Lecture 4
Single View Metrology

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Computational Vision and Geometry Lab

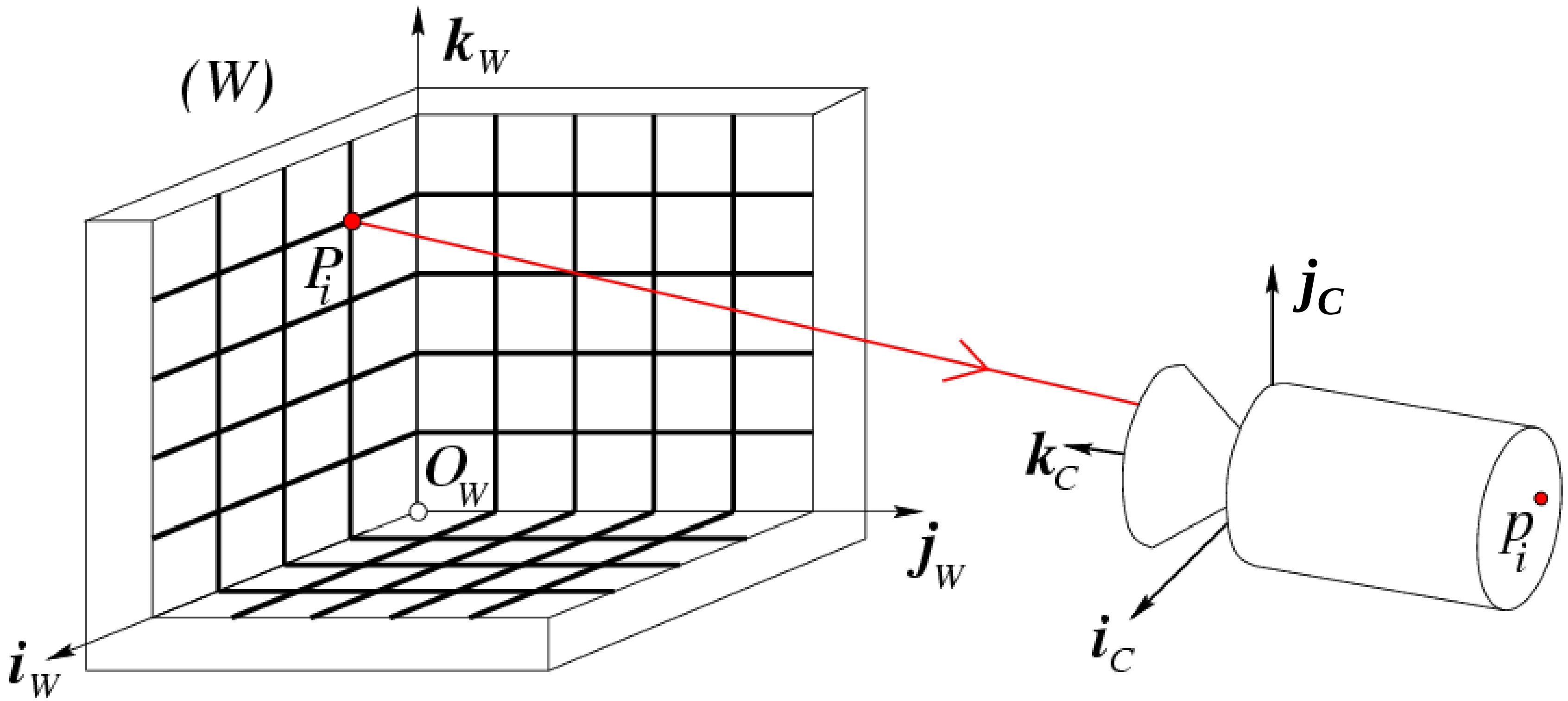
Lecture 4

Single View Metrology

- Review calibration
- Vanishing points and lines
- Estimating geometry from a single image
- Extensions

Reading:
[HZ] Chapter 2 “Projective Geometry and Transformation in 3D”
[HZ] Chapter 3 “Projective Geometry and Transformation in 3D”
[HZ] Chapter 8 “More Single View Geometry”
[Hoeim & Savarese] Chapter 2

Calibration Problem



$$M = K\begin{bmatrix} R & T \end{bmatrix}$$

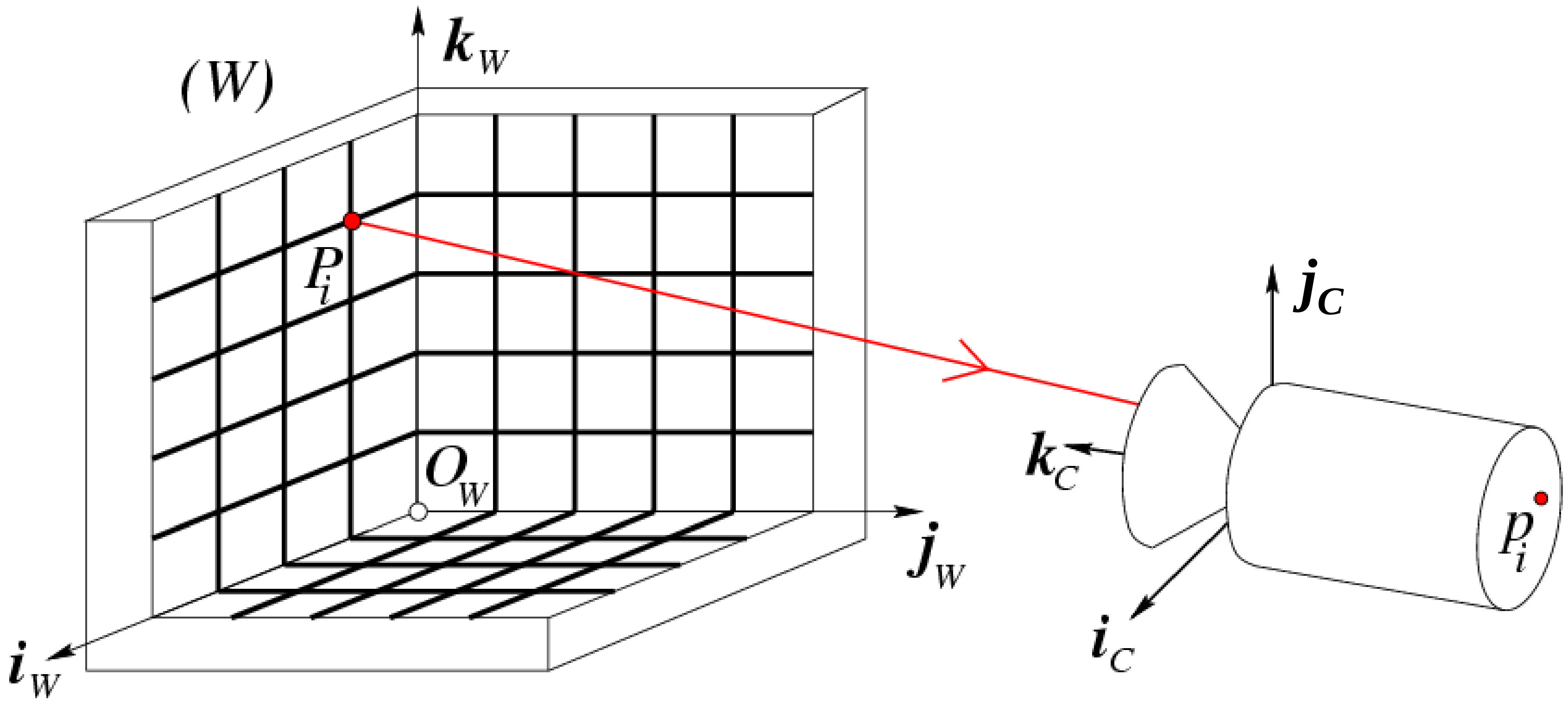
$$K = \begin{bmatrix} \alpha & -\alpha \cot\theta & u_o \\ 0 & \dfrac{\beta}{\sin\theta} & v_o \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_i \rightarrow M\,P_i \rightarrow p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix}$$

World ref. system

In pixels

Calibration Problem



$$M = K\begin{bmatrix} R & T \end{bmatrix}$$

$$P_i \rightarrow M\,P_i \rightarrow p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix}$$

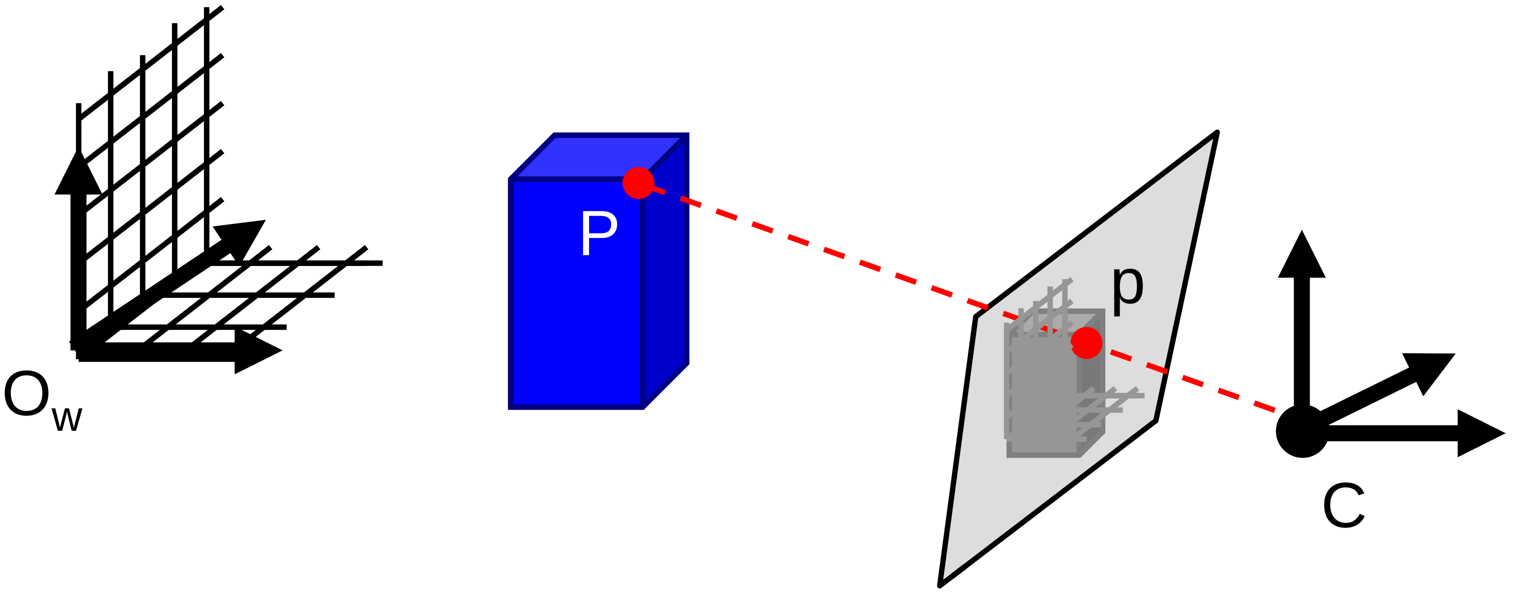
11 unknown

Need at least 6 correspondences

World ref. system

In pixels

Once the camera is calibrated...



