymbol{O}| = 14$$

Algorithm:

1. Select random sample of minimum required size to fit model [?]
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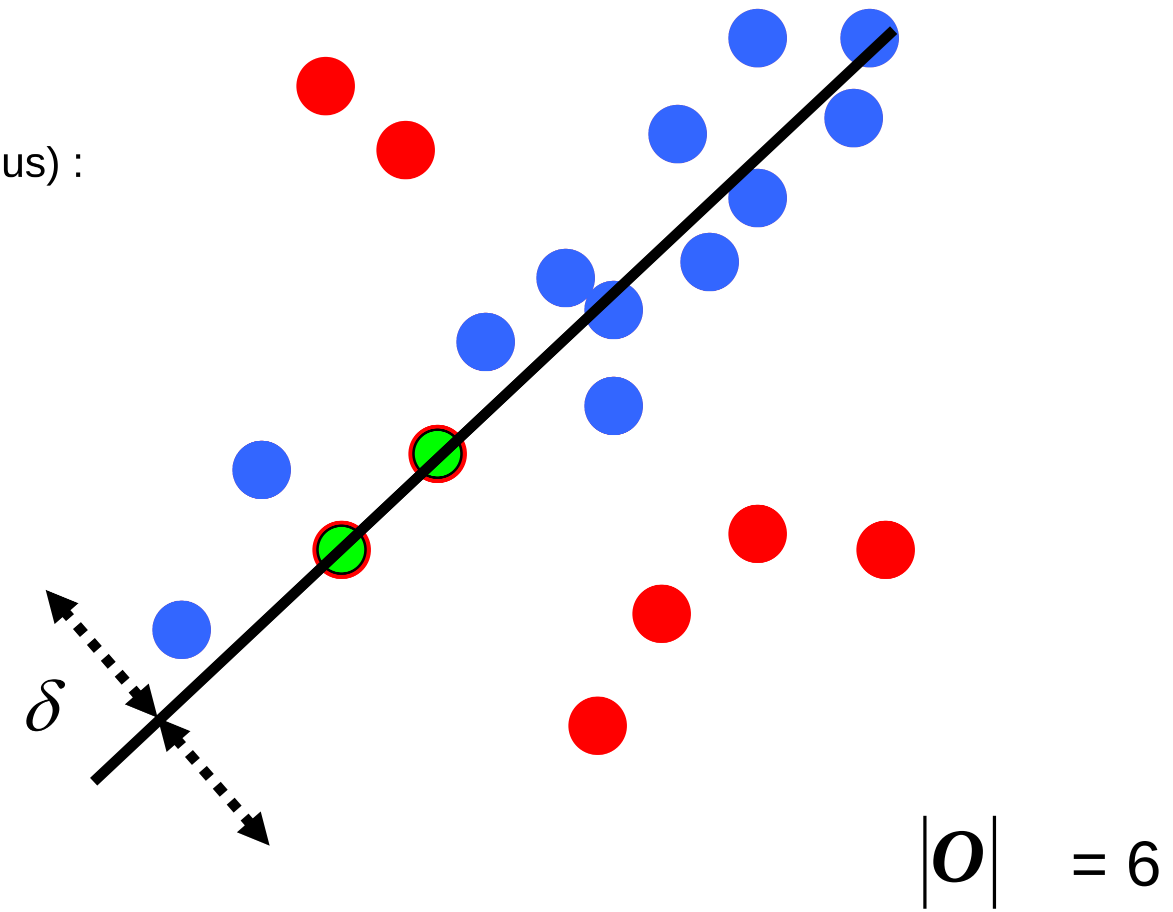
RANSAC

(RANdom SAmple Consensus) :

Fischler & Bolles in ‘81.



Algorithm:

1. Select random sample of minimum required size to fit model [?]
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3. Compute the set of inliers to this model from whole data set

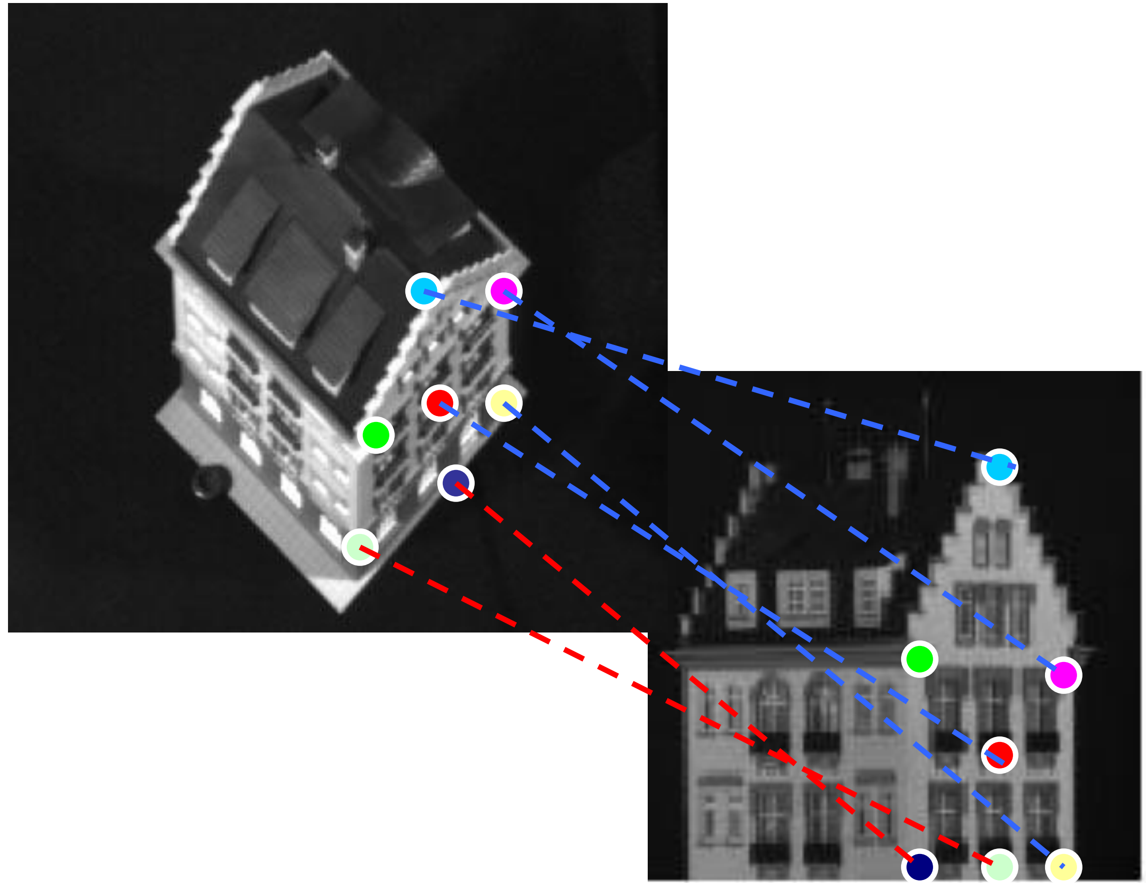
Repeat 1-3 until model with the most inliers over all samples is found

How many samples?

