Lecture 3

Camera Calibration

Professor Silvio Savarese

Computational Vision and Geometry Lab

Lecture 3

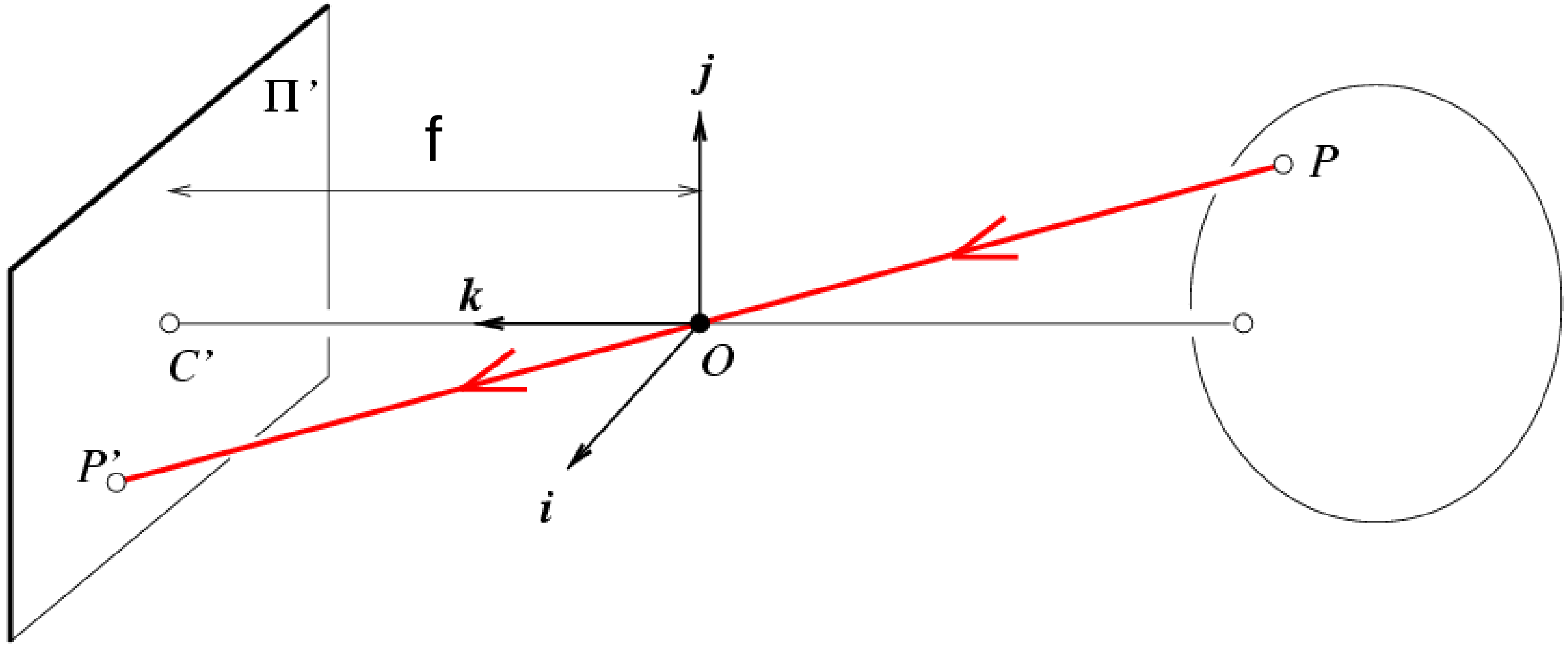
Camera Calibration

- Recap of camera models
- Camera calibration problem
- Camera calibration with radial distortion
- Example

Reading: [FP] Chapter 3 “Geometric Camera Calibration”
[HZ] Chapter 7 “Computation of Camera Matrix P”

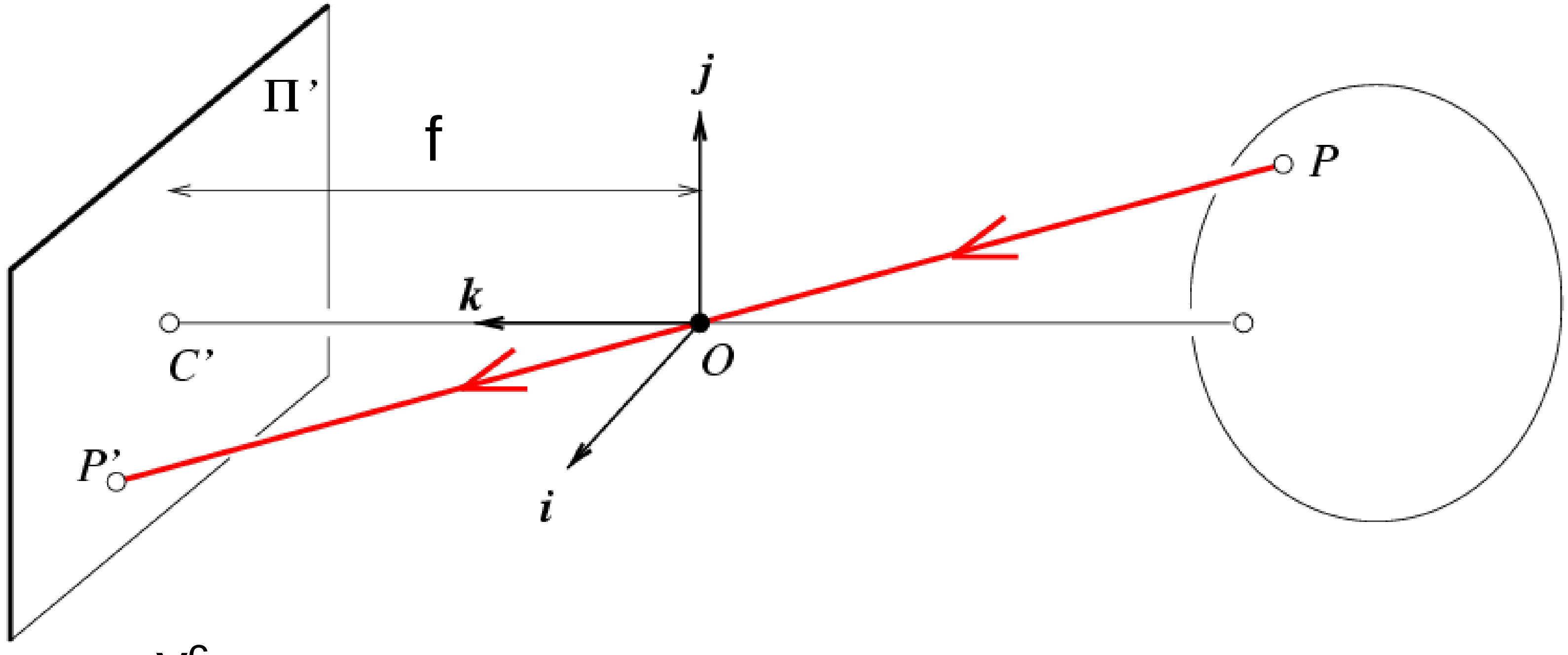
Some slides in this lecture are courtesy to Profs. J. Ponce, F-F Li