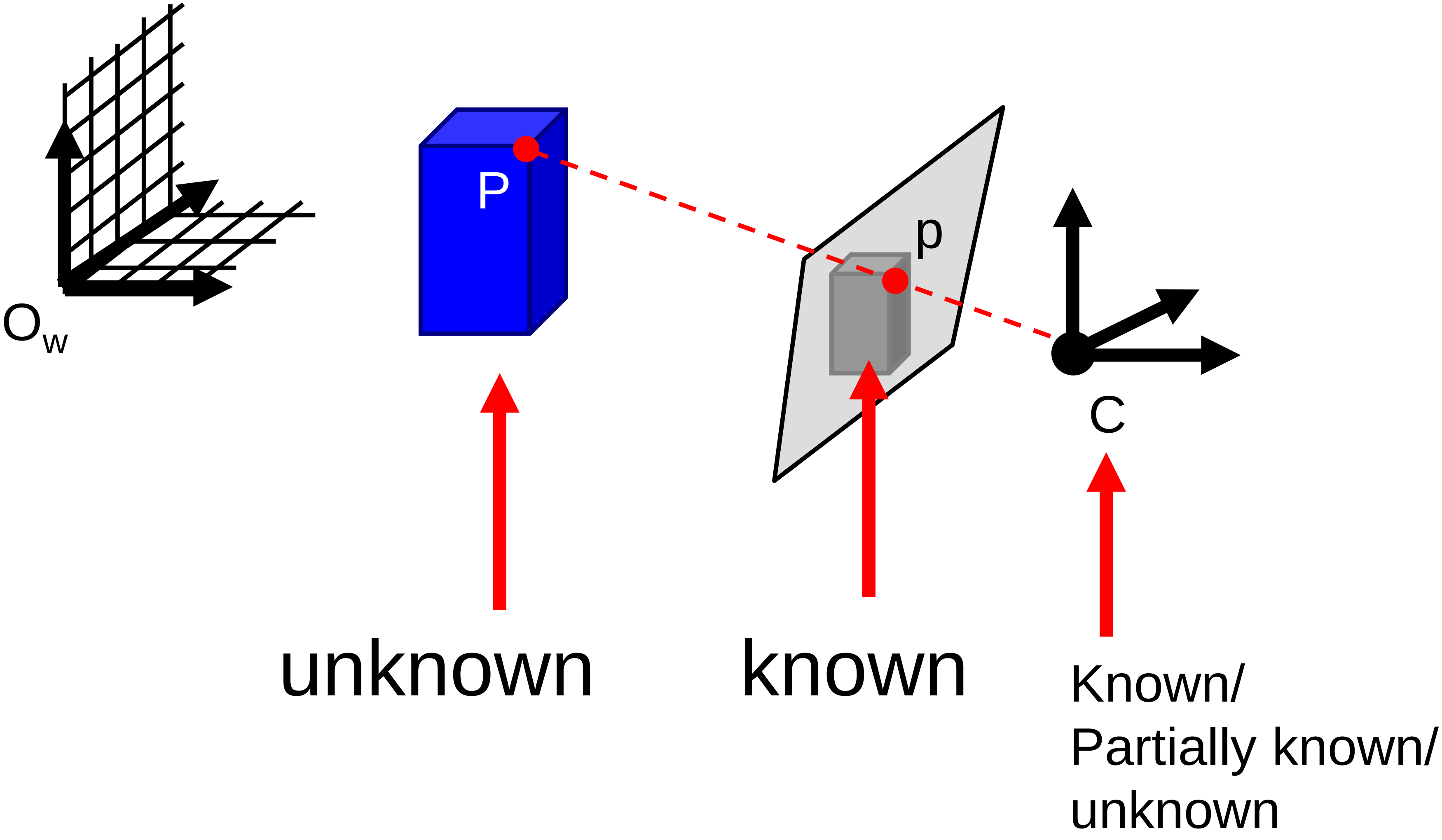
$$M = K\begin{bmatrix} R & T \end{bmatrix}$$

-Internal parameters K are known
-R, T are known – but these can only relate C to the calibration rig

Can I estimate P from the measurement p from a single image?

No - in general ☹ [P can be anywhere along the line defined by C and p]

Recovering structure from a single view



Recovering structure from a single view

http://www.robots.ox.ac.uk/~vgg/projects/SingleView/models/hut/hutme.wrl

Transformation in 2D

-Isometries

-Similarities

-Affinity

-Projective

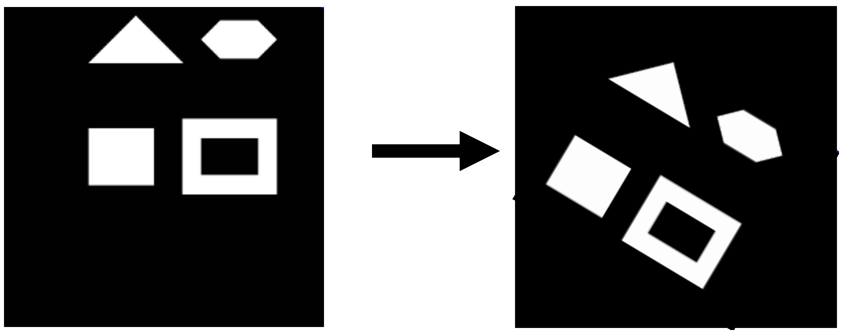
Transformation in 2D

Isometries:

[Euclideans]

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_e \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Preserve distance (areas)
- 3 DOF
- Regulate motion of rigid object

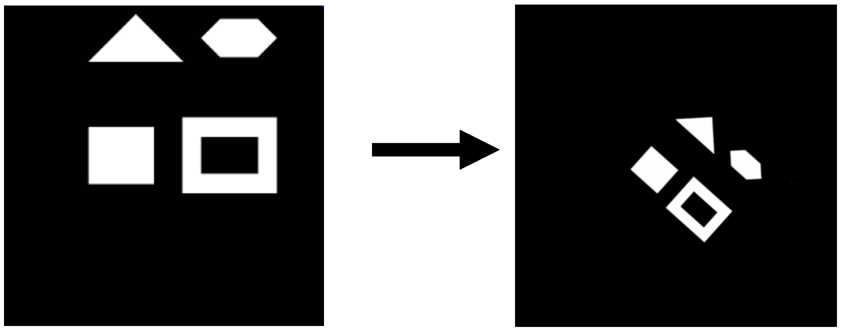


Transformation in 2D

Similarities:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s\,R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_s \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

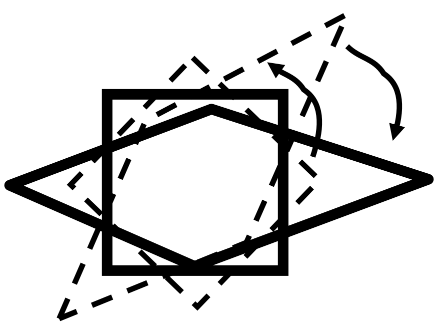
- Preserve
    - ratio of lengths
    - angles
- 4 DOF

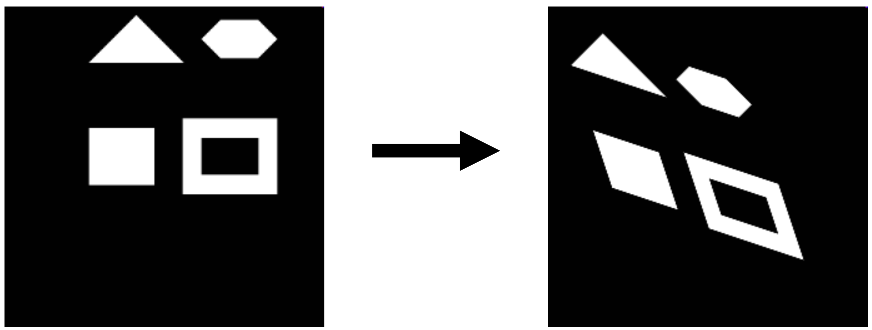


Transformation in 2D

Affinities: $$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_a \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = R(\theta) \cdot R(-\phi) \cdot D \cdot R(\phi) \qquad D = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$





Transformation in 2D

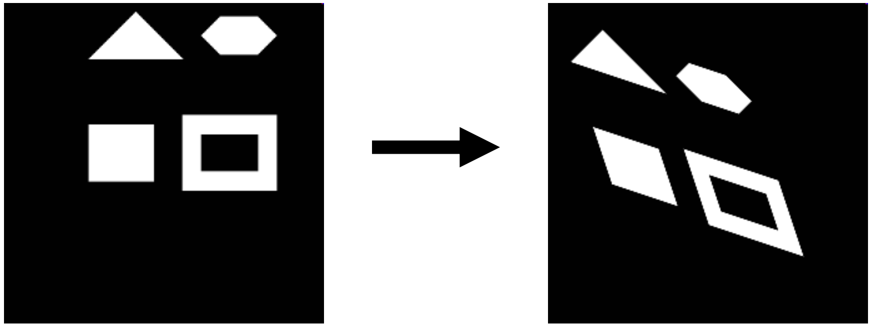
Affinities:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_a \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = R(\theta) \cdot R(-\phi) \cdot D \cdot R(\phi) \qquad D = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

-Preserve:
- Parallel lines
- Ratio of areas
- Ratio of lengths on collinear lines
- others…

- 6 DOF

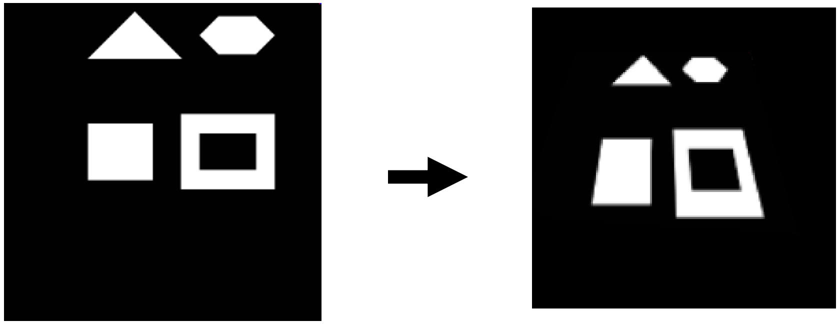


Transformation in 2D

Projective:

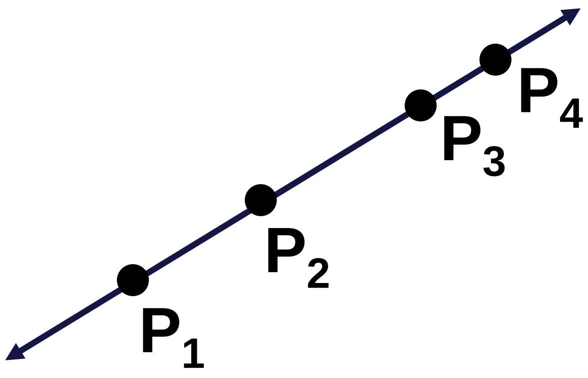
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A & t \\ v & b \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_p \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- 8 DOF
- Preserve:
  - cross ratio of 4 collinear points
  - collinearity
  - and a few others…



The cross ratio

The cross-ratio of 4 collinear points



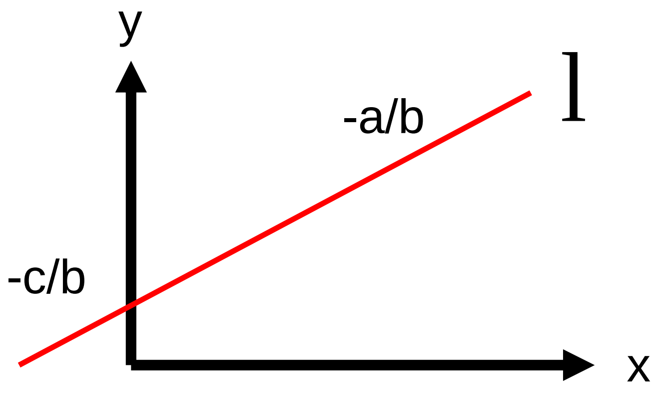
$$\frac{\|\mathbf{P}_3 - \mathbf{P}_1\| \, \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_3 - \mathbf{P}_2\| \, \|\mathbf{P}_4 - \mathbf{P}_1\|}$$

$$\mathbf{P}_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

Can permute the point ordering $\frac{\|\mathbf{P}_1 - \mathbf{P}_3\| \, \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_1 - \mathbf{P}_2\| \, \|\mathbf{P}_4 - \mathbf{P}_3\|}$