- Number of samples *N*
  - p = probability at least one random sample is free from outliers (e.g. p=0.99)
  - e = outlier ratio
  - s = minimum number needed to fit the model

	proportion of outliers e						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

Estimating H by RANSAC



- H → 8 DOF
- Need 4 correspondences

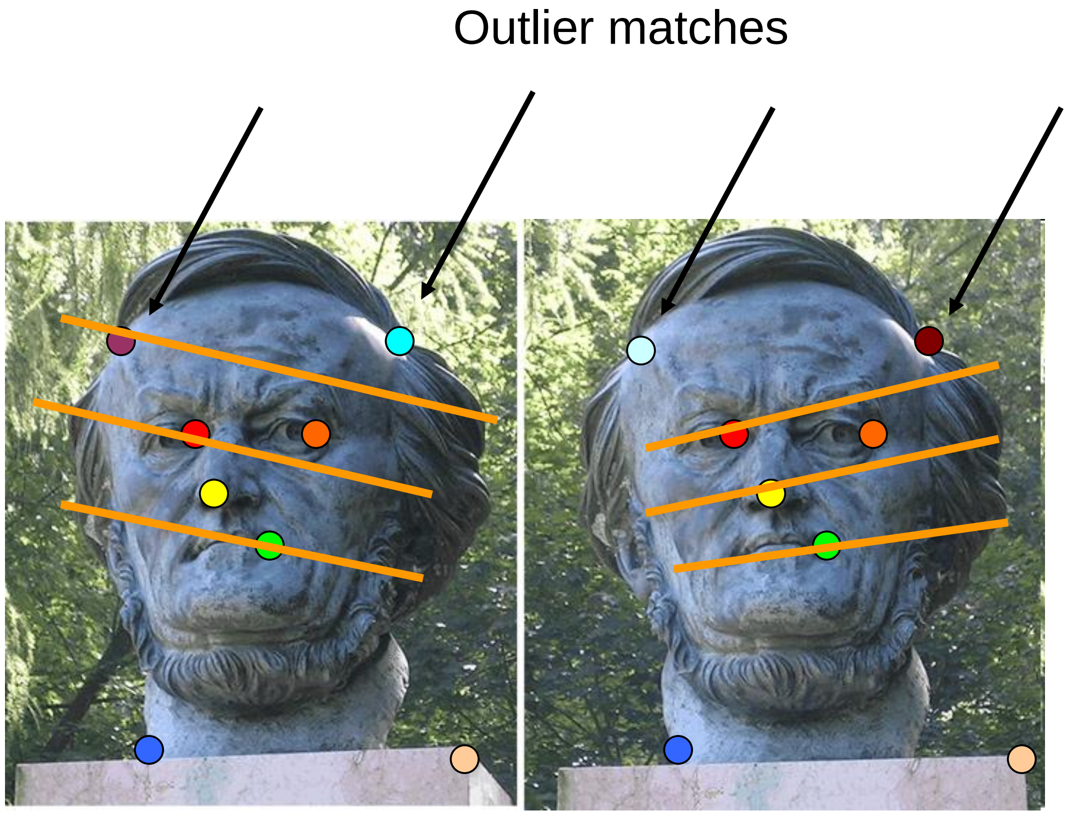
Sample set = set of matches between 2 images

Algorithm:

1. Select a random sample of minimum required size [?]
2. Compute a putative model from these
3. Compute the set of inliers to this model from whole sample space

Repeat 1-3 until model with the most inliers over all samples is found

Estimating F by RANSAC



- F $\rightarrow$ 7 DOF
- Need 7 (8) correspondences

Sample set = set of matches between 2 images

Algorithm:

1. Select a random sample of minimum required size [?]
2. Compute a putative model from these
3. Compute the set of inliers to this model from whole sample space

Repeat 1-3 until model with the most inliers over all samples is found

RANSAC - conclusions

Good:

- Simple and easily implementable
- Successful in different contexts

Bad:

- Many parameters to tune
- Trade-off accuracy-vs-time
- Cannot be used if ratio inliers/outliers is too small

Fitting

Goal: Choose a parametric model to fit a certain quantity from data

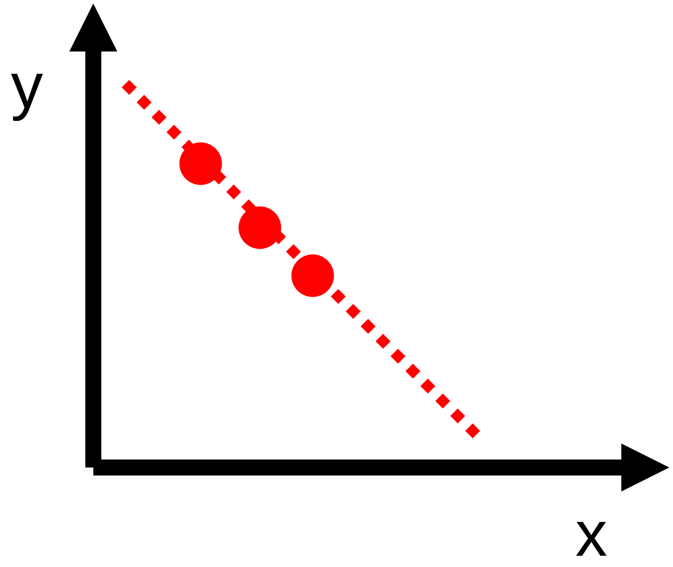
Techniques:

- Least square methods
- RANSAC
- Hough transform

Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Given a set of points, find the curve or line that explains the data points best



Hough transform

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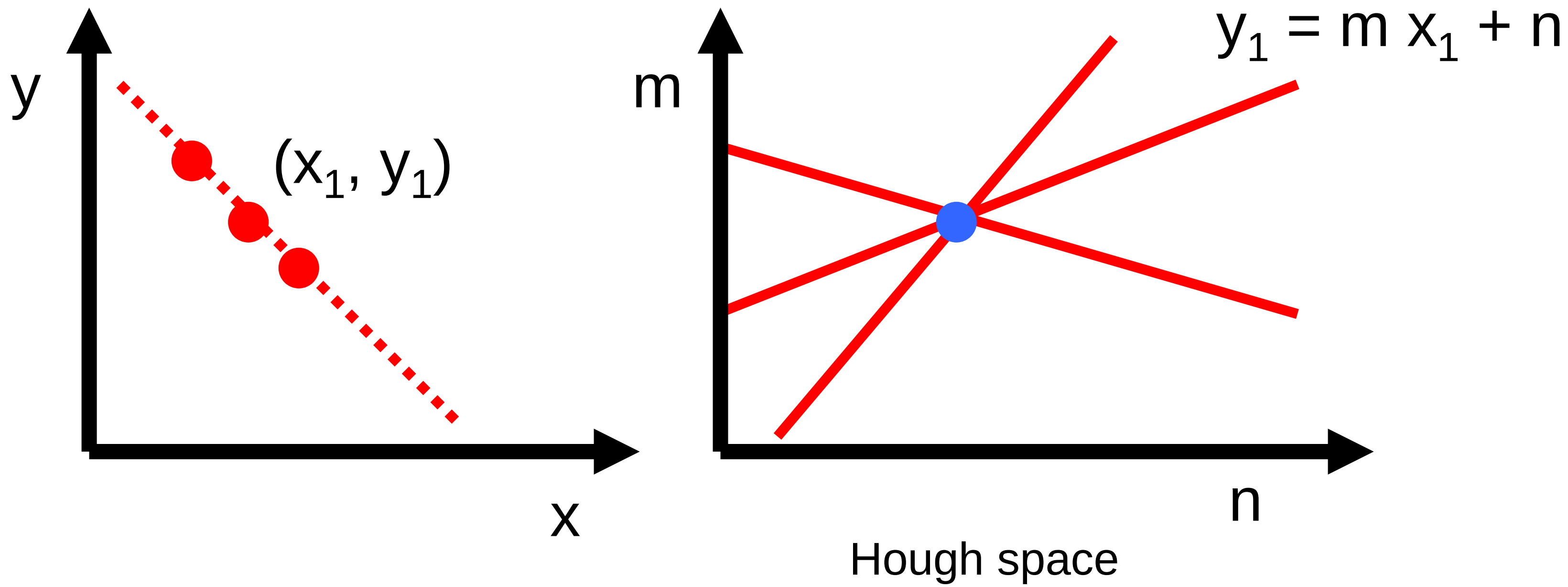