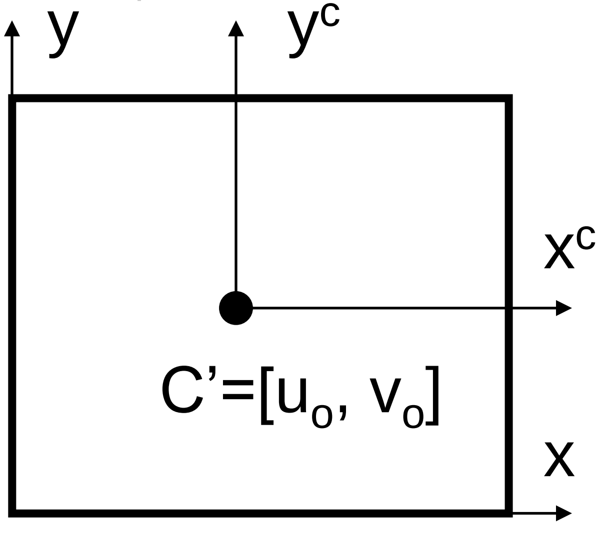
Projective camera



f = focal length

Projective camera

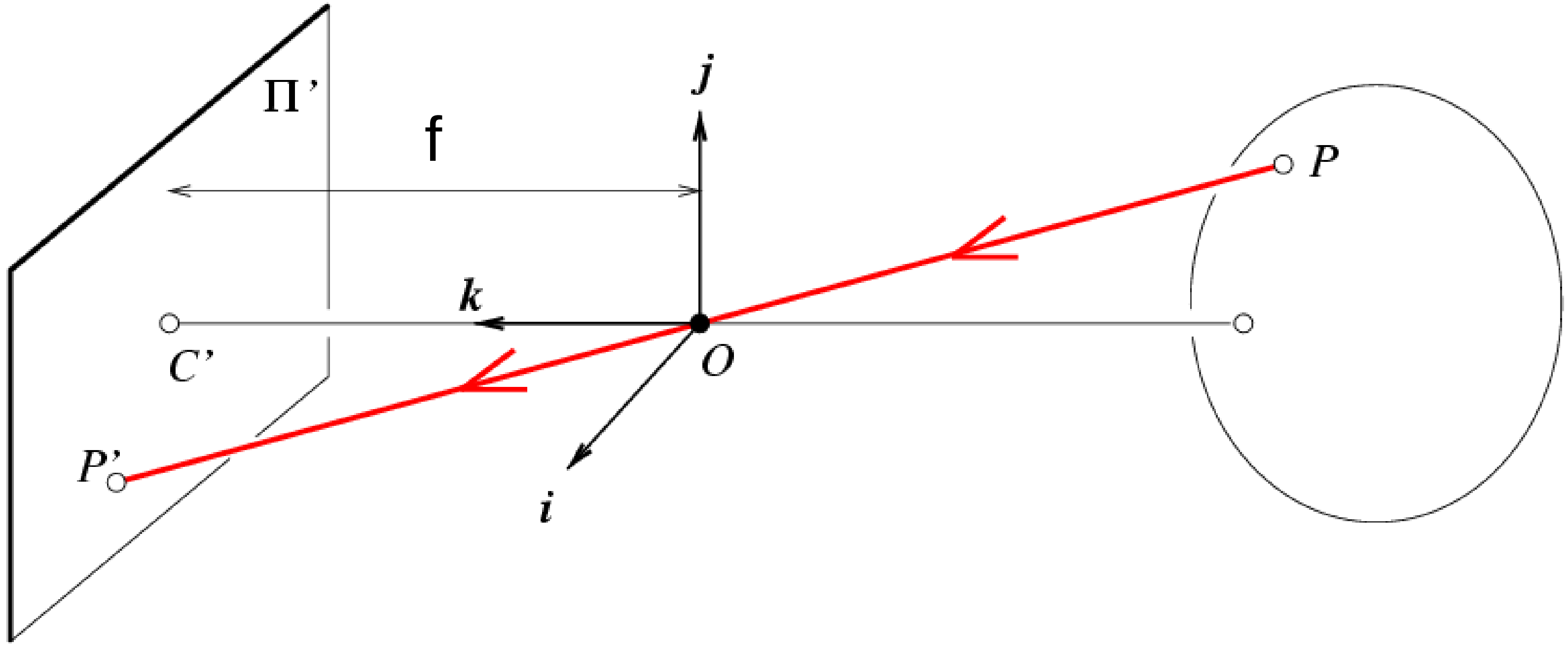




f = focal length

u_o, v_o = offset

Projective camera



Units: k,l [pixel/m]

f [m]

Non-square pixels

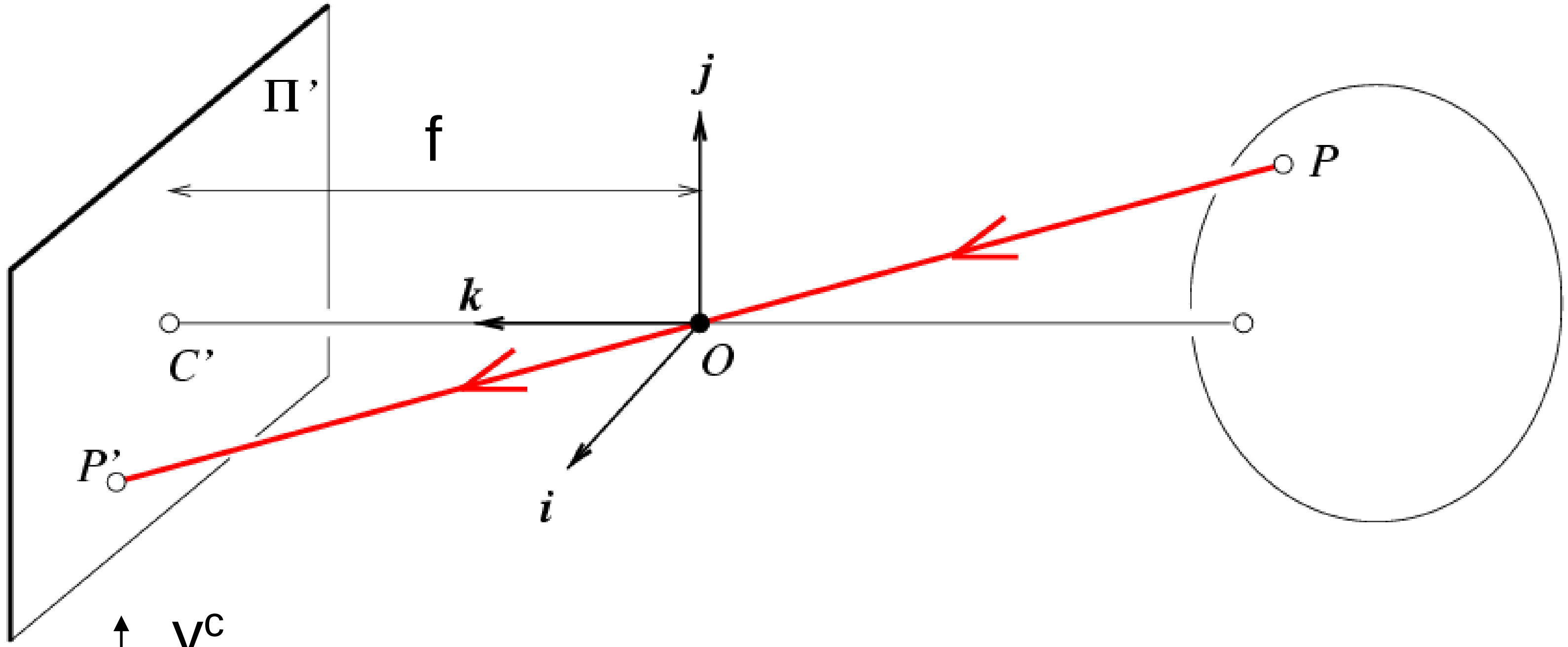
α, β [pixel]