Lines in a 2D plane

$$ax + by + c = 0$$



$$\mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

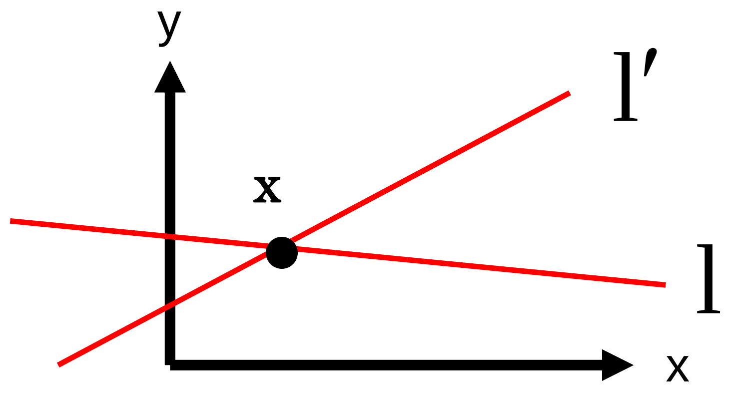
If $\mathbf{x} = [\, x_1, x_2]^T \in \mathbf{l}$

$$\begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}^T \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

Lines in a 2D plane

Intersecting lines

$$\mathrm{x} = \mathrm{l} \times \mathrm{l}'$$



Proof

$$\mathrm{l} \times \mathrm{l}' \perp \mathrm{l} \quad \rightarrow (\mathrm{l} \times \mathrm{l}') \cdot \mathrm{l} = 0 \quad \rightarrow x \in l$$

$$\mathrm{l} \times \mathrm{l}' \perp \mathrm{l}' \quad \rightarrow \underbrace{(\mathrm{l} \times \mathrm{l}')}_{\mathrm{x}} \cdot \mathrm{l}' = 0 \quad \rightarrow x \in l'$$

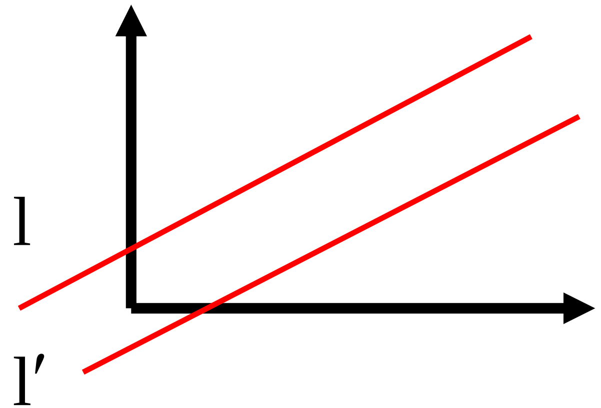
$\rightarrow$ x is the intersecting point

2D Points at infinity (ideal points)

$$\mathrm{x} = \begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \end{bmatrix}, \mathrm{x}_3 \neq 0$$

$$-a/b = -a'/b'$$

$$\mathrm{l} = \begin{bmatrix} \mathrm{a} \\ \mathrm{b} \\ \mathrm{c} \end{bmatrix}$$



$$x_\infty = \begin{bmatrix} x'_1 \\ x'_2 \\ 0 \end{bmatrix}$$

$$l' = \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix}$$

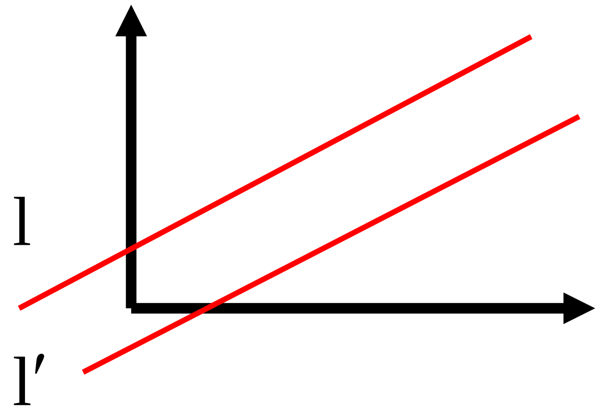
Let's intersect two parallel lines: $\rightarrow l \times l' \propto \begin{bmatrix} b \\ -a \\ 0 \end{bmatrix} = x_\infty$

- In Euclidian coordinates this point is at infinity
- Agree with the general idea of two lines intersecting at infinity

2D Points at infinity (ideal points)

$$\mathrm{x} = \begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \end{bmatrix}, \mathrm{x}_3 \neq 0$$

$$-a/b = -a'/b'$$



$$\mathrm{l} = \begin{bmatrix} \mathrm{a} \\ \mathrm{b} \\ \mathrm{c} \end{bmatrix}$$

$$l' = \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix}$$

Note: the line l = [a b c]^T pass trough the ideal point x_∞

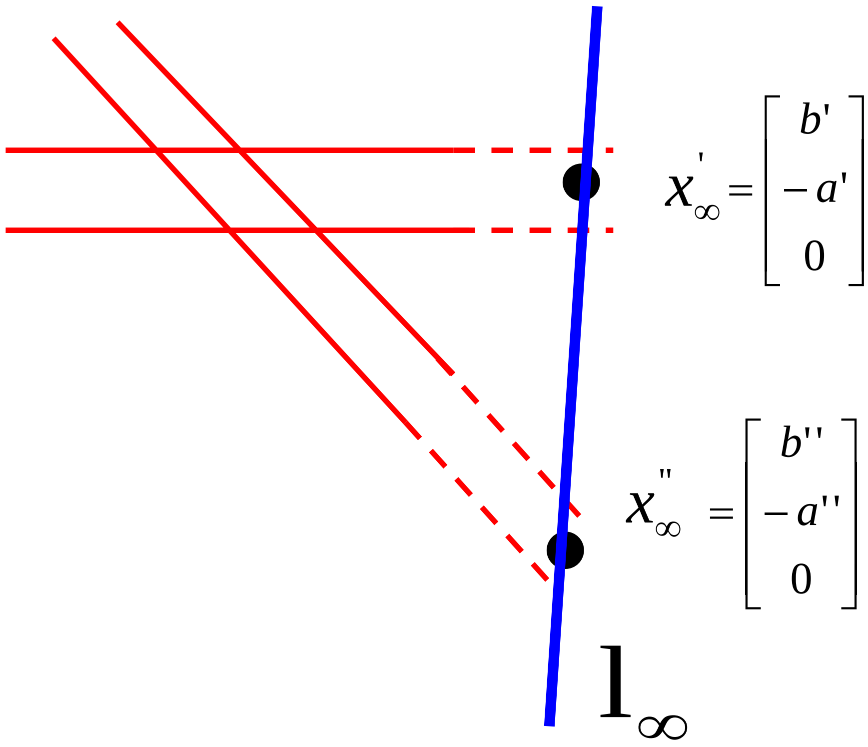
$$\mathrm{l}^{\mathrm{T}}\ x_\infty = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} b \\ -a \\ 0 \end{bmatrix} = 0$$

So does the line l’ since a b’ = a’ b

Lines infinity $\mathbf{l}_\infty$

Set of ideal points lies on a line called the line at infinity
How does it look like?

$$\mathbf{l}_\infty = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

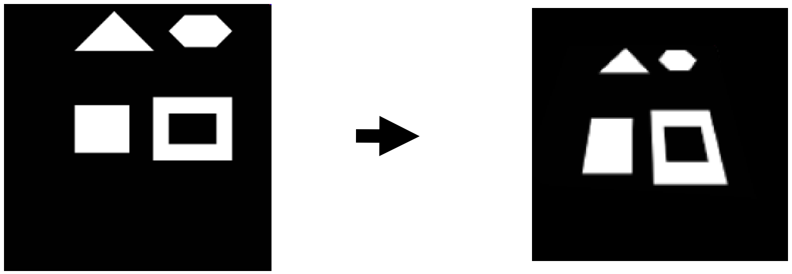


Indeed: $\begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix}^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$

A line at infinity can thought of the set of "directions" of lines in the plane

Projective transformation of a point at infinity

$$H = \begin{bmatrix} A & t \\ v & b \end{bmatrix}$$



$$p' = H\ p$$

is it a point at infinity?

$$H\ p_\infty = ? = \begin{bmatrix} A & t \\ v & b \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} p'_x \\ p'_y \\ p'_z \end{bmatrix}$$

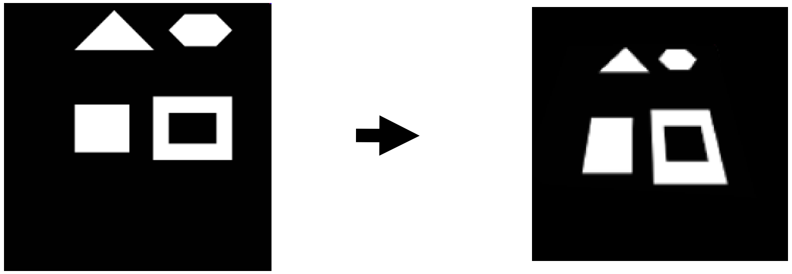
…no!

$$H_A\ p_\infty = ? = \begin{bmatrix} A & t \\ 0 & b \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} p'_x \\ p'_y \\ 0 \end{bmatrix}$$

An affine transformation of a point at infinity is still a point at infinity

Projective transformation of a line (in 2D)

$$\mathrm{H} = \begin{bmatrix} \mathrm{A} & \mathrm{t} \\ \mathrm{v} & \mathrm{b} \end{bmatrix}$$



$$\mathrm{l}' = \mathrm{H}^{-\mathrm{T}}\,\mathrm{l}$$

is it a line at infinity?

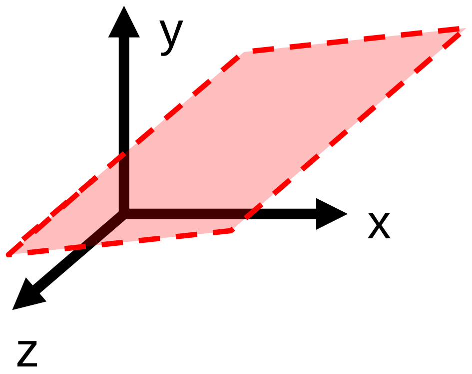
$$\mathrm{H}^{-\mathrm{T}}\,\mathrm{l}_\infty = ? = \begin{bmatrix} \mathrm{A} & \mathrm{t} \\ \mathrm{v} & \mathrm{b} \end{bmatrix}^{-\mathrm{T}} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \mathrm{t}_x \\ \mathrm{t}_y \\ \mathrm{b} \end{bmatrix}$$

…no!

$$\mathrm{H}_\mathrm{A}^{-\mathrm{T}}\,\mathrm{l}_\infty = ? \quad = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix}^{-T} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} A^{-T} & 0 \\ -t^T A^{-T} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Points and planes in 3D

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ 1 \end{bmatrix} \qquad \Pi = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$



$$x \in \Pi \leftrightarrow x^T \Pi = 0 \qquad ax + by + cz + d = 0$$

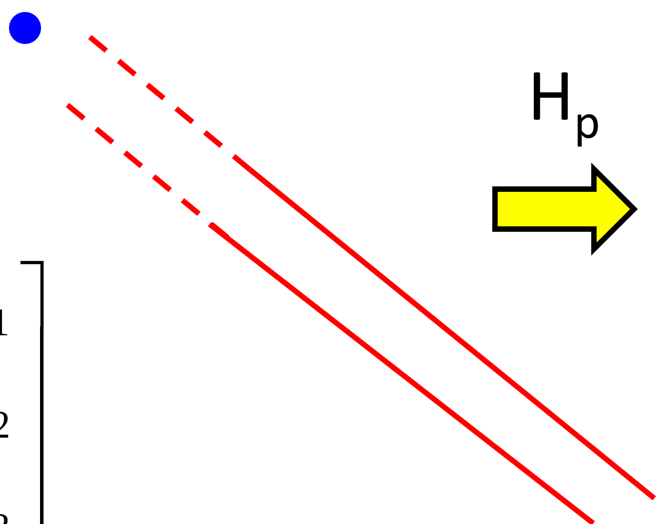
How about lines in 3D?