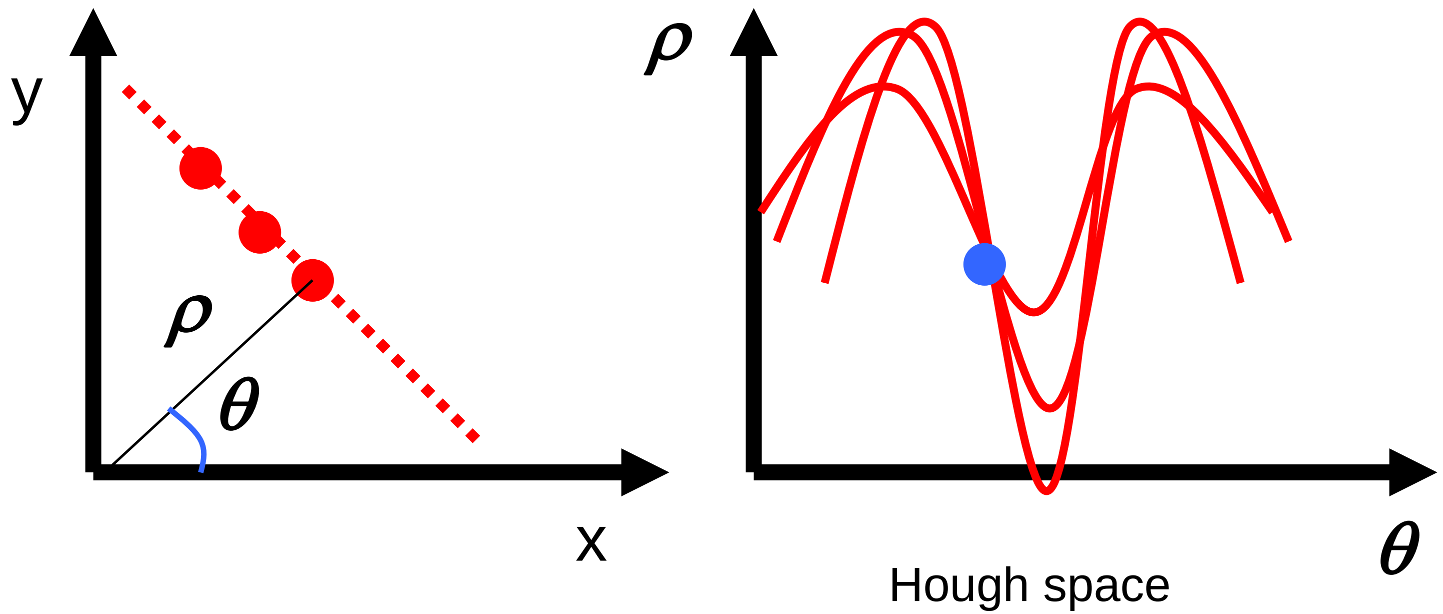
$$y = m\ x + n$$

Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

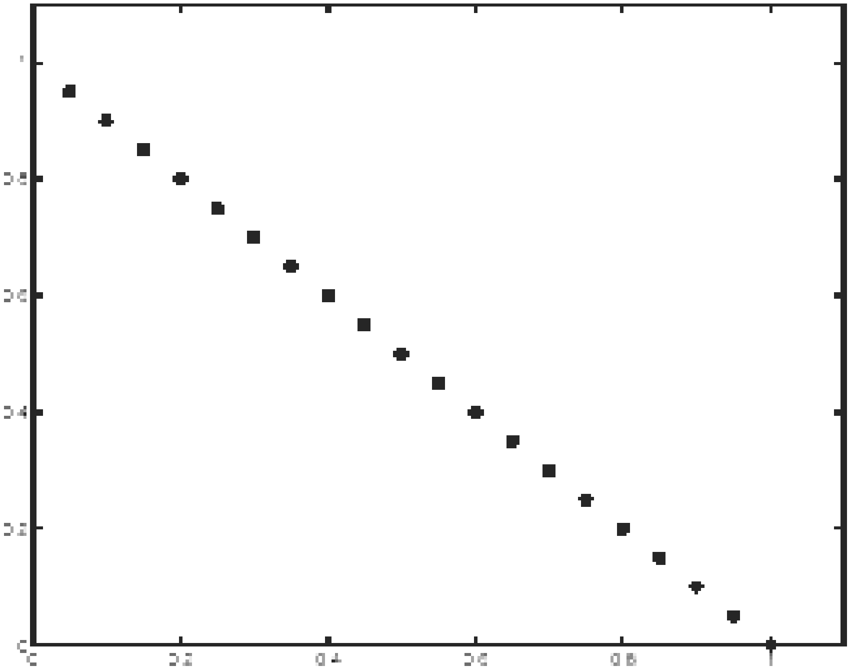
Issue : parameter space [m,n] is unbounded…