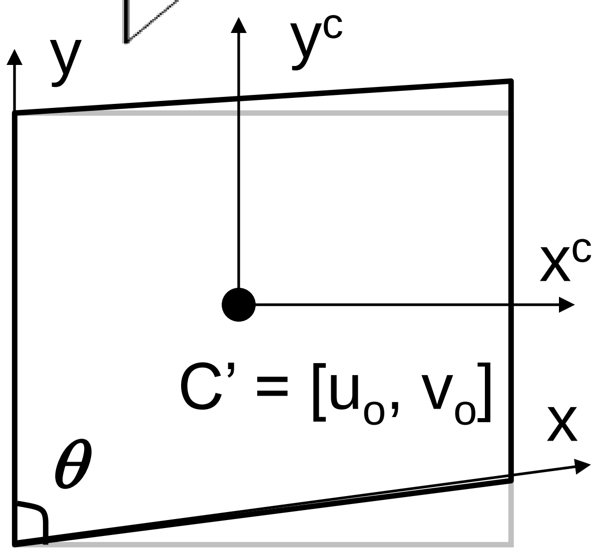
f = focal length

u_o, v_o = offset

$\alpha, \beta \rightarrow$ non-square pixels

Projective camera





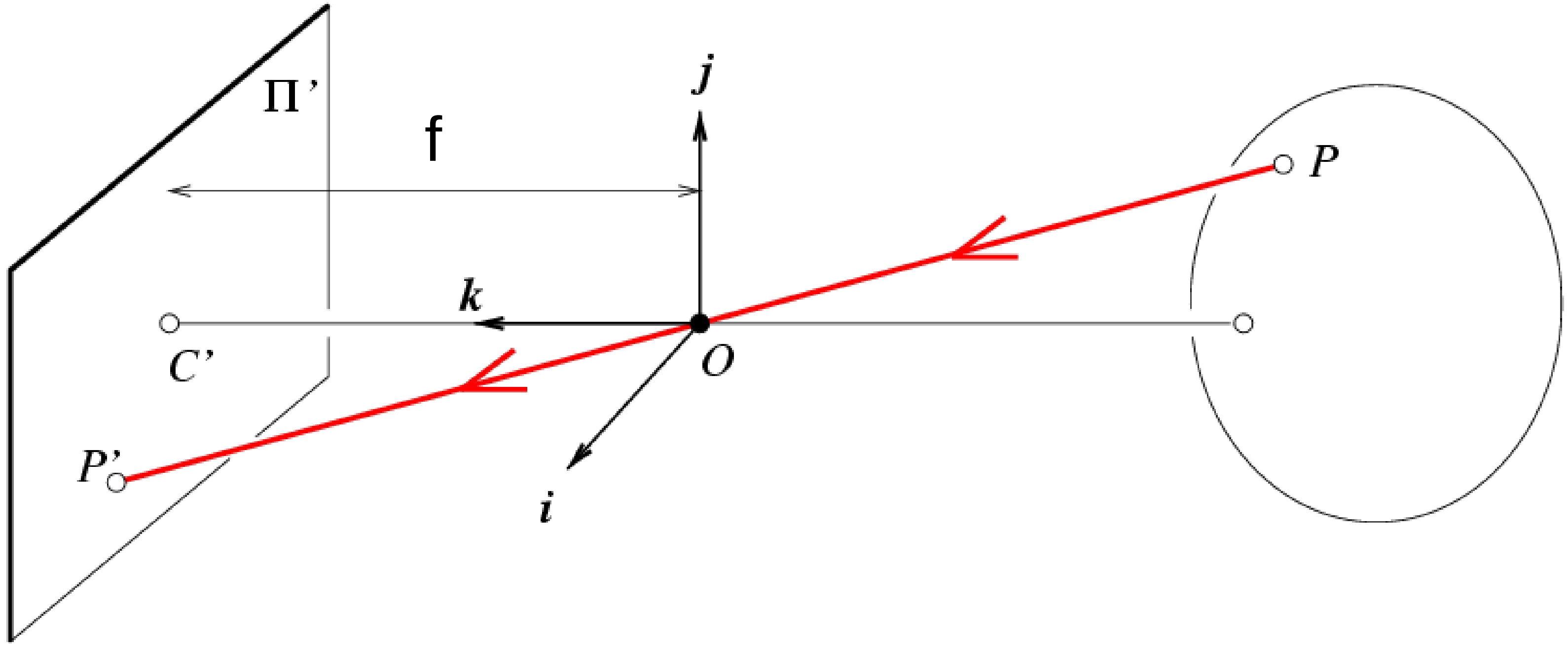
f = focal length

u_o, v_o = offset

$\alpha, \beta \rightarrow$ non-square pixels

θ = skew angle

Projective camera



$$P' = \begin{bmatrix} \alpha & -\alpha \cot\theta & u_o & 0 \\ 0 & \dfrac{\beta}{\sin\theta} & v_o & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