- Lines have 4 degrees of freedom - hard to represent in 3D-space
- Can be defined as intersection of 2 planes

Vanishing points

In 3D, vanishing points are the equivalent of ideal points in 2D

Points where parallel lines intersect in 3D



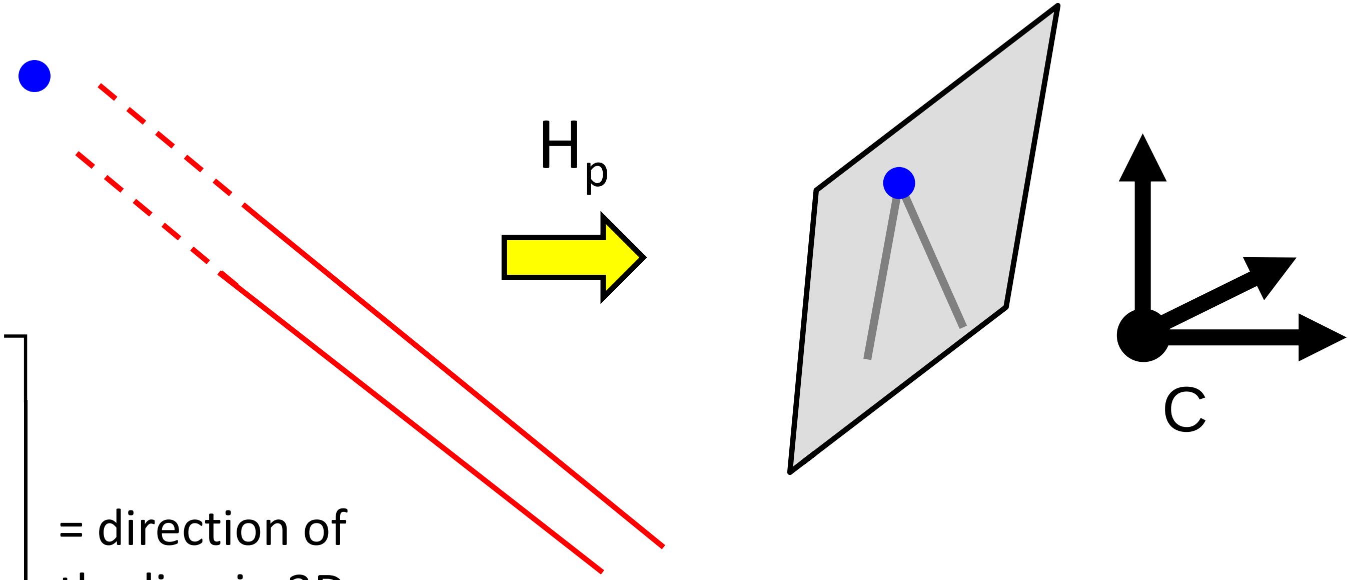
$$x_\infty = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ 0 \end{bmatrix}$$

$$p_\infty = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$$

Vanishing points

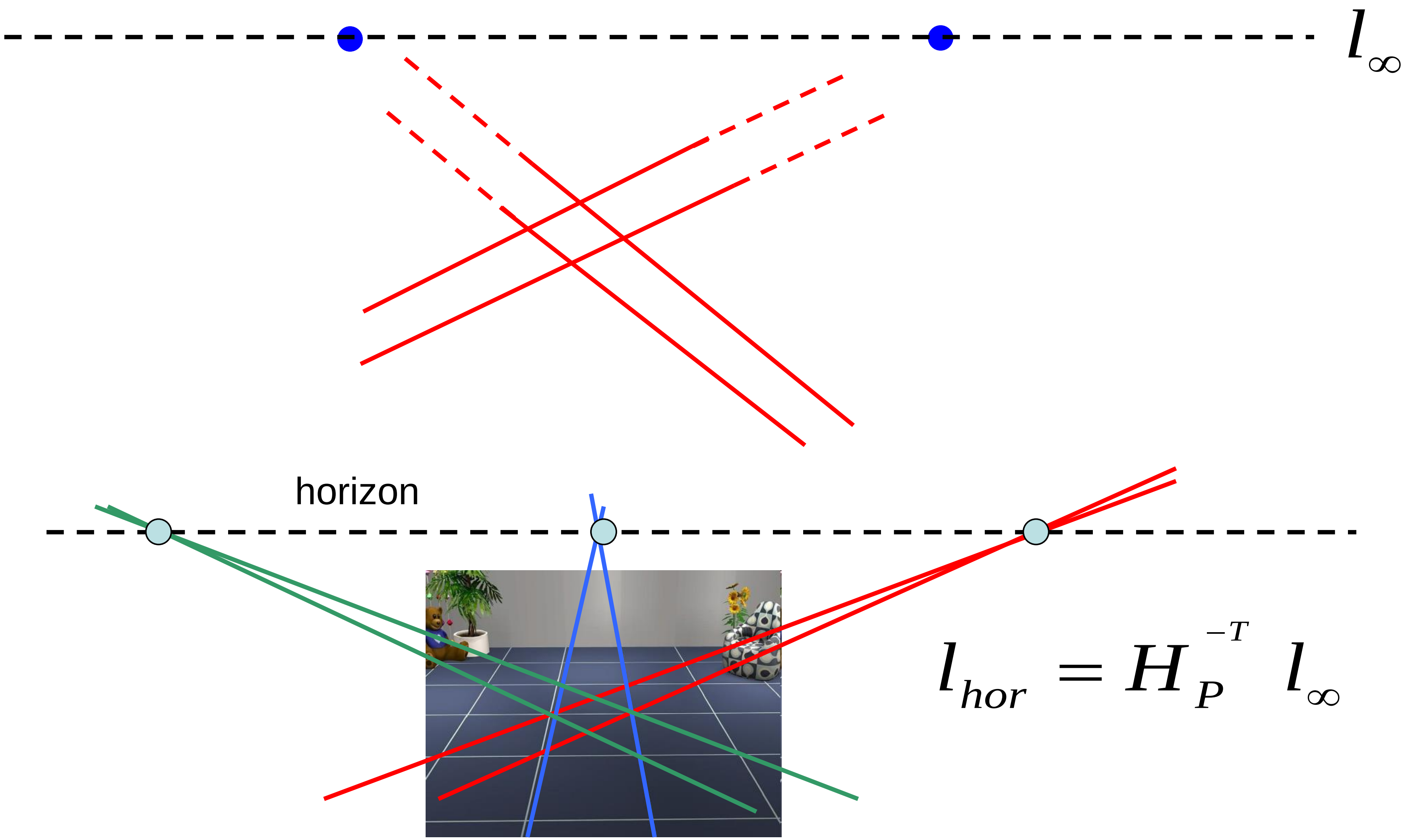
In 3D, vanishing points are the equivalent of ideal points in 2D

Points where parallel lines intersect in 3D



$$x_\infty = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ 0 \end{bmatrix}$$ = direction of the line in 3D

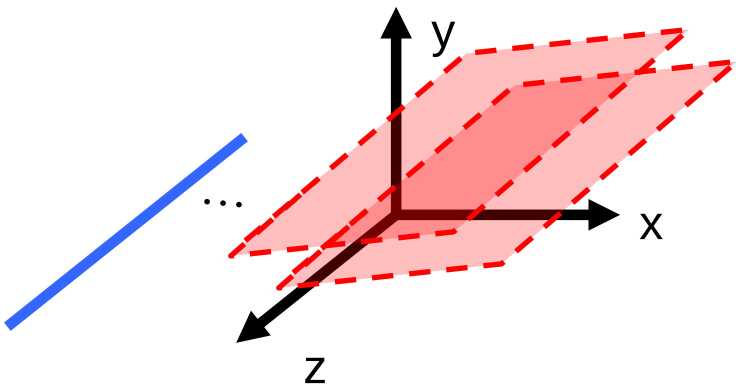
The horizon line



$$l_{hor} = H_P^{-T} \, l_\infty$$

The horizon line

Planes at infinity & vanishing lines

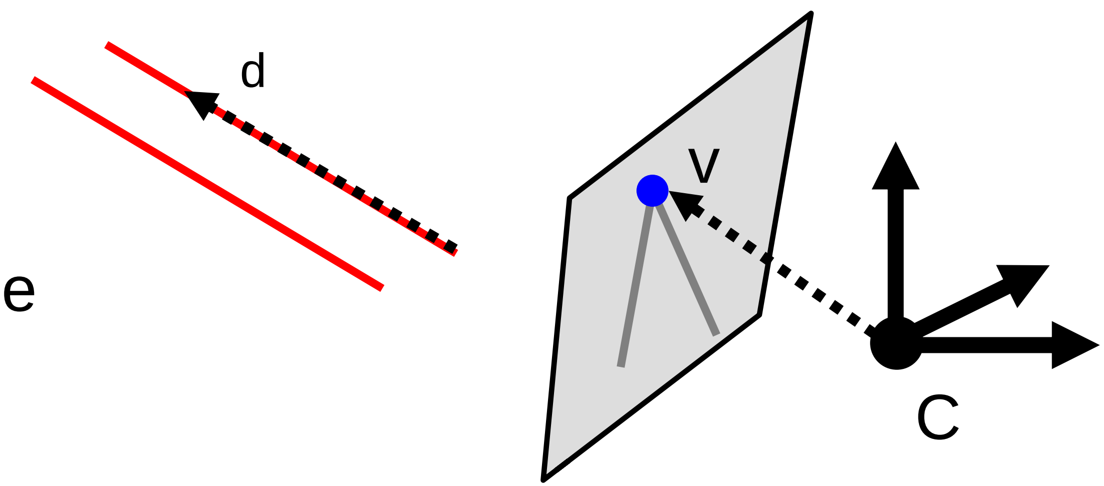


$$\Pi_\infty = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

plane at infinity

- Parallel planes intersect the plane at infinity in a common line **– the vanishing line (→ horizon)**
- A set of vanishing lines defines the plane at infinity Π_∞
- 2 planes are parallel iff their intersections is a line that belongs to Π_∞

Vanishing points and their image



d = direction of the line

$$\mathbf{v} = K\,\mathbf{d}$$

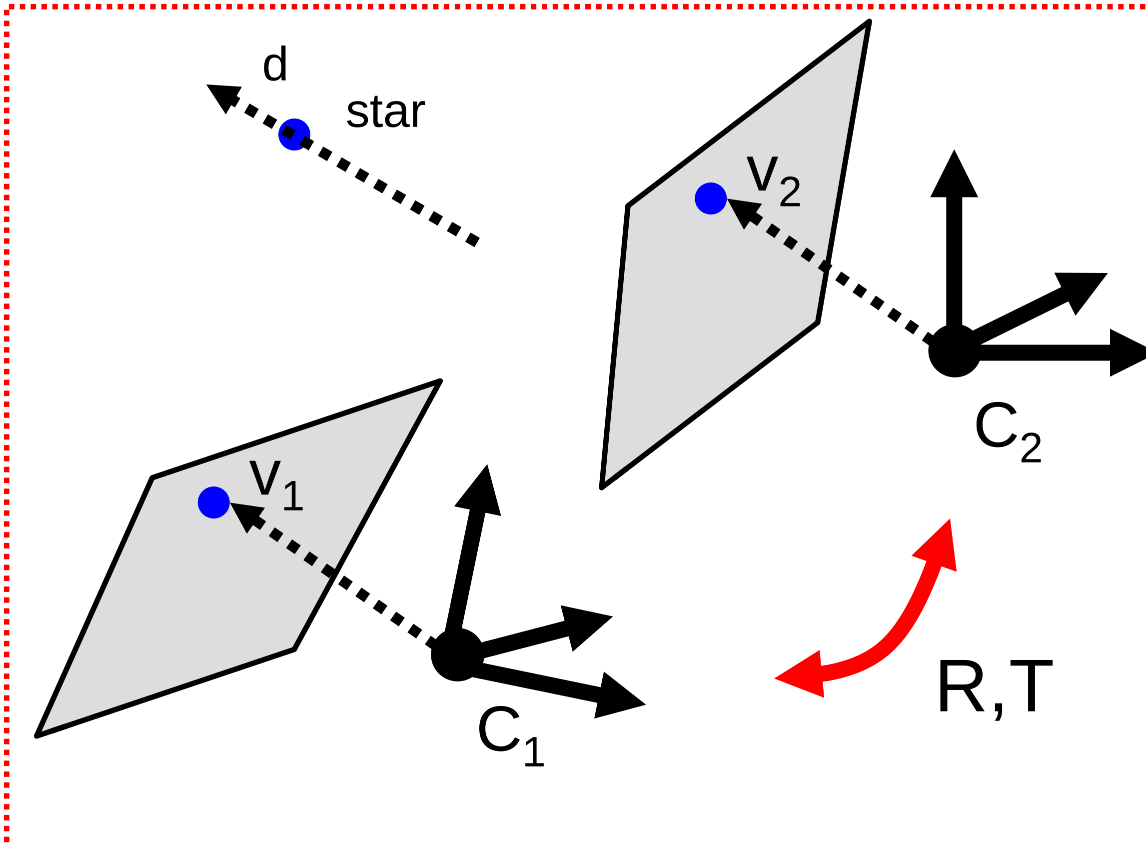
$$\mathbf{x}_\infty = \underbrace{\begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix}}_{\mathbf{d}} \xrightarrow{M} \mathbf{v} = \mathbf{X}_\infty\,\mathbf{M} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix} = \mathbf{K}\begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Vanishing points - example

v1, v2: measurements
K = known and constant

Can I compute R?