- Use a polar representation for the parameter space

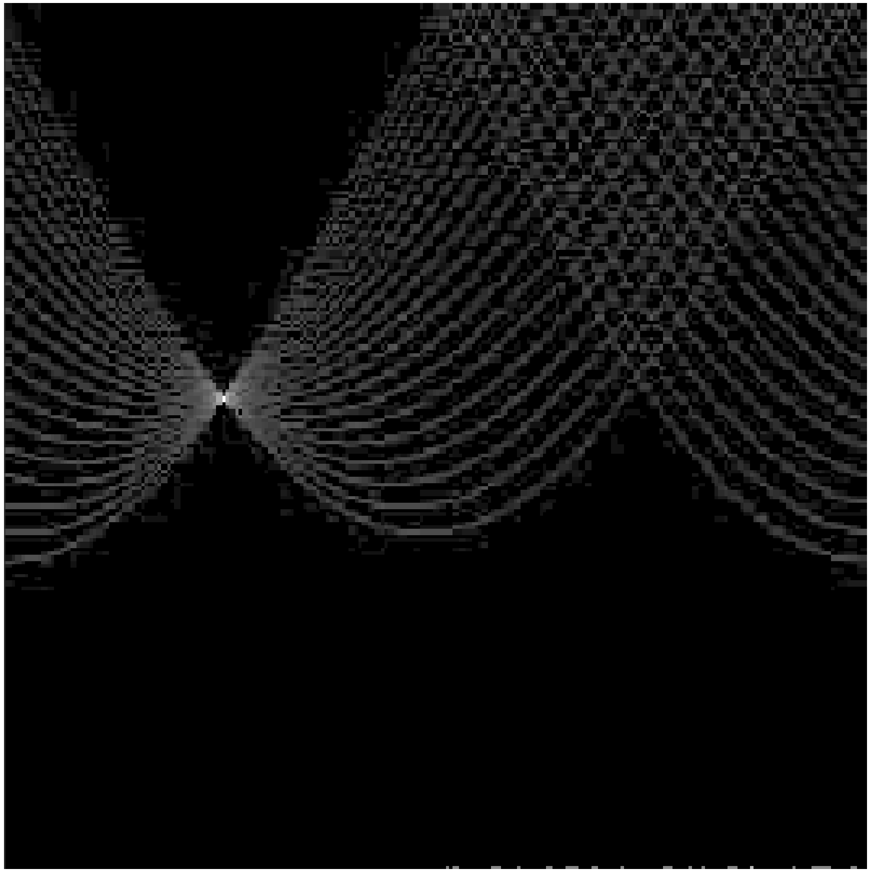


$$x\cos\theta + y\sin\theta = \rho$$

Hough transform - experiments



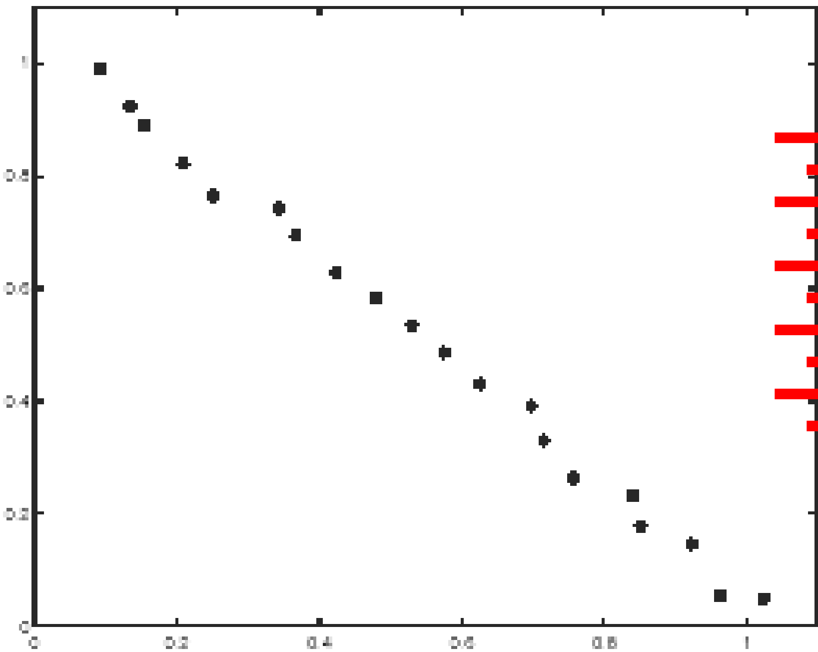
features



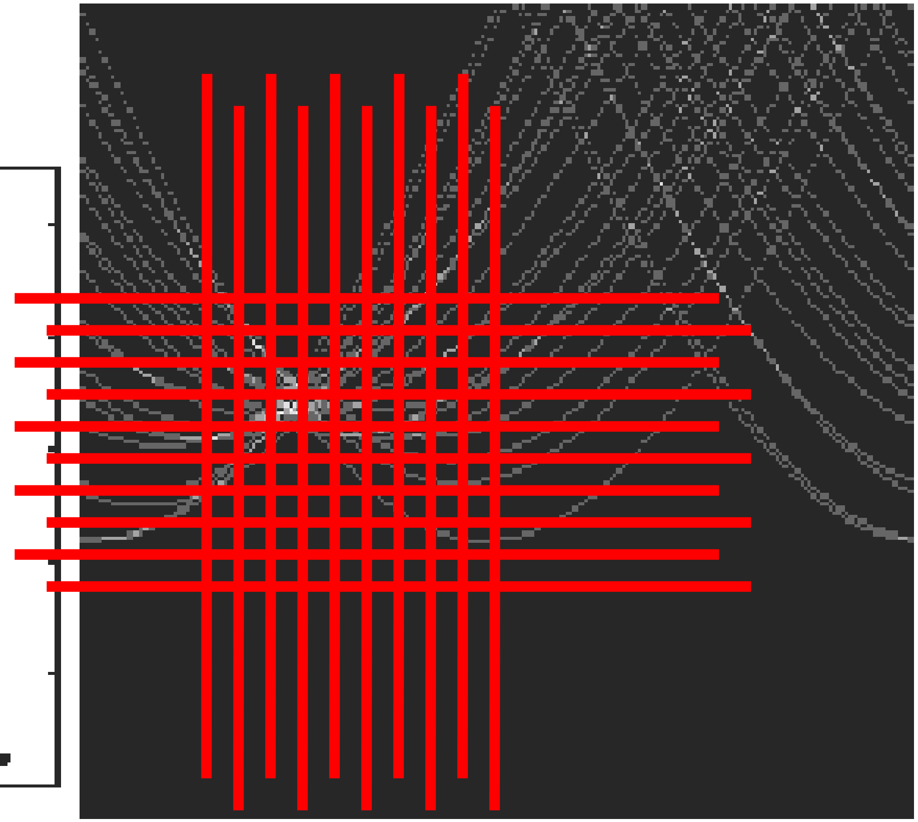
votes

Hough transform - experiments

Noisy data



features



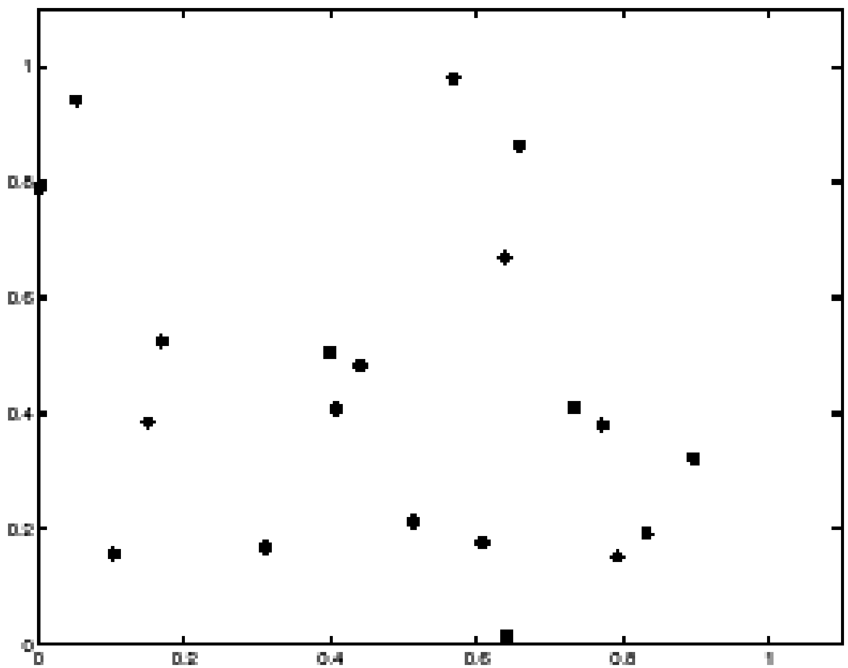
votes

How to compute the intersection point?