f = focal length

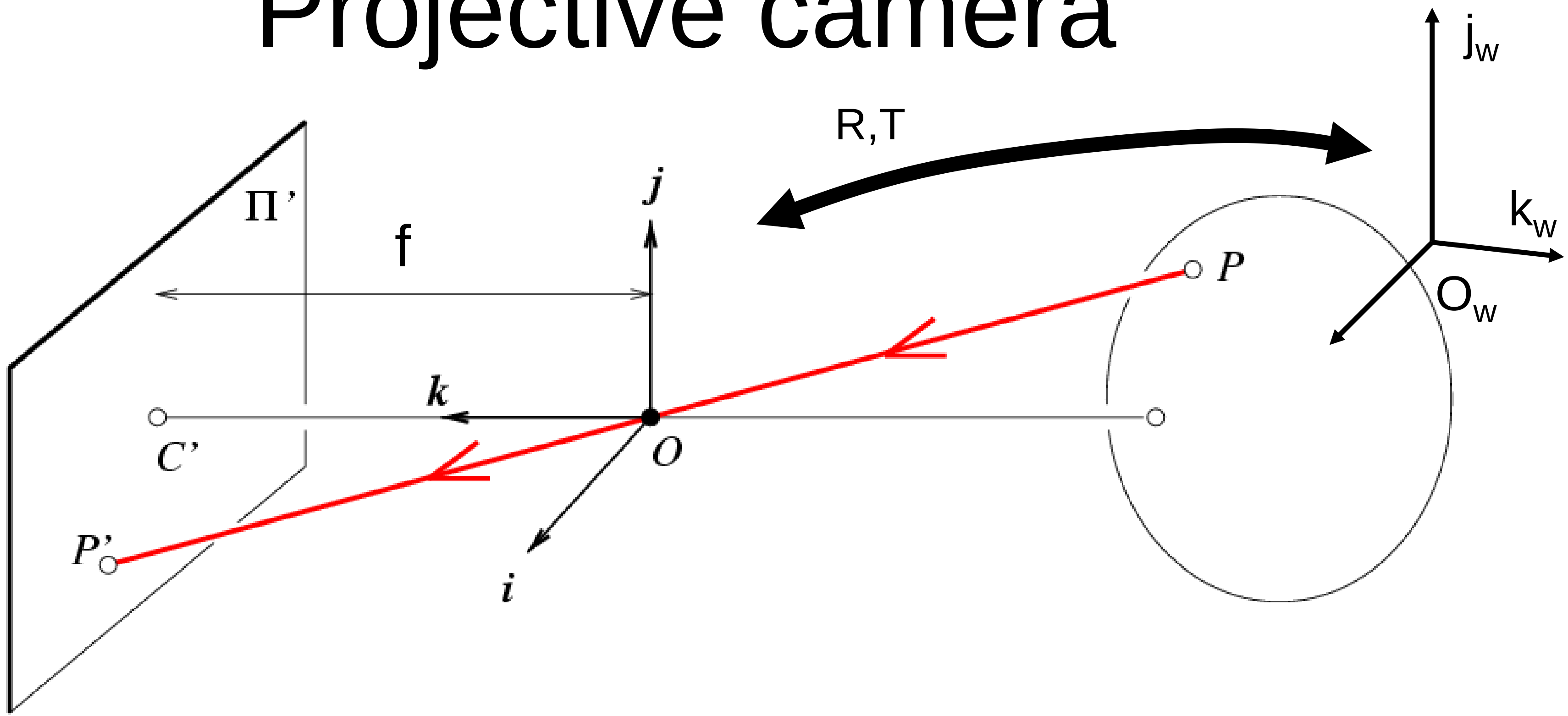
u_o, v_o = offset

$\alpha, \beta \rightarrow$ non-square pixels

θ = skew angle

K has 5 degrees of freedom!

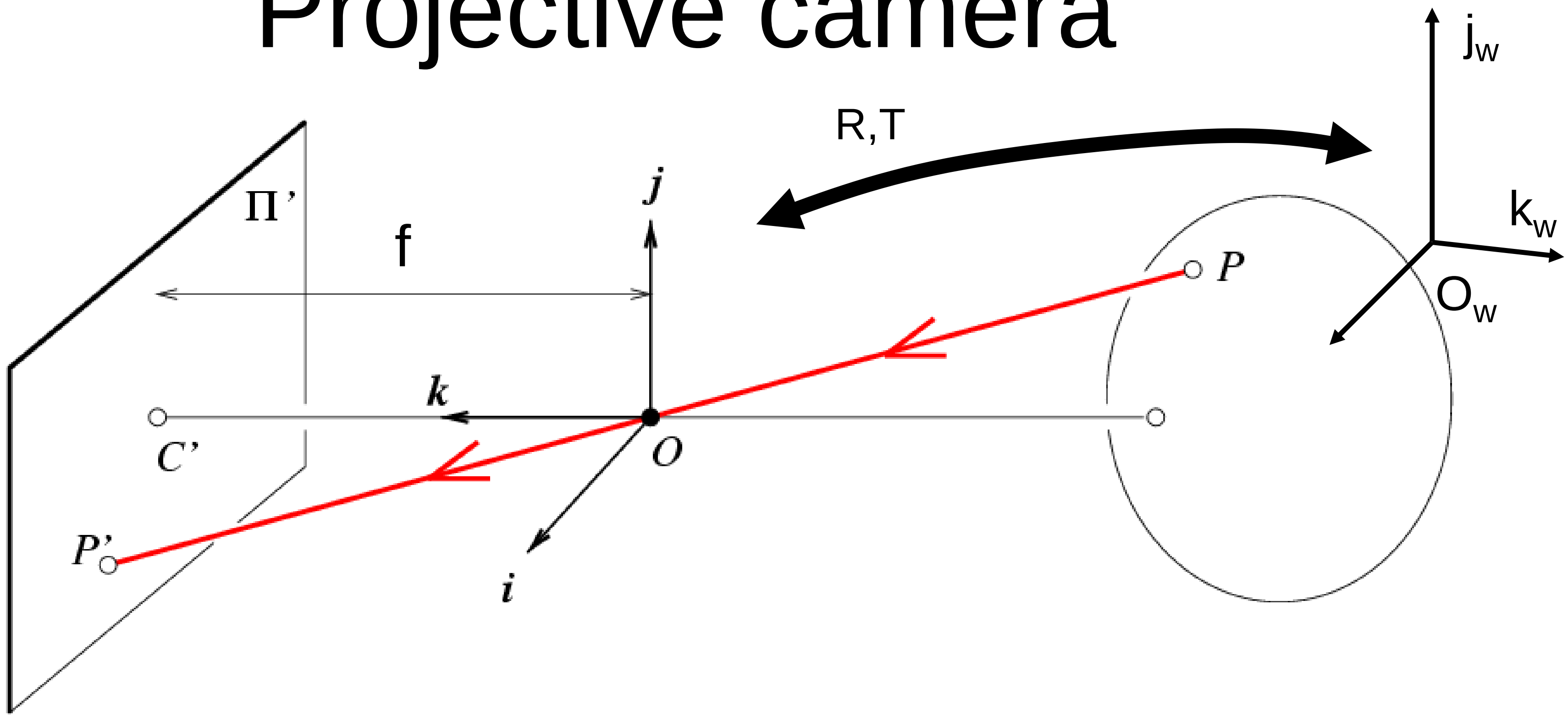
Projective camera



$$P = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}_{4\times 4} P_w$$

f = focal length
u_o, v_o = offset
$\alpha, \beta \rightarrow$ non-square pixels
θ = skew angle
R,T = rotation, translation