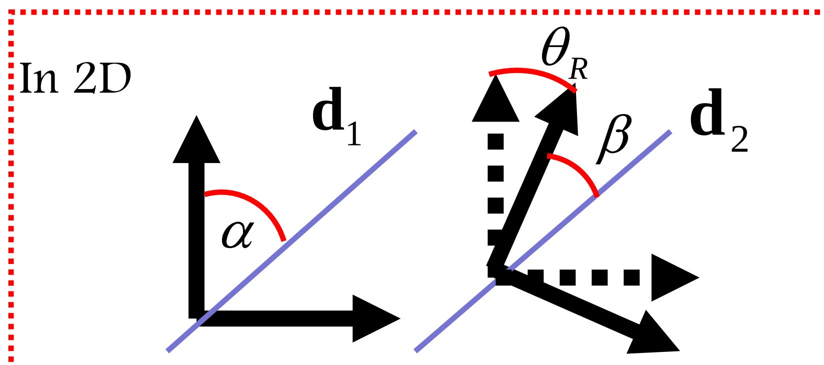
No rotation around z



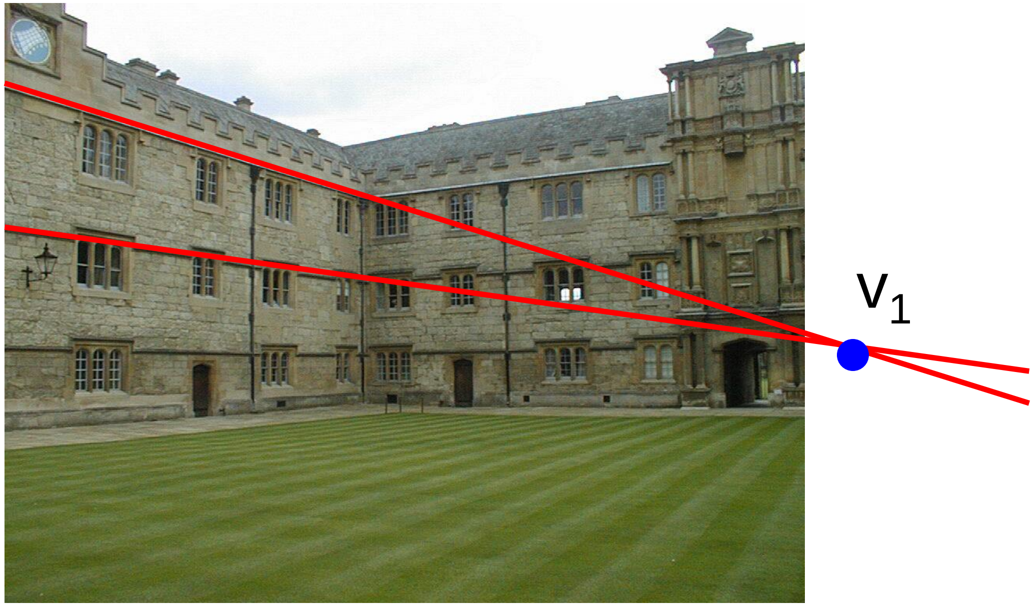
$$\mathbf{d}_1 = \frac{K^{-1}\,\mathbf{v}_1}{\left\| K^{-1}\,\mathbf{v}_1 \right\|}$$

$$\mathbf{d}_2 = \frac{K^{-1}\,\mathbf{v}_2}{\left\| K^{-1}\,\mathbf{v}_2 \right\|}$$

$$R\,\mathbf{d}_1 = \mathbf{d}_2 \longrightarrow R$$



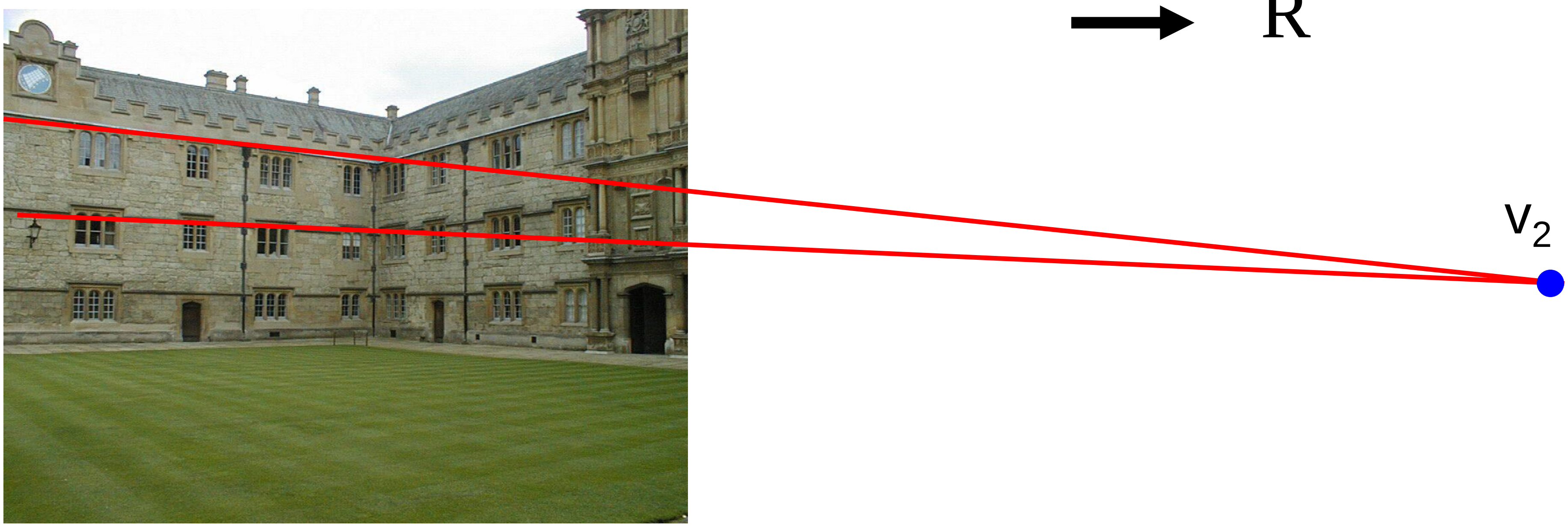
$$\theta_R = \alpha - \beta$$



$$\mathbf{d}_1 = \frac{K^{-1}\,\mathbf{v}_1}{\left\| K^{-1}\,\mathbf{v}_1 \right\|}$$

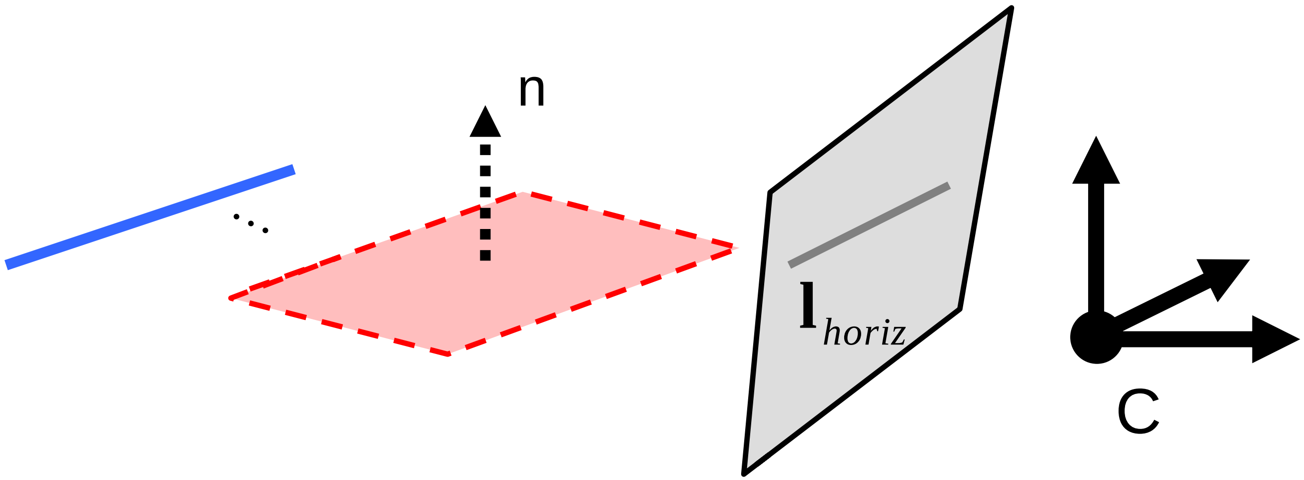
$$\mathbf{d}_2 = \frac{K^{-1}\,\mathbf{v}_2}{\left\| K^{-1}\,\mathbf{v}_2 \right\|}$$

$$\longrightarrow \quad R$$

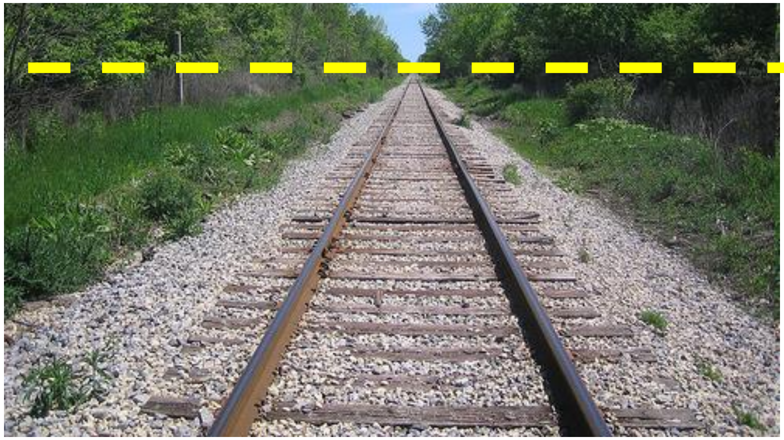


Vanishing lines and their images

Parallel planes intersect the plane at infinity
in a common line – the vanishing line (horizon)



$$\mathbf{n} = K^T \mathbf{l}_{\mathrm{horiz}}$$



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- Estimating geometry from a single image
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Reading:
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[HZ] Chapter 8 “More Single View Geometry”
[Hoeim & Savarese] Chapter 2

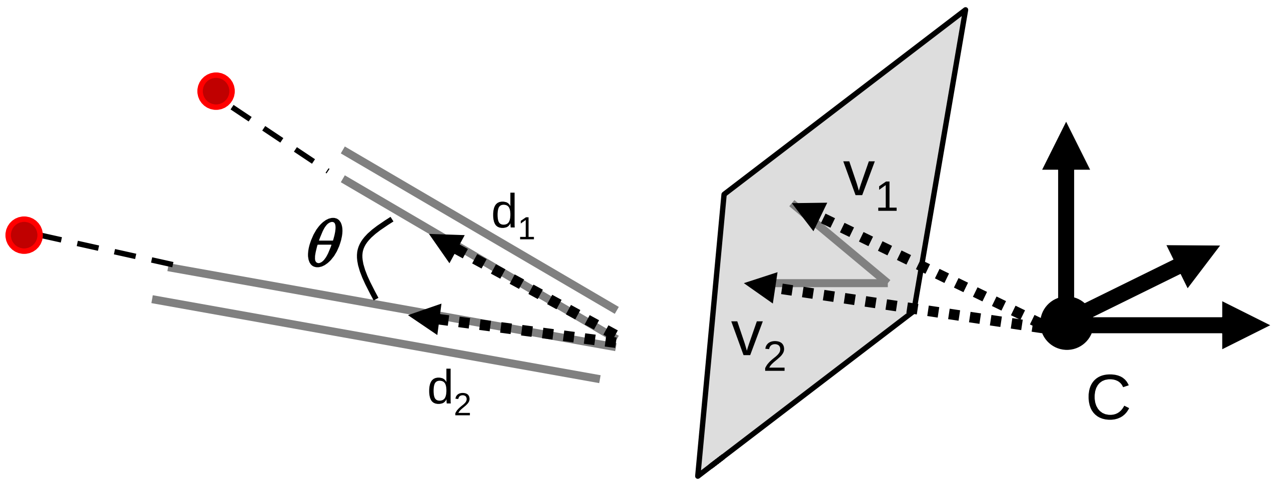
Estimating geometry & calibrating the camera from a single image

Are these two lines parallel or not?