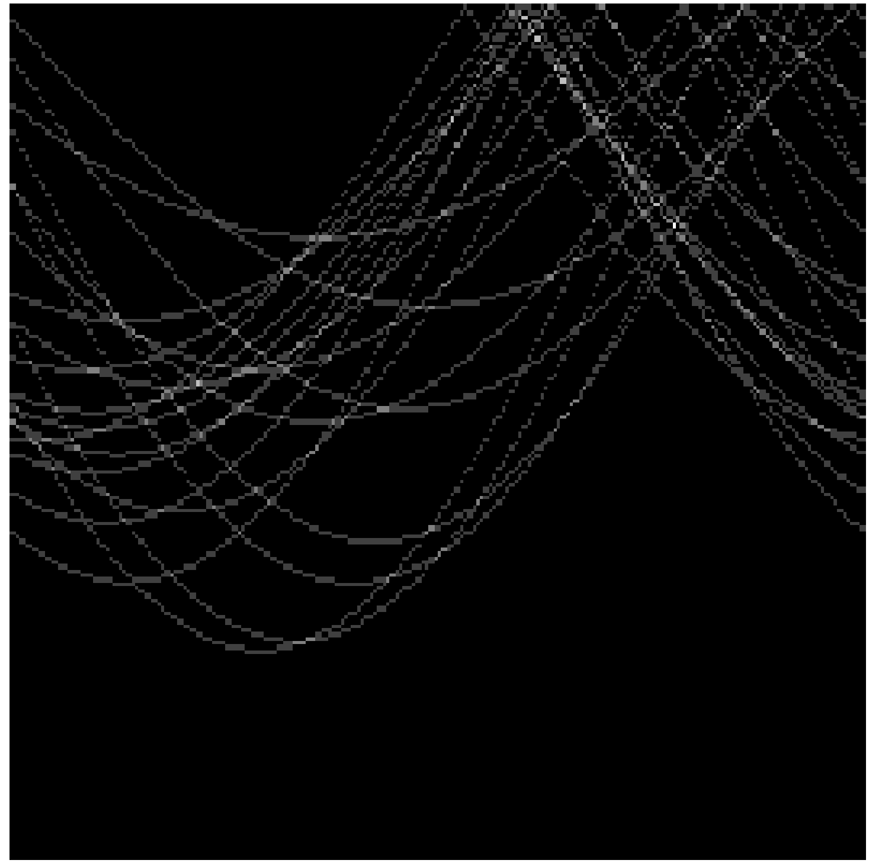
IDEA: introduce a grid a count intersection points in each cell

Issue: Grid size needs to be adjusted…

Hough transform - experiments



features



votes

Issue: spurious peaks due to uniform noise

Hough transform - conclusions

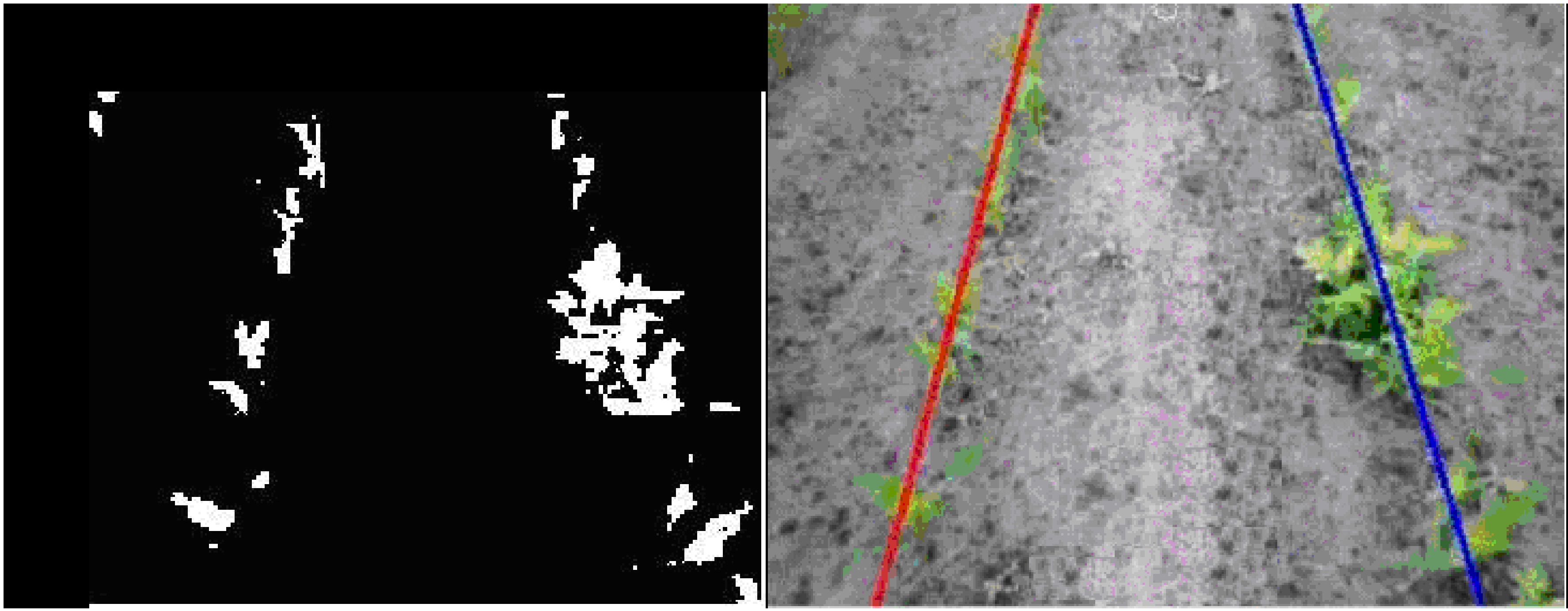
Good:

- All points are processed independently, so can cope with occlusion/outliers
- Some robustness to noise: noise points unlikely to contribute consistently to any single bin

Bad:

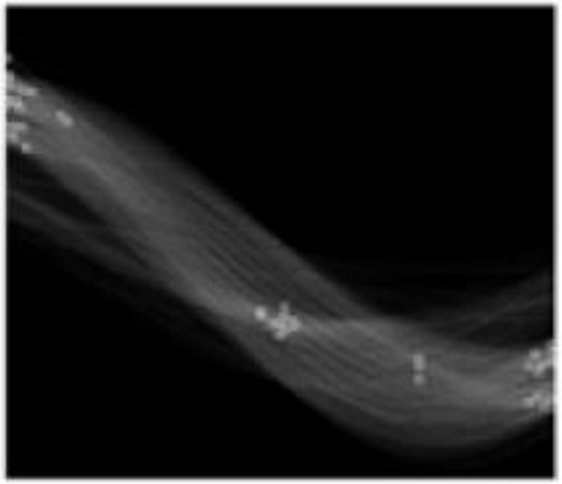
- Spurious peaks due to uniform noise
- Trade-off noise-grid size (hard to find sweet point)

Hough transform - experiments



Courtesy of TKK Automation Technology Laboratory



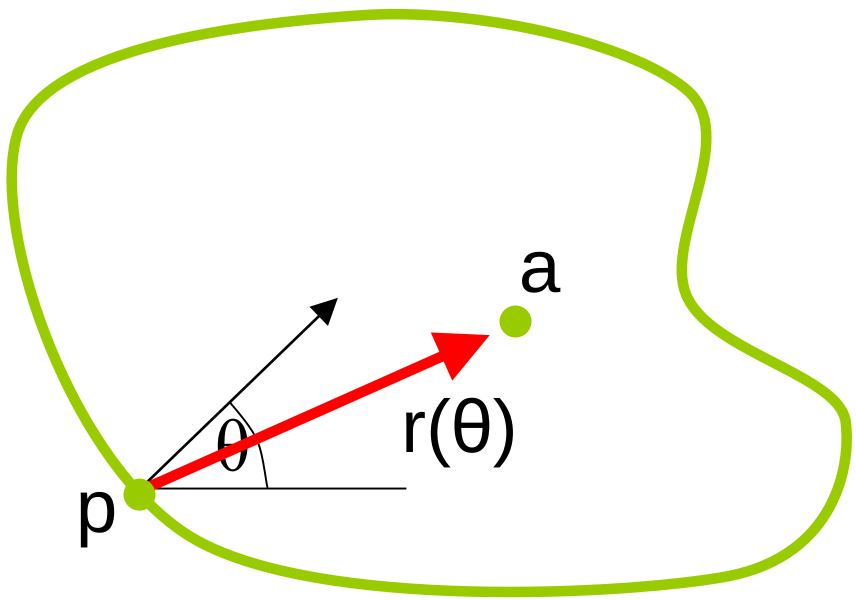
Credit slide: C. *Grauman*

Generalized Hough transform

[more on forthcoming lectures]