Projective camera



$$P' = M \, P_w$$

$$= K \begin{bmatrix} R & T \end{bmatrix} P_w$$

Internal parameters

External parameters

f = focal length

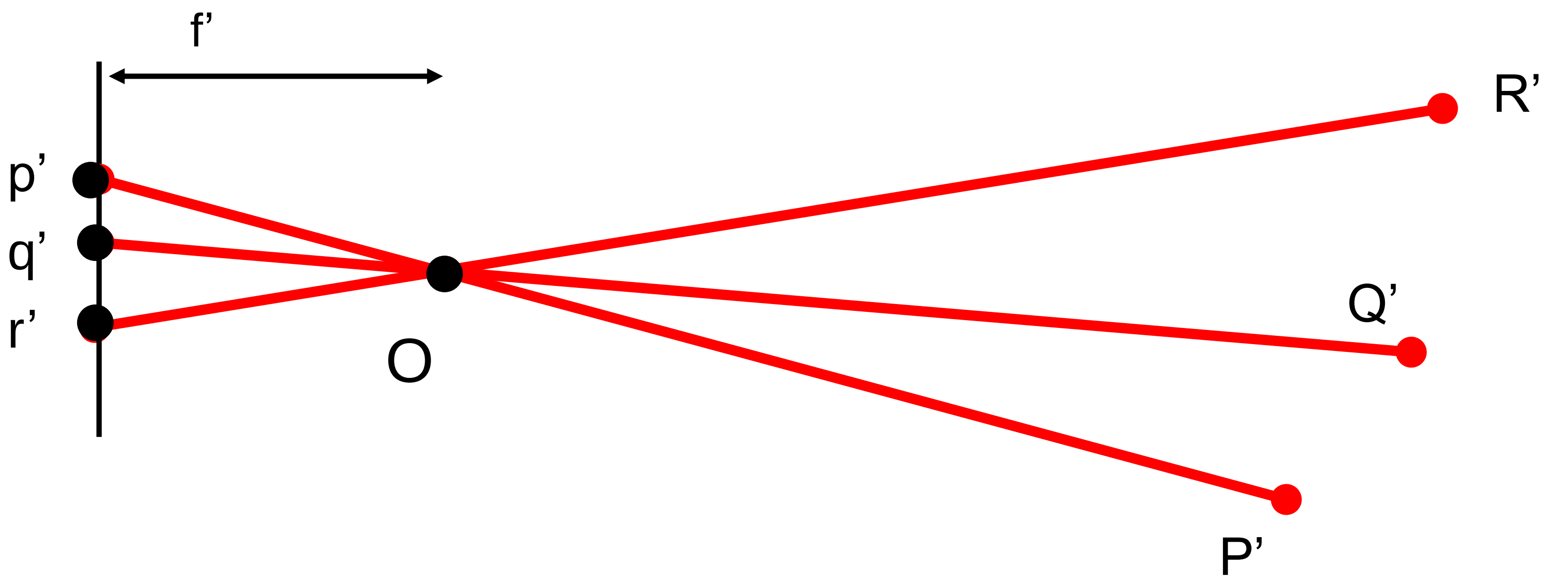
u_o, v_o = offset

$\alpha, \beta \rightarrow$ non-square pixels

θ = skew angle

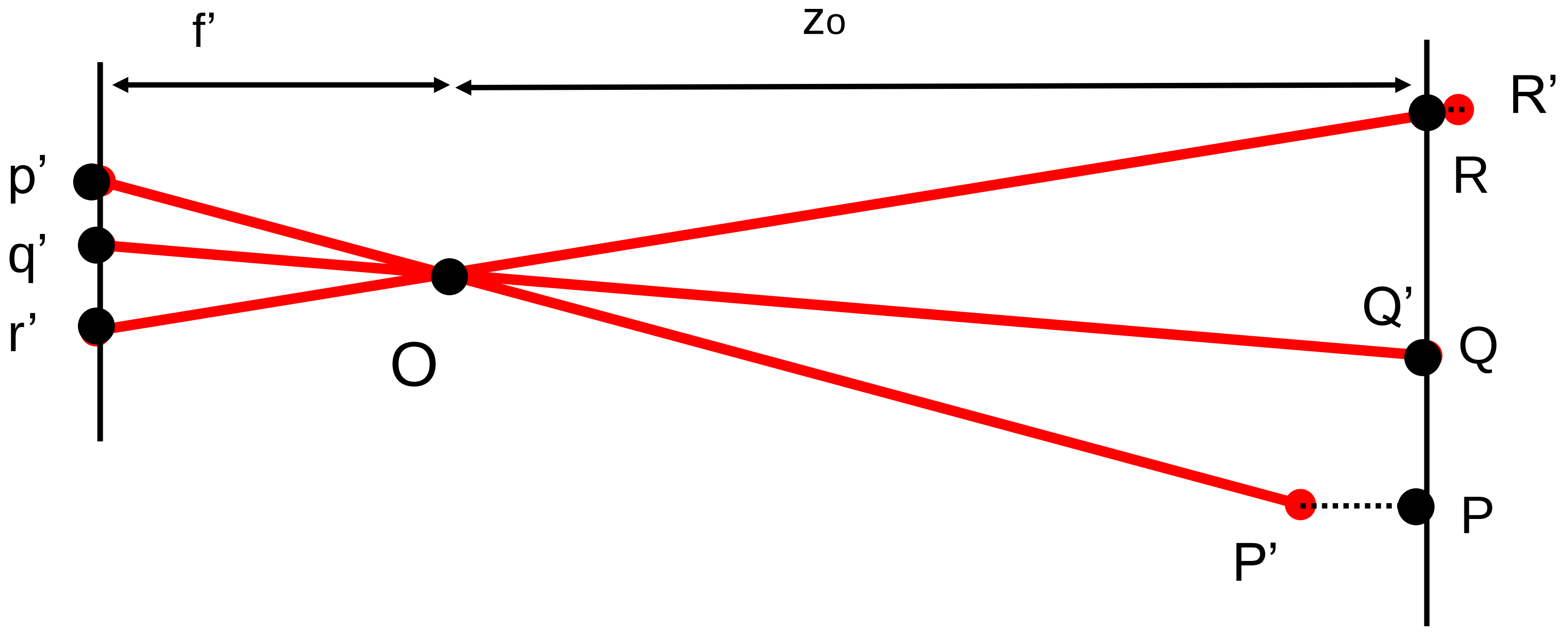
R,T = rotation, translation

Projective camera



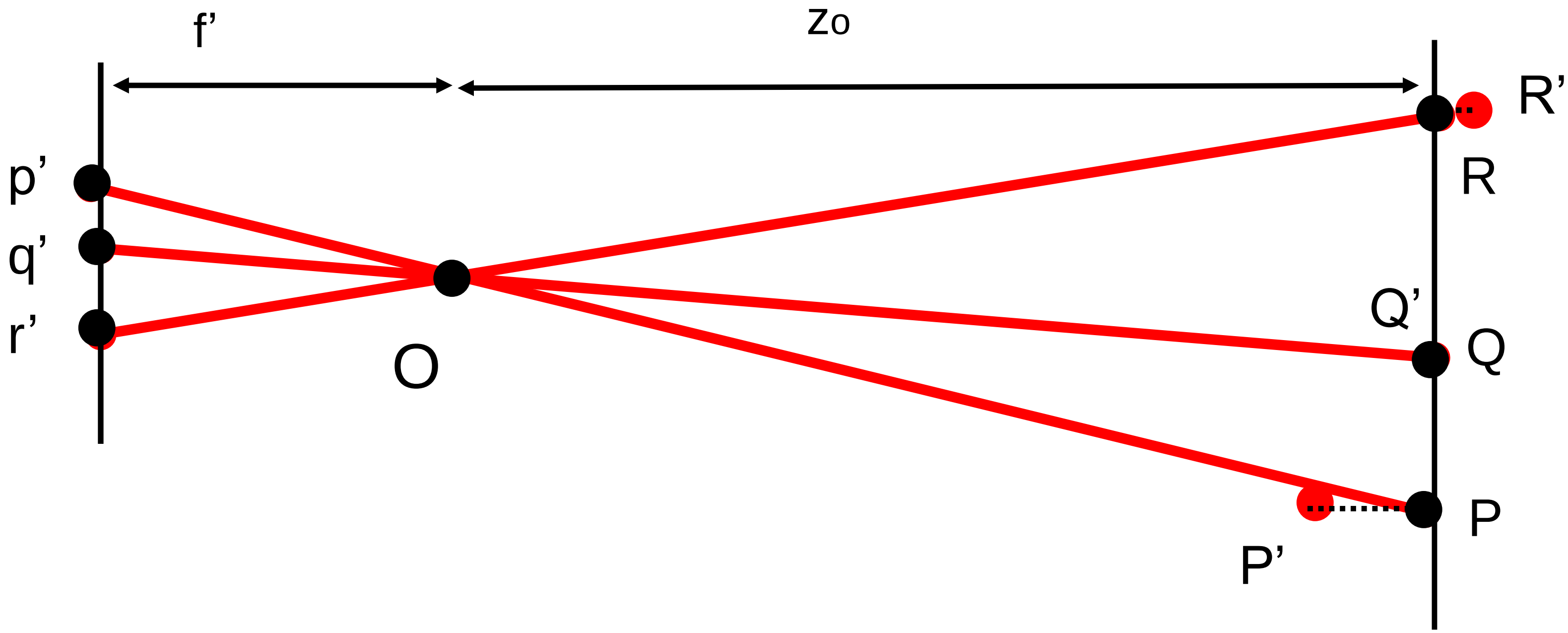
Weak perspective projection

When the relative scene depth is small compared to its distance from the camera



Weak perspective projection

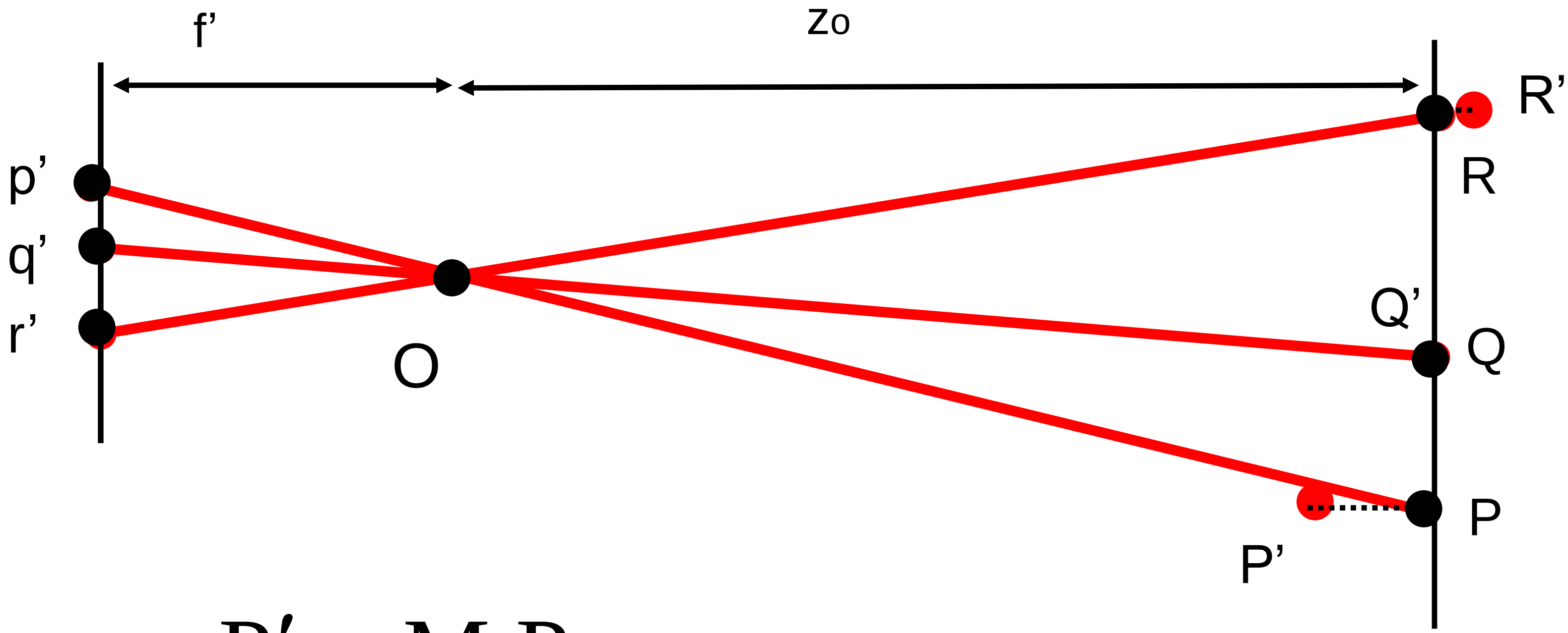
When the relative scene depth is small compared to its distance from the camera



$$\begin{cases} x' = -\dfrac{f'}{z}x \\ y' = -\dfrac{f'}{z}y \end{cases} \rightarrow \begin{cases} x' = -\boxed{\dfrac{f'}{z_0}}x \\ y' = -\dfrac{f'}{z_0}y \end{cases}$$

Magnification m

Weak perspective projection



$$P' = M\ P_w$$

$$M = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$$

Instead of

$$M = K\begin{bmatrix} R & T \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{v} & \mathbf{1} \end{bmatrix}$$

$$P' = M\,P_w = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} P_W = \begin{bmatrix} \mathbf{m}_1 P_W \\ \mathbf{m}_2 P_W \\ \mathbf{m}_3 P_W \end{bmatrix} \qquad M = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{v} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

$$\overset{E}{\rightarrow} \left(\frac{\mathbf{m}_1 P_w}{\mathbf{m}_3 P_w}, \frac{\mathbf{m}_2 P_w}{\mathbf{m}_3 P_w}\right)$$