- Recognize the horizon line
- Measure if the 2 lines meet at the horizon
- if yes, these 2 lines are // in 3D

•Recognition helps reconstruction!
•Humans have learnt this

Angle between 2 vanishing points



$$\cos\boldsymbol{\theta} = \frac{\mathrm{v}_1^T \boldsymbol{\omega}\, \mathrm{v}_2}{\sqrt{\mathrm{v}_1^T \boldsymbol{\omega}\, \mathrm{v}_1}\sqrt{\mathrm{v}_2^T \boldsymbol{\omega}\, \mathrm{v}_2}} \qquad \omega = (K\ K^T)^{-1}$$

If $\boldsymbol{\theta} = 90 \quad \rightarrow \quad \boxed{\mathrm{v}_1^T \boldsymbol{\omega}\, \mathrm{v}_2 = 0}$

Projective transformation of Ω_∞

Absolute conic

$$\boldsymbol{\omega} = P^{-T}\Omega_\infty P^{-1} = (K\,K^T)^{-1}$$

$$P = K\begin{bmatrix} R & T \end{bmatrix}$$

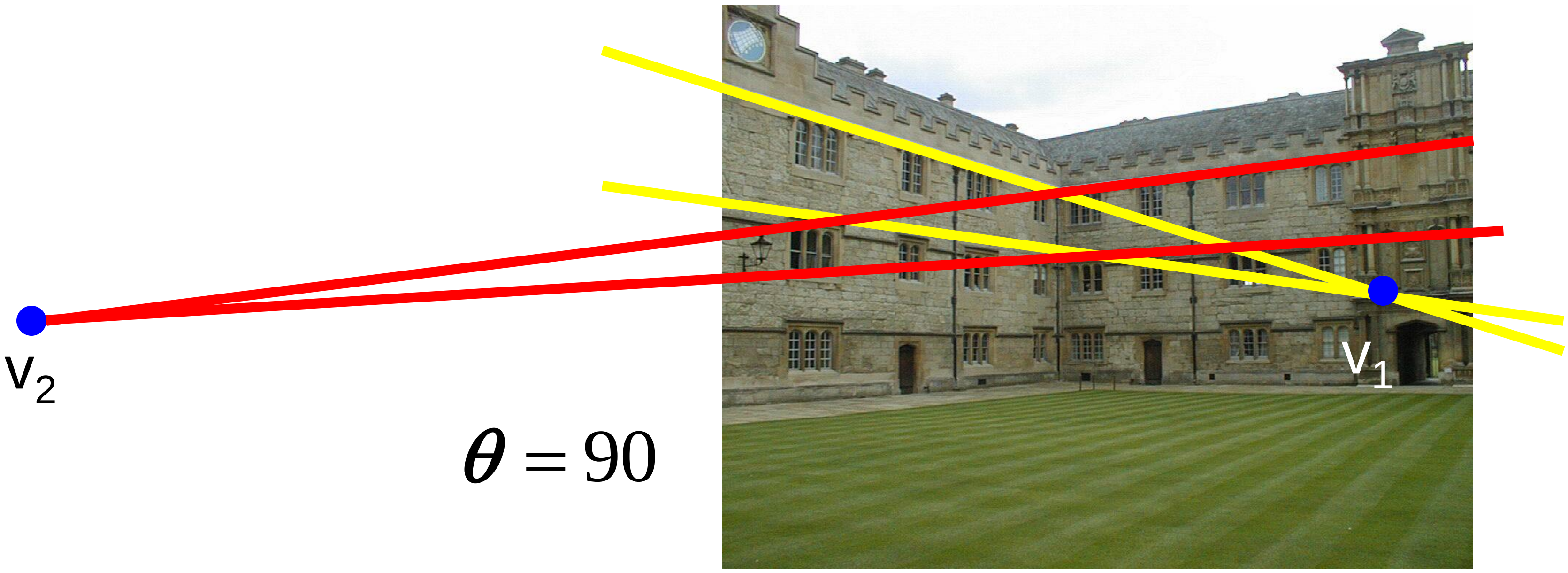
1. It is not function of R, T

2. $$\omega = \begin{bmatrix} \omega_1 & \omega_2 & \omega_4 \\ \omega_2 & \omega_3 & \omega_5 \\ \omega_4 & \omega_5 & \omega_6 \end{bmatrix}$$ symmetric

3. $\omega_2 = 0$ zero-skew

4. $\omega_2 = 0$, $\omega_1 = \omega_3$ square pixel

Angle between 2 scene lines



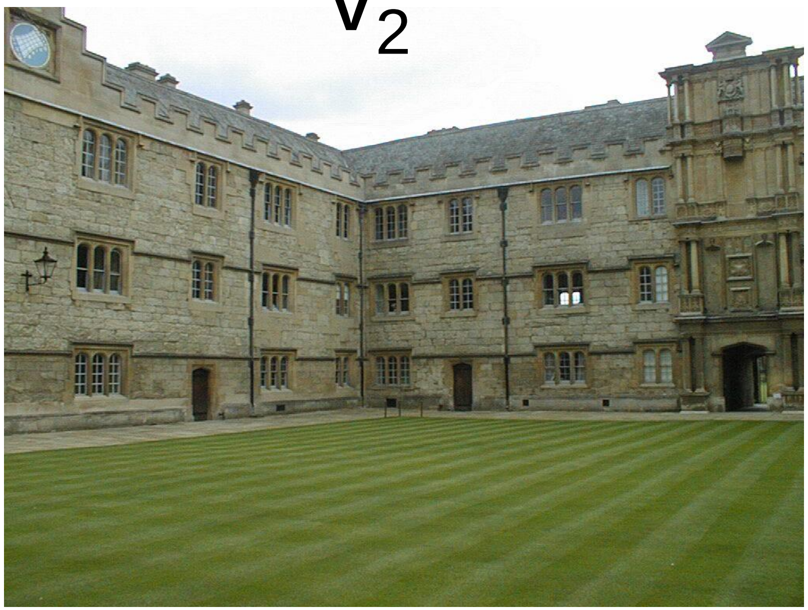
$$\theta = 90$$

$$\begin{cases} \mathrm{v}_1^{\mathrm{T}} \boldsymbol{\omega}\, \mathrm{v}_2 = 0 \\ \omega = (K\ K^T)^{-1} \end{cases}$$

→ Constraint on K

Single view calibration - example

v_2



v_2

v_3

known up to scale

$$\omega = \begin{bmatrix} \omega_1 & \omega_2 & \omega_4 \\ \omega_2 & \omega_3 & \omega_5 \\ \omega_4 & \omega_5 & \omega_6 \end{bmatrix}$$

$$\begin{cases} v_1^T \omega\, v_2 = 0 \\ v_1^T \omega\, v_3 = 0 \\ v_2^T \omega\, v_3 = 0 \end{cases}$$

$$\omega_2 = 0$$

$$\omega_1 = \omega_3$$

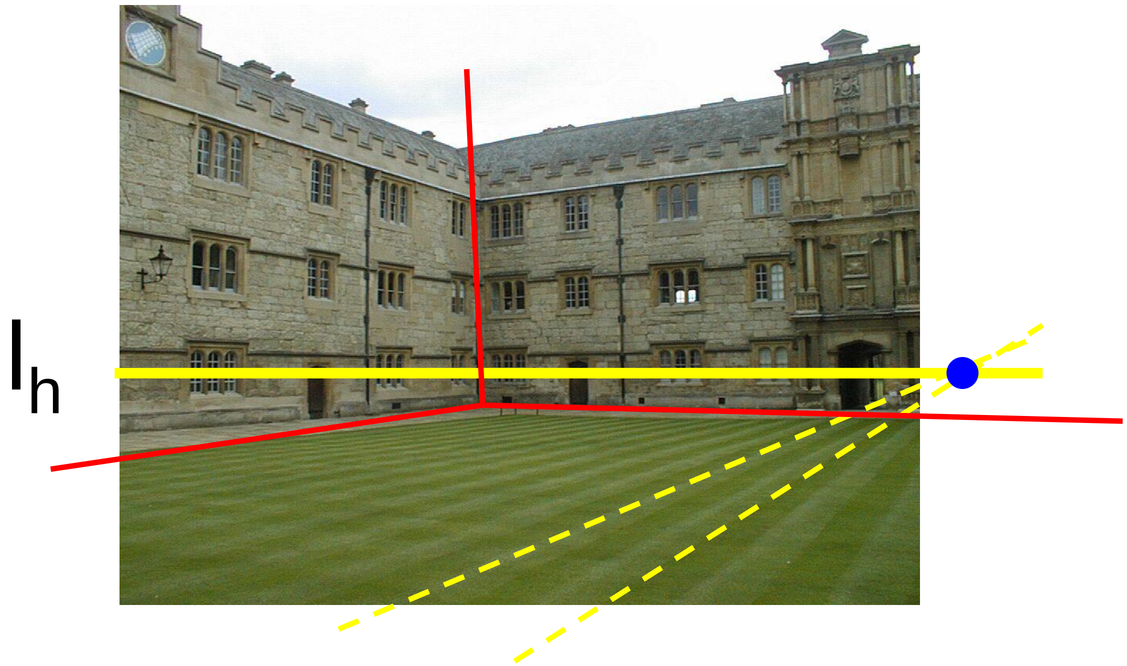
→ Compute ω:

Once ω is calculated, we get K:

$$\omega = (K\ K^T)^{-1} \rightarrow K$$

(Cholesky factorization; HZ pag 582)

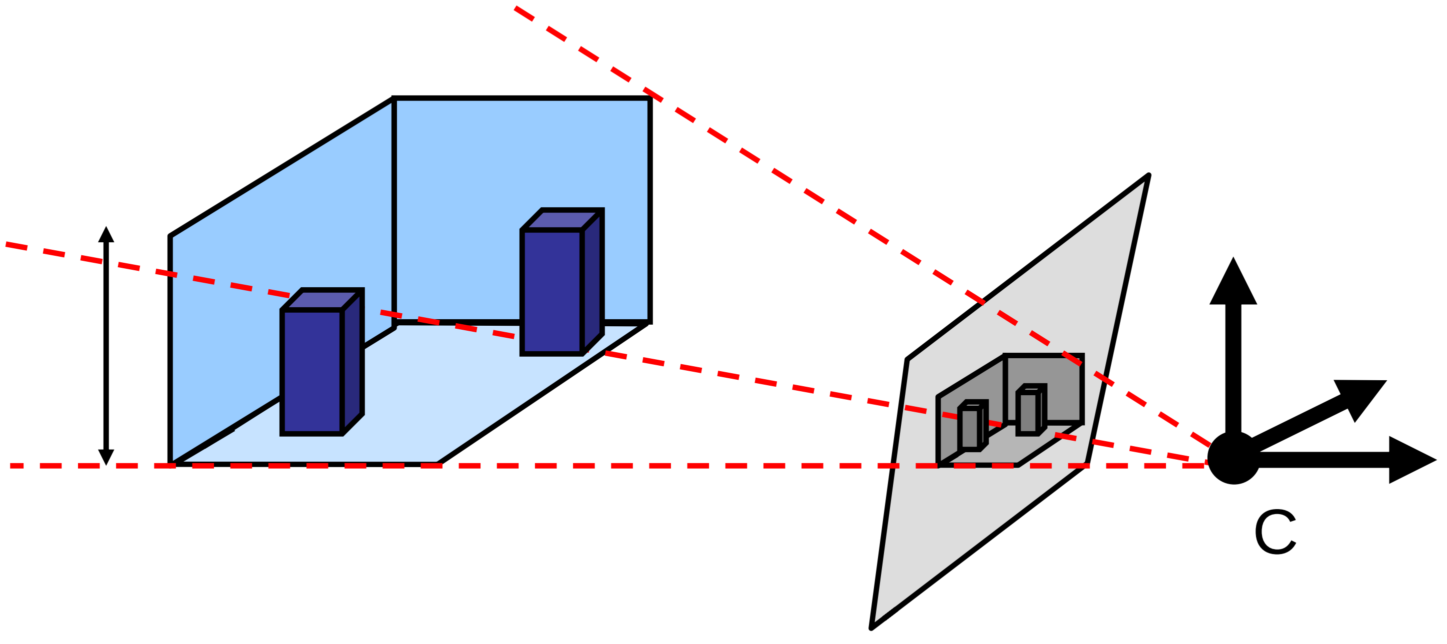
Single view reconstruction - example



K known $\rightarrow$ $\mathbf{n} = K^T \mathbf{l}_{horiz}$ = Scene plane orientation in the camera reference system