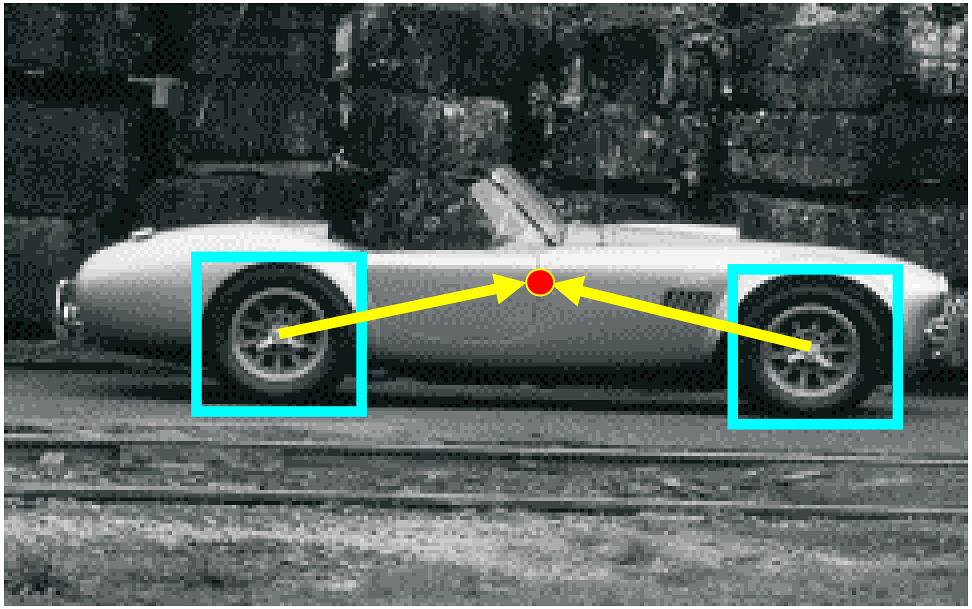
D. Ballard, Generalizing the Hough Transform to Detect Arbitrary Shapes, Pattern Recognition 13(2), 1981

- Identify a shape model by measuring the location of its parts and shape centroid
- Measurements: orientation theta, location of p
- Each measurement casts a vote in the Hough space: $p + r(\theta)$



Generalized Hough transform

B. Leibe, A. Leonardis, and B. Schiele, Combined Object Categorization and Segmentation with an Implicit Shape Model, ECCV Workshop on Statistical Learning in Computer Vision 2004



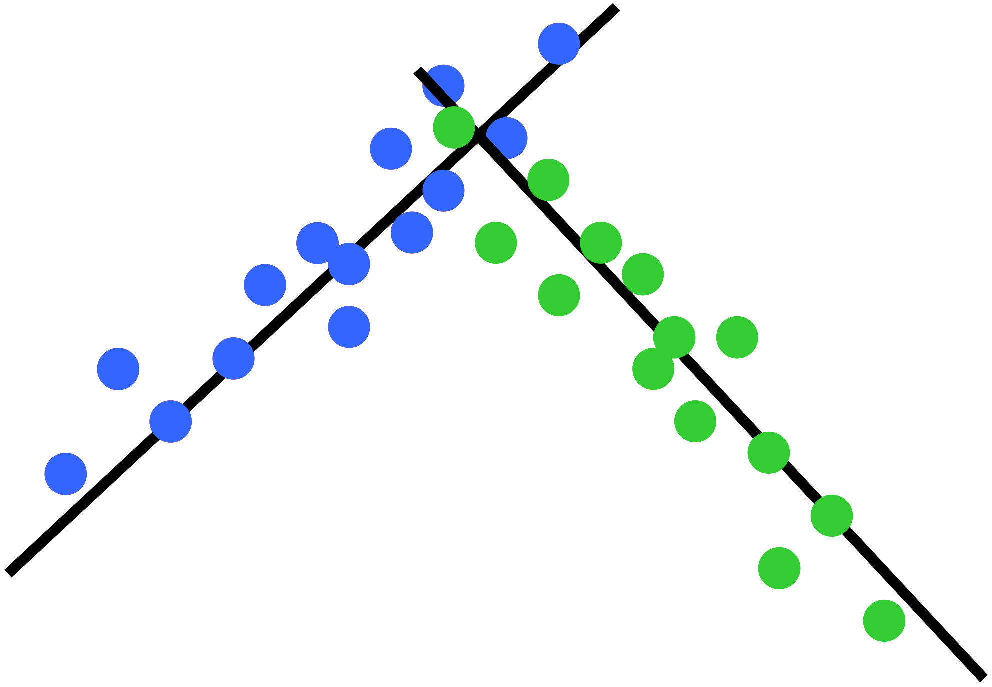
Lecture 9

Fitting and Matching

- Problem formulation
- Least square methods
- RANSAC
- Hough transforms
- Multi-model fitting
- Fitting helps matching!

Reading:
[HZ] Chapter: 4 "Estimation – 2D projective transformation",
Chapter 11 "Computation of the fundamental matrix F"
[FP] Chapters: 16 "Segmentation and fitting using probabilistic methods"

Fitting multiple models



- Incremental fitting
- E.M. (probabilistic fitting)
- Hough transform

Incremental line fitting

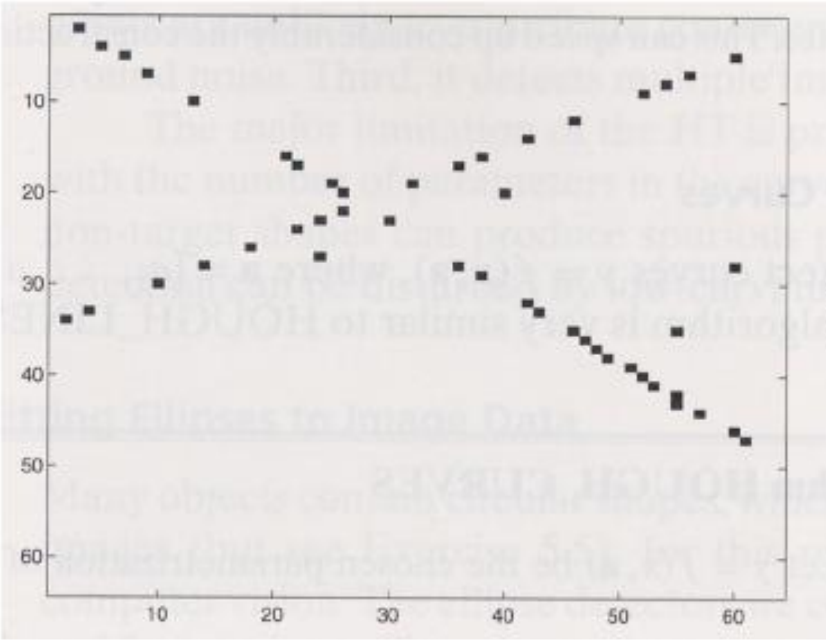
Scan data point sequentially (using locality constraints)