Perspective

$$P' = M\,P_w = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} P_W = \begin{bmatrix} \mathbf{m}_1 P_W \\ \mathbf{m}_2 P_W \\ 1 \end{bmatrix} \qquad M = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} = \begin{bmatrix} & \mathbf{m}_1 & & \\ & \mathbf{m}_2 & & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

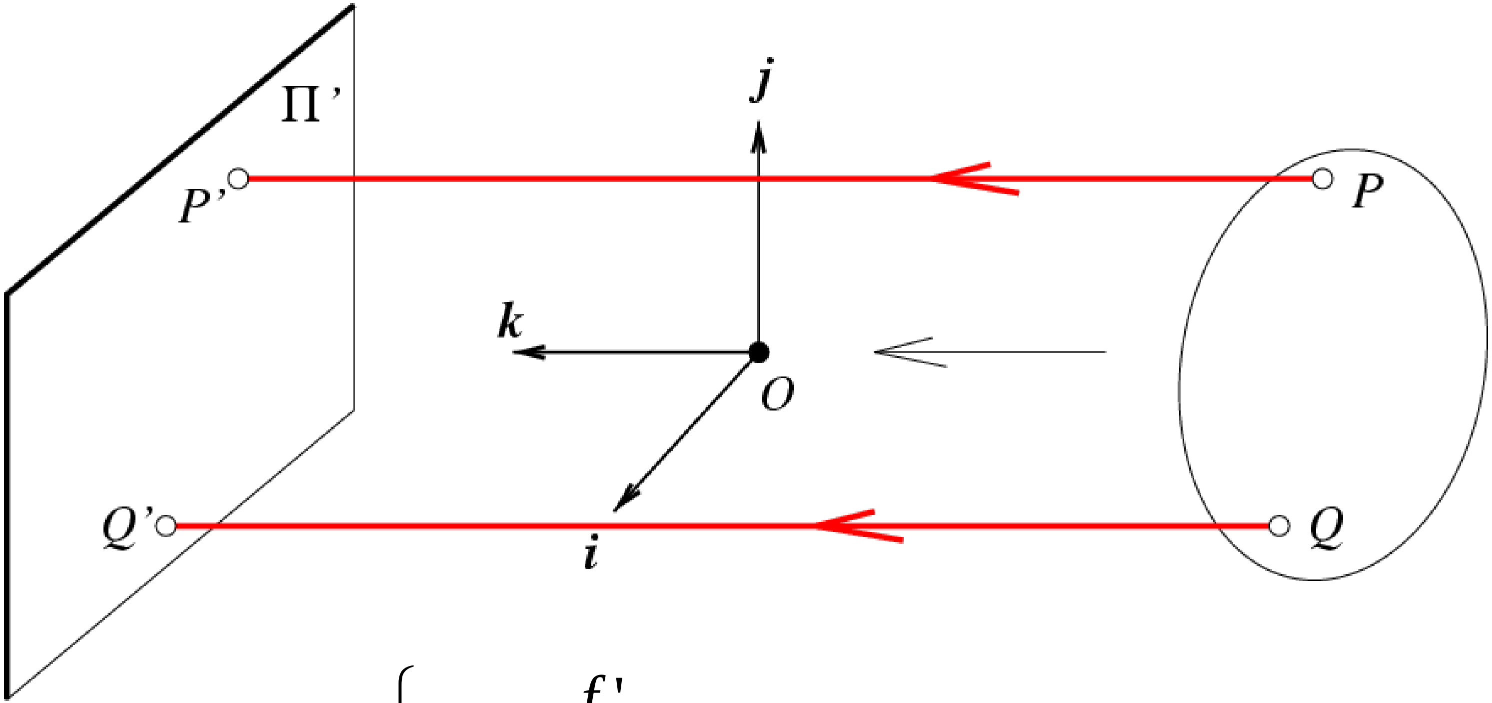
$$\overset{E}{\rightarrow} (\mathbf{m}_1 P_w, \mathbf{m}_2 P_w)$$

magnification

Weak perspective

Orthographic (affine) projection

Distance from center of projection to image plane is infinite



$$\begin{cases} x' = -\dfrac{f'}{z}x \\ y' = -\dfrac{f'}{z}y \end{cases} \rightarrow \begin{cases} x' = -x \\ y' = -y \end{cases}$$

Pros and Cons of These Models

- Weak perspective much simpler math.
  - Accurate when object is small and distant.
  - Most useful for recognition.

- Pinhole perspective much more accurate for scenes.
  - Used in structure from motion.

Weak perspective projection

The Kangxi Emperor's Southern Inspection Tour (1691-1698) By Wang Hui

Weak perspective projection

The Kangxi Emperor's Southern Inspection Tour (1691-1698) By Wang Hui

Lecture 3
Camera Calibration

- Recap of camera models
- Camera calibration problem
- Camera calibration with radial distortion
- Example

Reading: [FP] Chapter 3
[HZ] Chapter 7

Some slides in this lecture are courtesy to Profs. J. Ponce, F-F Li

Projective camera

$$P' = M\,P_w = K\begin{bmatrix} R & T \end{bmatrix} P_w$$

Internal parameters

External parameters

$$\mathcal{M} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha\cot\theta\,\boldsymbol{r}_2^T + u_0\boldsymbol{r}_3^T & \alpha t_x - \alpha\cot\theta\, t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta}\boldsymbol{r}_2^T + v_0\boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta}t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix}_{3\times 4}$$

$$K = \begin{bmatrix} \boldsymbol{\alpha} & -\boldsymbol{\alpha}\cot\boldsymbol{\theta} & u_o \\ 0 & \dfrac{\boldsymbol{\beta}}{\sin\boldsymbol{\theta}} & v_o \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix} \qquad T = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Goal of calibration