Select orientation discontinuities

Single view reconstruction - example



Recover the structure within the camera reference system

Notice: the actual scale of the scene is NOT recovered

- Recognition helps reconstruction!
- Humans have learnt this

Are these two lines parallel or not?
- Recognize the horizon line
- Measure if the 2 lines meet at the horizon
- if yes, these 2 lines are // in 3D

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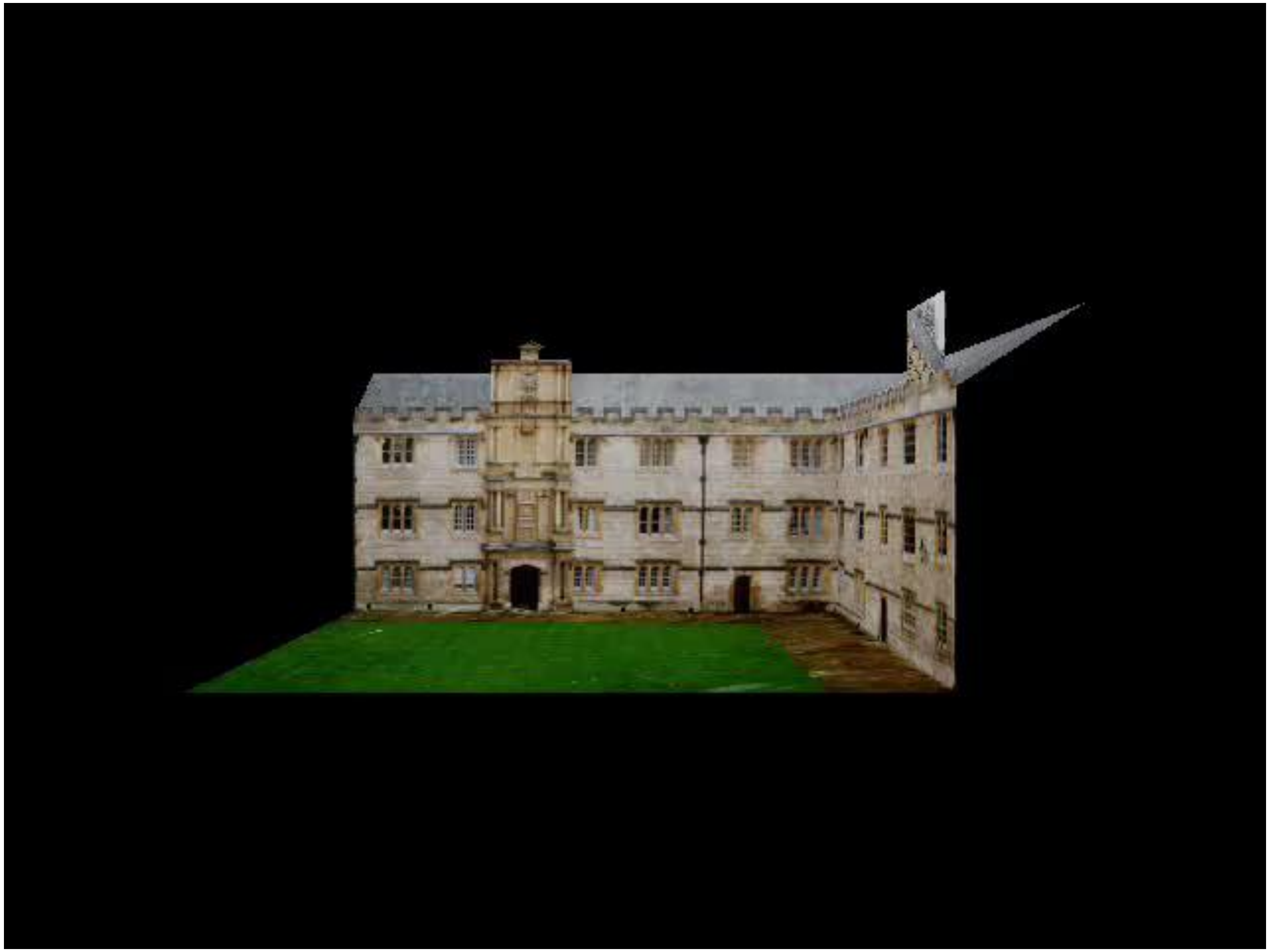
- Review calibration
- Vanishing points and lines
- Estimating geometry from a single image
- Extensions

Reading:
[HZ] Chapter 2 "Projective Geometry and Transformation in 3D"
[HZ] Chapter 3 "Projective Geometry and Transformation in 3D"
[HZ] Chapter 8 "More Single View Geometry"
[Hoeim & Savarese] Chapter 2

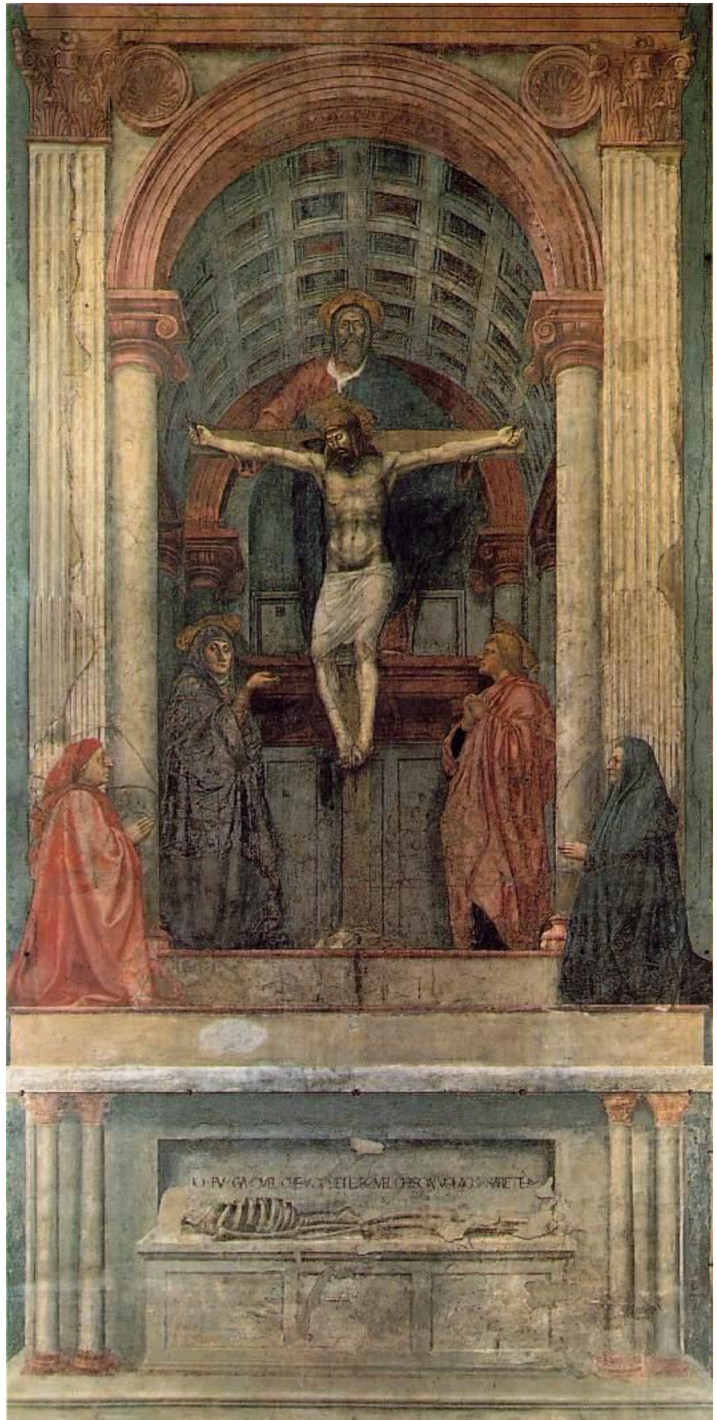
Criminisi & Zisserman, 99

http://www.robots.ox.ac.uk/~vgg/projects/SingleView/models/merton/merton.wrl

Criminisi & Zisserman, 99



http://www.robots.ox.ac.uk/~vgg/projects/SingleView/models/merton/merton.wrl

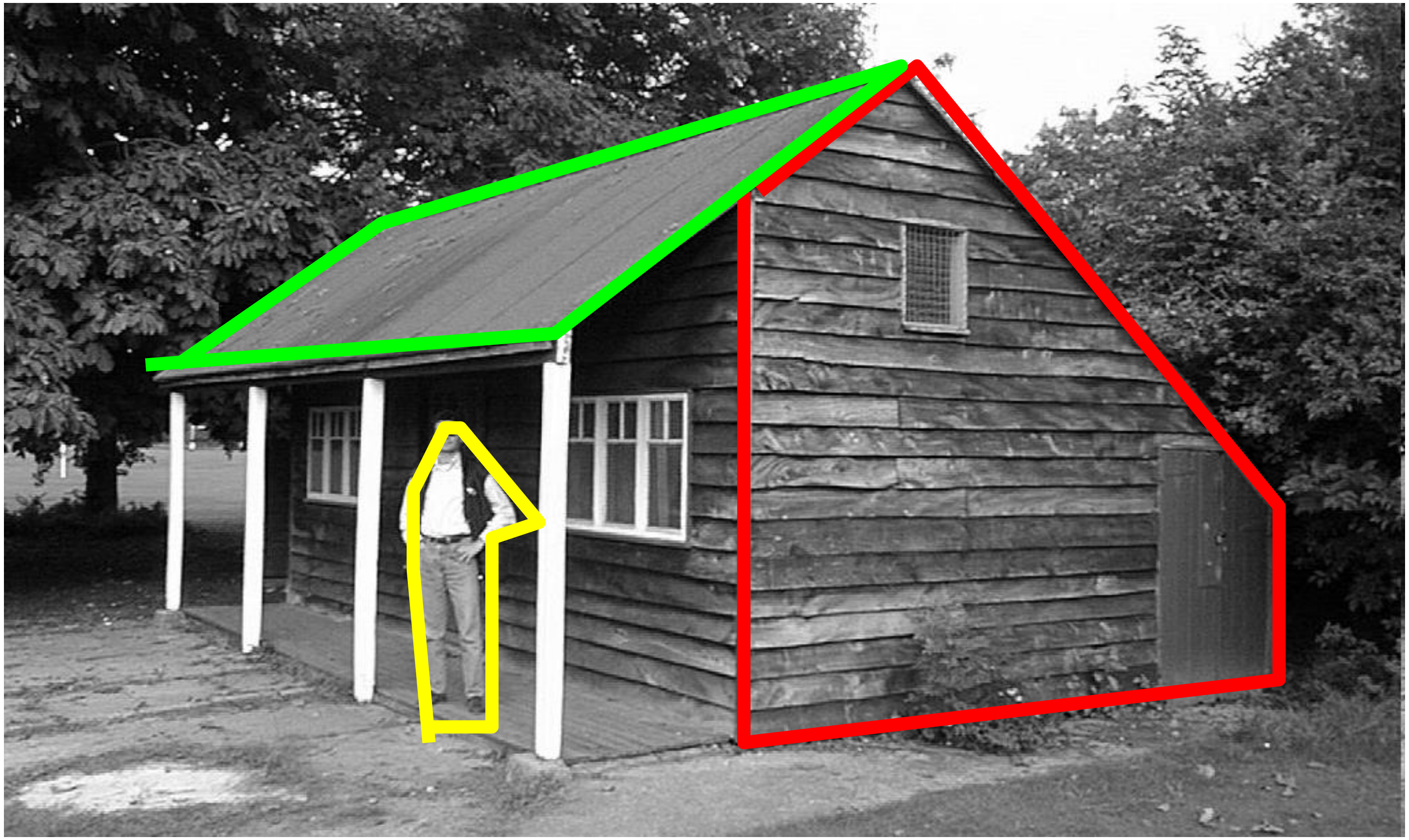


La Trinita' (1426)
Firenze, Santa Maria
Novella; by Masaccio
(1401-1428)

La Trinita' (1426)
Firenze, Santa Maria
Novella; by Masaccio
(1401-1428)

http://www.robots.ox.ac.uk/~vgg/projects/SingleView/models/hut/hutme.wrl

Single view reconstruction - drawbacks



Manually select:

- Vanishing points and lines;
- Planar surfaces;
- Occluding boundaries;
- Etc..