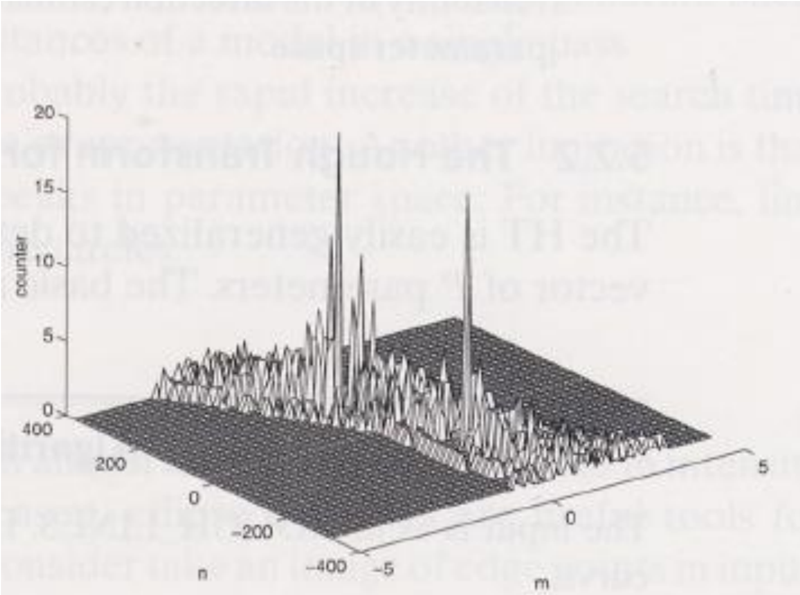
Perform following loop:

1. Select N point and fit line to N points
2. Compute residual R_N
3. Add a new point, re-fit line and re-compute R_{N+1}
4. Continue while line fitting residual is small enough,

- ➢ When residual exceeds a threshold, start fitting new model (line)

Hough transform





Courtesy of unknown

Same cons and pros as before…

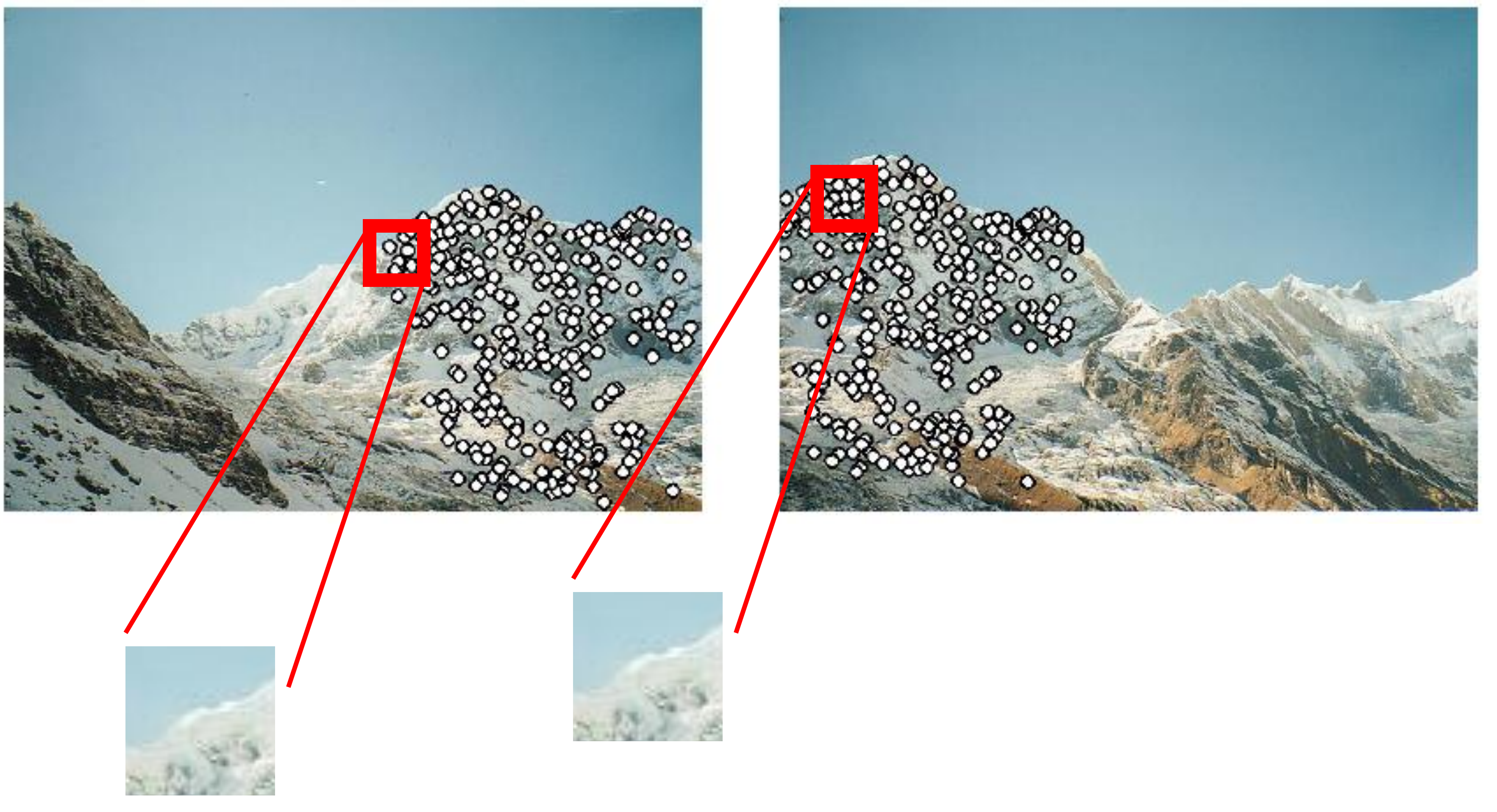
Lecture 9

Fitting and Matching

- Problem formulation
- Least square methods
- RANSAC
- Hough transforms
- Multi-model fitting
- Fitting helps matching!

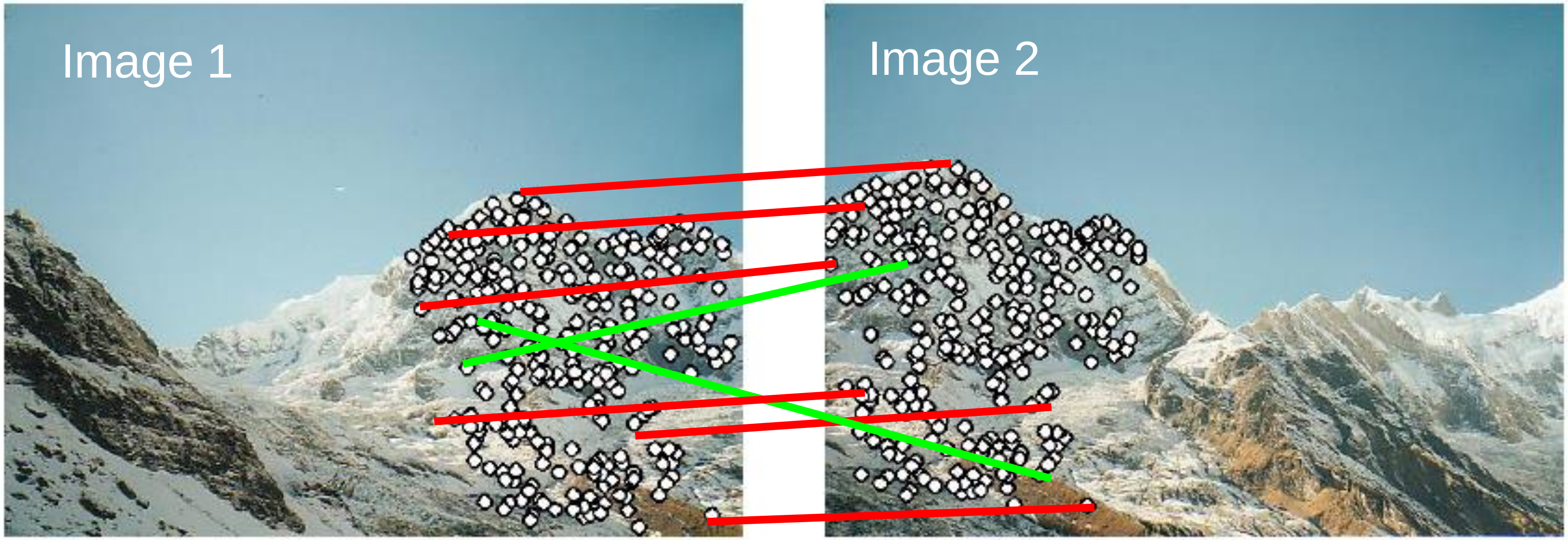
Reading:
[HZ] Chapter: 4 “Estimation – 2D projective transformation”,
Chapter 11 “Computation of the fundamental matrix F”
[FP] Chapters: 16 “Segmentation and fitting using probabilistic methods”