Automatic Photo Pop-up

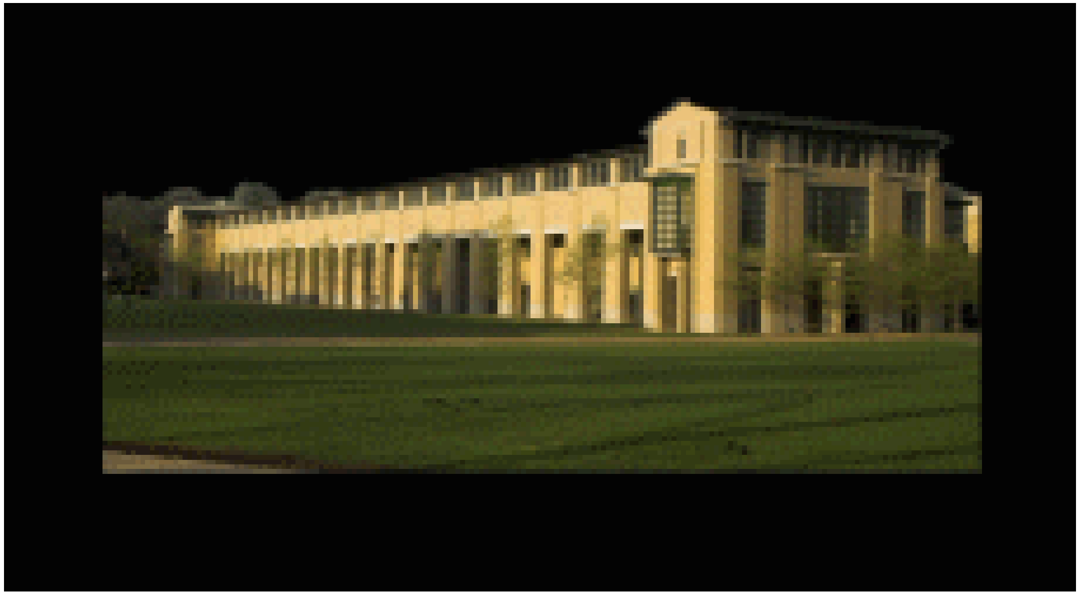
Hoiem et al, 05

Automatic Photo Pop-up

Hoiem et al, 05…

Automatic Photo Pop-up

Hoiem et al, 05…



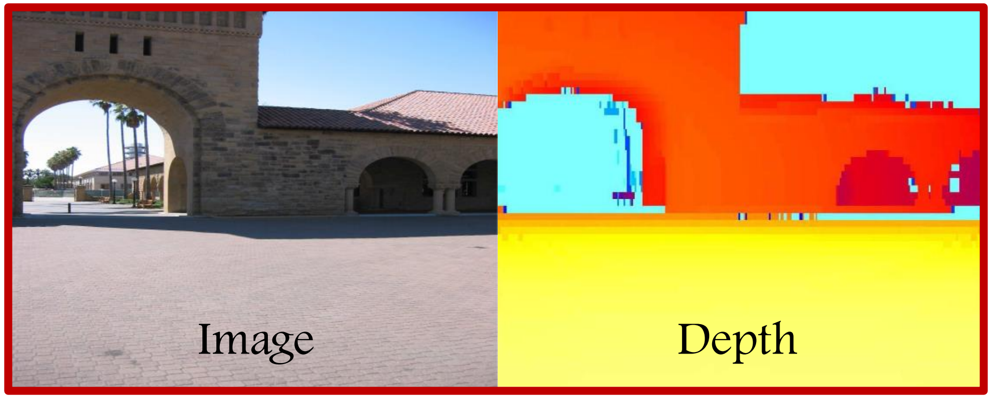
Software:

http://www.cs.uiuc.edu/homes/dhoiem/projects/software.html

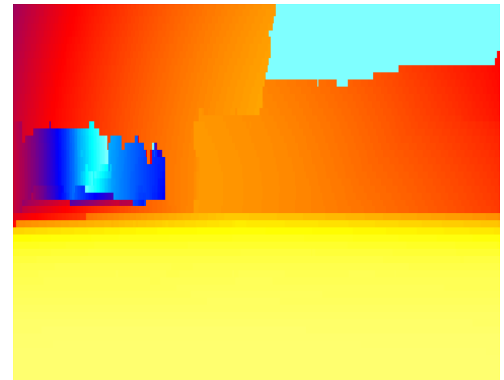
Make3D

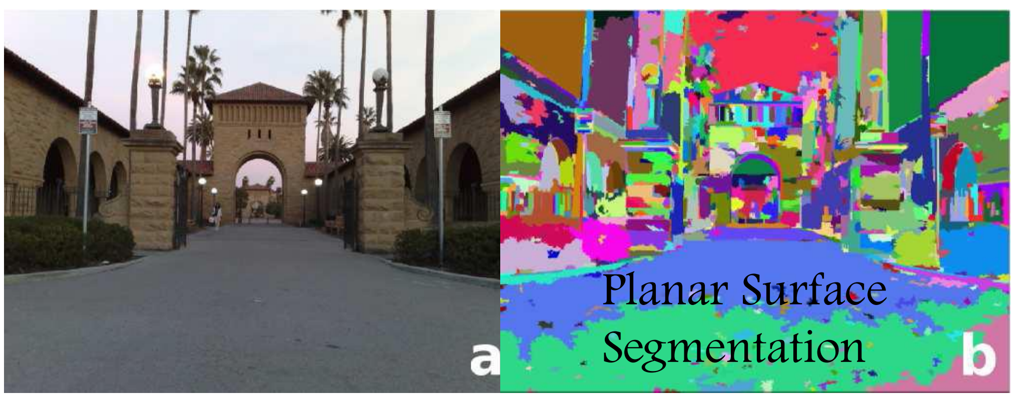
Saxena, Sun, Ng, 05…

Training



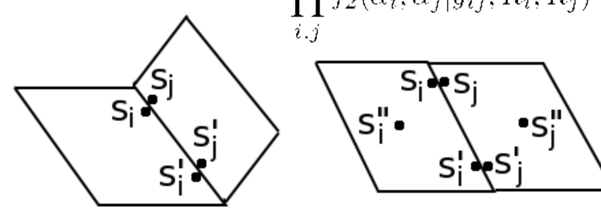
Prediction





Plane Parameter MRF

$$P(\alpha|X,\nu,y,R;\theta) = \frac{1}{Z}\prod_i f_1(\alpha_i|X_i,\nu_i,R_i;\theta) \prod_{i,j} f_2(\alpha_i,\alpha_j|y_{ij},R_i,R_j)$$



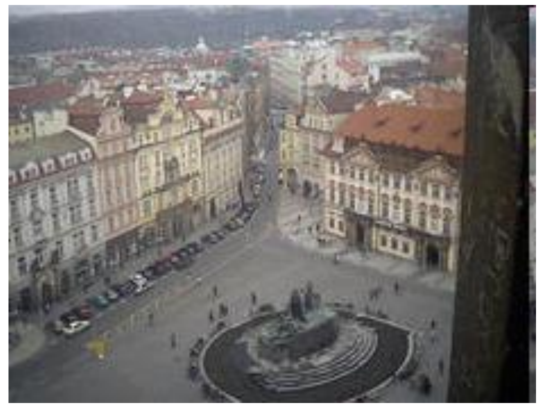
(a) Connectivity

(b) Co-Planarity

youtube

Single Image Depth Reconstruction

Saxena, Sun, Ng, 05…

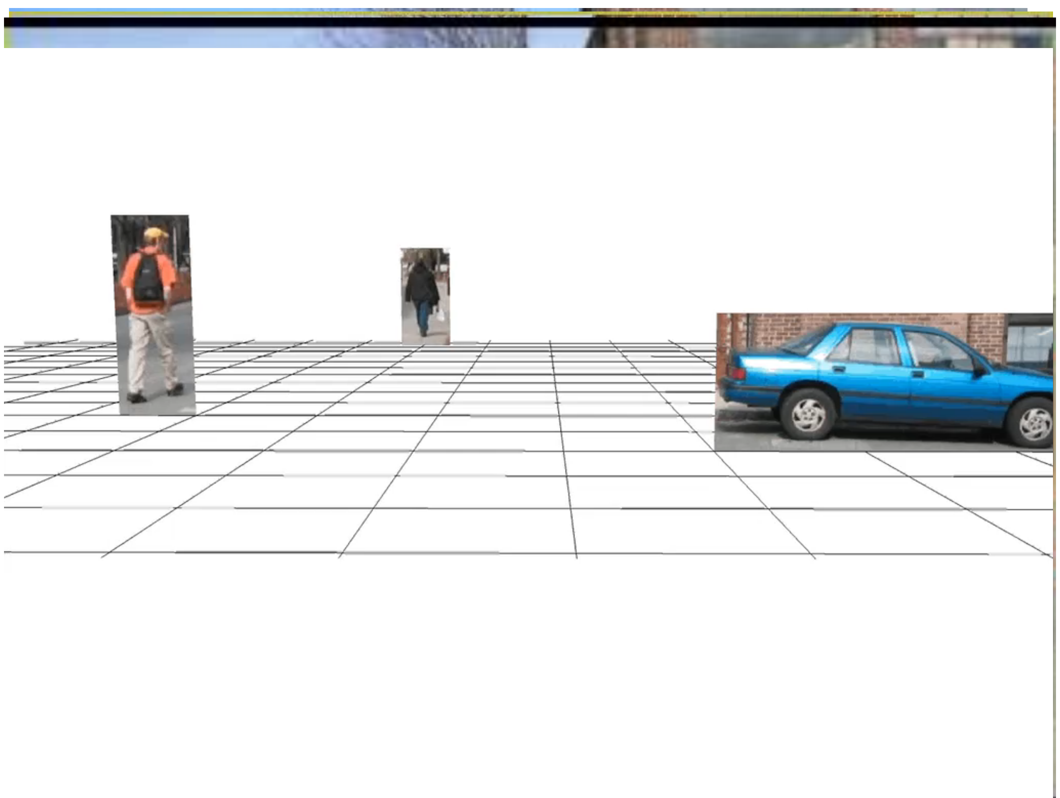
  

A software: **Make3D**
"Convert your image into 3d model" http://make3d.stanford.edu/

http://make3d.stanford.edu/images/view3D/185
http://make3d.stanford.edu/images/view3D/931?noforward=true
http://make3d.stanford.edu/images/view3D/108

Coherent object detection and scene layout estimation from a single image

Y. Bao, M. Sun, S. Savarese, CVPR 2010, BMVC 2010



Next lecture:

Multi-view geometry (epipolar geometry)