Fitting helps matching!



Features are matched (for instance, based on correlation)

Fitting helps matching!



Matches bases on appearance only

Red: good matches

Green: bad matches

Idea:

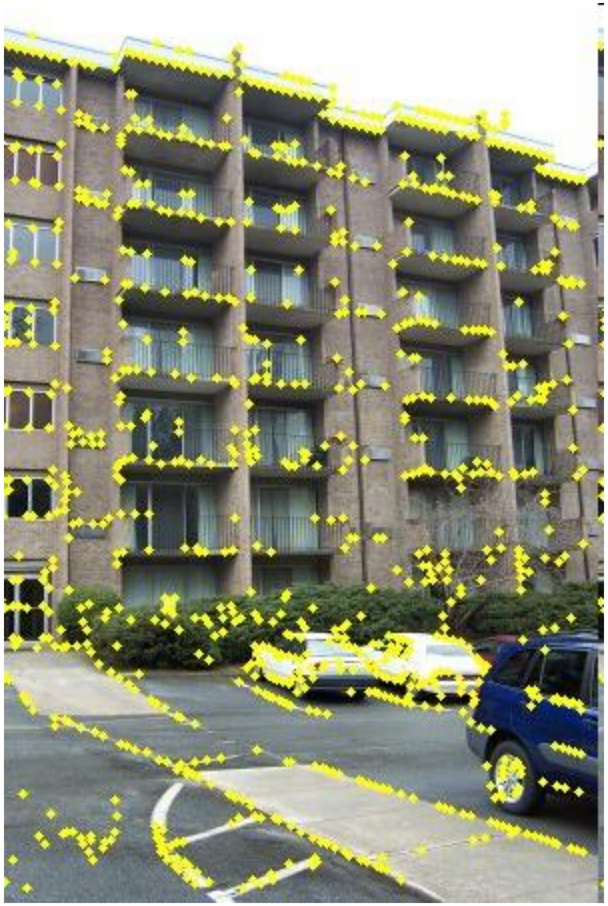
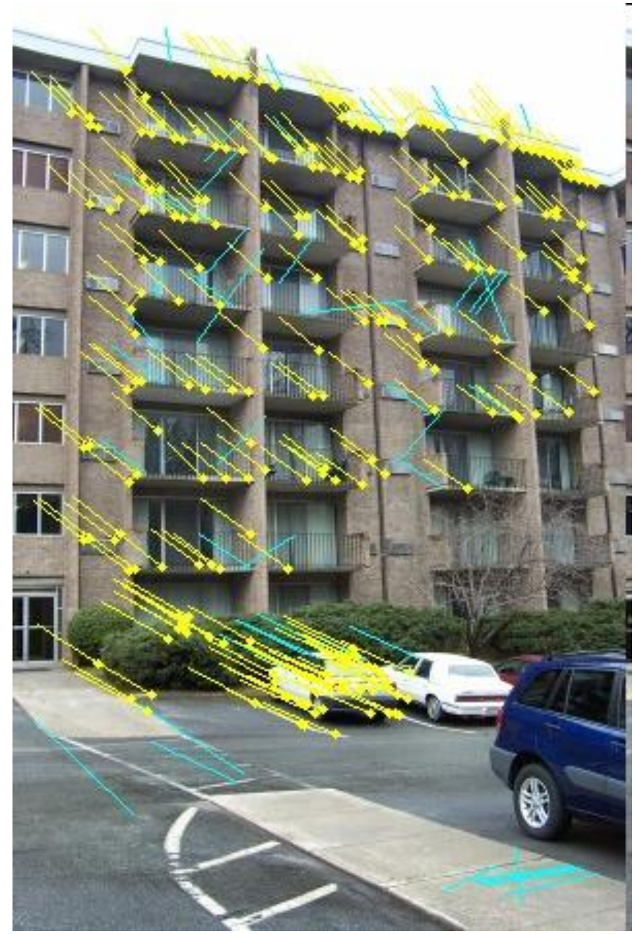
- Fitting an homography H (by RANSAC) mapping features from images 1 to 2
- Bad matches will be labeled as outliers (hence rejected)!

Fitting helps matching!

Recognising Panoramas

M. Brown and D. G. Lowe. Recognising Panoramas. In Proceedings of the *9th International Conference on Computer Vision -- ICCV2003*

Fitting helps matching!



Images courtesy of Brandon Lloyd

Next lecture:
Feature detectors and descriptors

Least squares methods
- fitting a line -

$$Ax = b$$

- More equations than unknowns

- Look for solution which minimizes $||Ax-b|| = (Ax-b)^T(Ax-b)$
- Solve $\frac{\partial (Ax-b)^T(Ax-b)}{\partial x_i} = 0$

- LS solution

$$x = (A^T A)^{-1} A^T b$$

Least squares methods

- fitting a line -

Solving $x = (A^t A)^{-1} A^t b$

$A^+ = (A^t A)^{-1} A^t$ = pseudo-inverse of A

$A = U \sum V^t$ = SVD decomposition of A

$A^{-1} = V \sum^{-1} U$

$A^+ = V \sum^+ U$

with $\sum^+$ equal to $\sum^{-1}$ for all nonzero singular values and zero otherwise

Least squares methods

- fitting an homography -

$$
\begin{aligned}
h_{11}x + h_{12}y + h_{13} - h_{31}xx' - h_{32}yx' - x' &= 0 \\
h_{21}x + h_{22}y + h_{23} - h_{31}xy' - h_{32}yy' - y' &= 0
\end{aligned}
$$

From n>=4 corresponding points: $\boxed{A\,h = 0}$

$$
\begin{pmatrix}
x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x'_1 & -y_1x'_1 & -x'_1 \\
0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y'_1 & -y_1y'_1 & -y'_1 \\
x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2x'_2 & -y_2x'_2 & -x'_2 \\
0 & 0 & 0 & x_2 & y_2 & 1 & -x_2y'_2 & -y_2y'_2 & -y'_2 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
x_n & y_n & 1 & 0 & 0 & 0 & -x_nx'_n & -y_nx'_n & -x'_n \\
0 & 0 & 0 & x_n & y_n & 1 & -x_ny'_n & -y_ny'_n & -y'_n
\end{pmatrix}
\begin{bmatrix}
h_{1,1} \\ h_{1,2} \\ \vdots \\ h_{3,3}
\end{bmatrix}
= 0
$$