Estimate intrinsic and extrinsic parameters from 1 or multiple images

$$P' = M\, P_w = K\begin{bmatrix} R & T \end{bmatrix} P_w$$

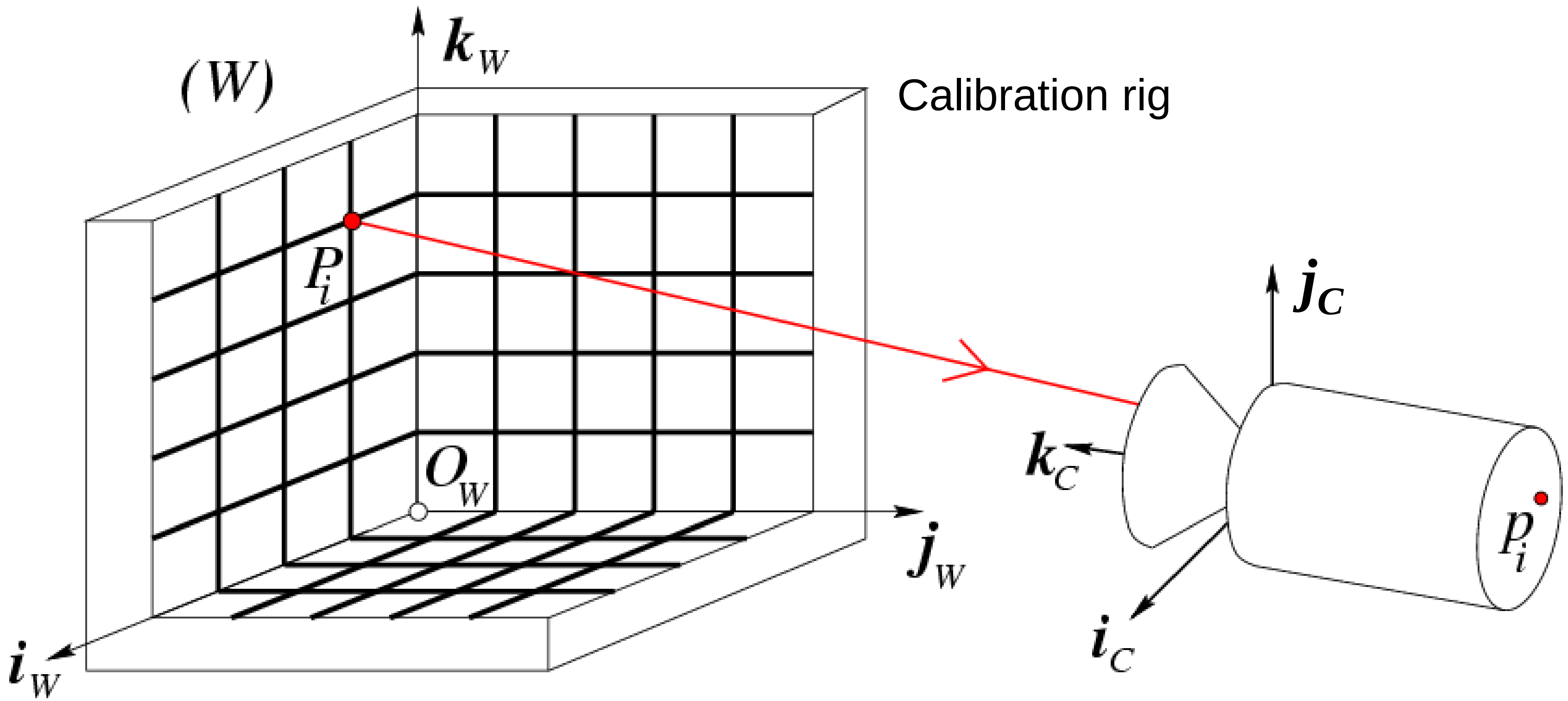
$$\mathcal{M} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \frac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \frac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix}_{3\times 4}$$

$$K = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_o \\ 0 & \frac{\beta}{\sin\theta} & v_o \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix} \qquad T = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Change notation:
$P = P_w$

$p = P'$

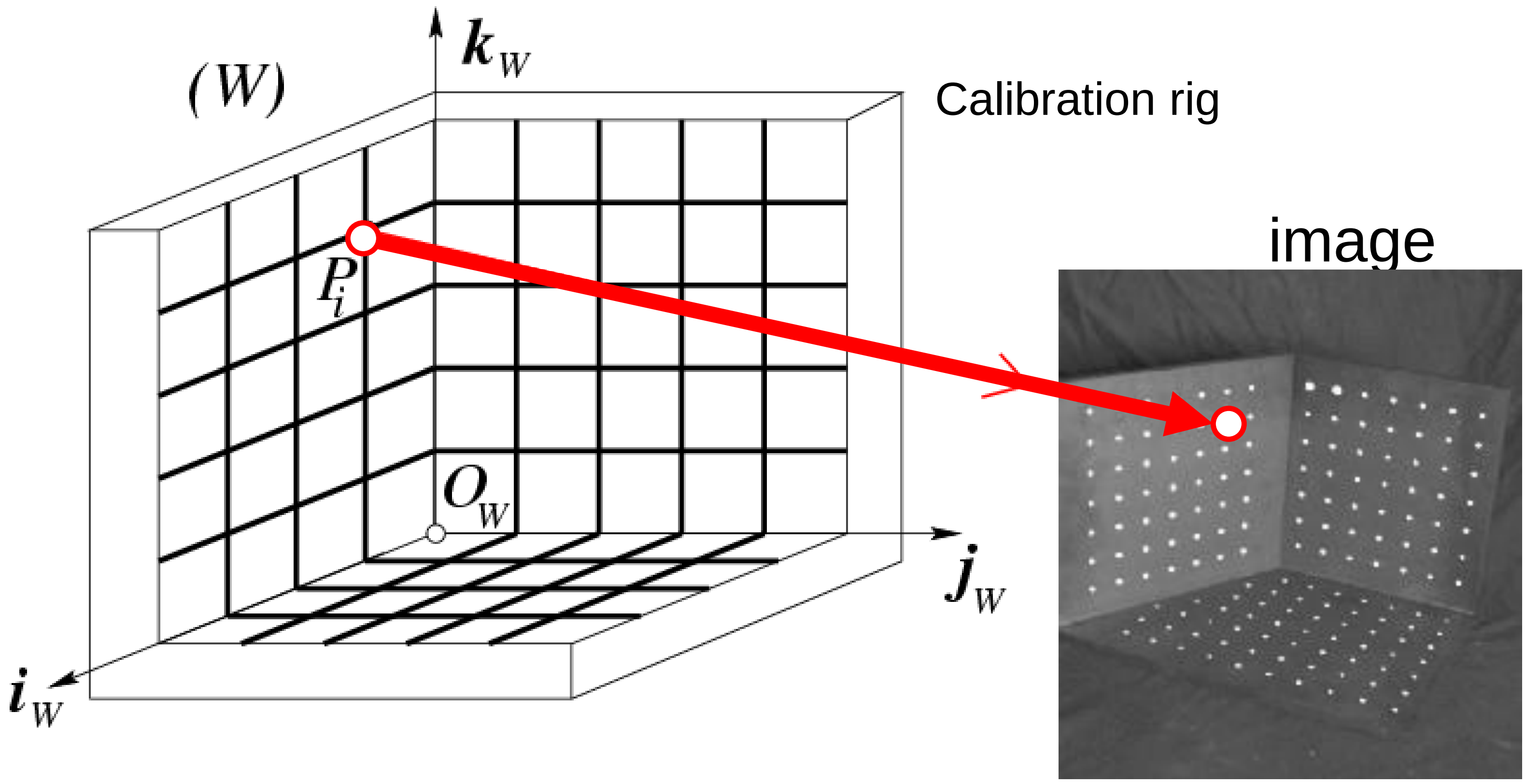
Calibration Problem



- $P_1 \ldots P_n$ with known positions in $[O_w, i_w, j_w, k_w]$
- $p_1, \ldots p_n$ known positions in the image

Goal: compute intrinsic and extrinsic parameters

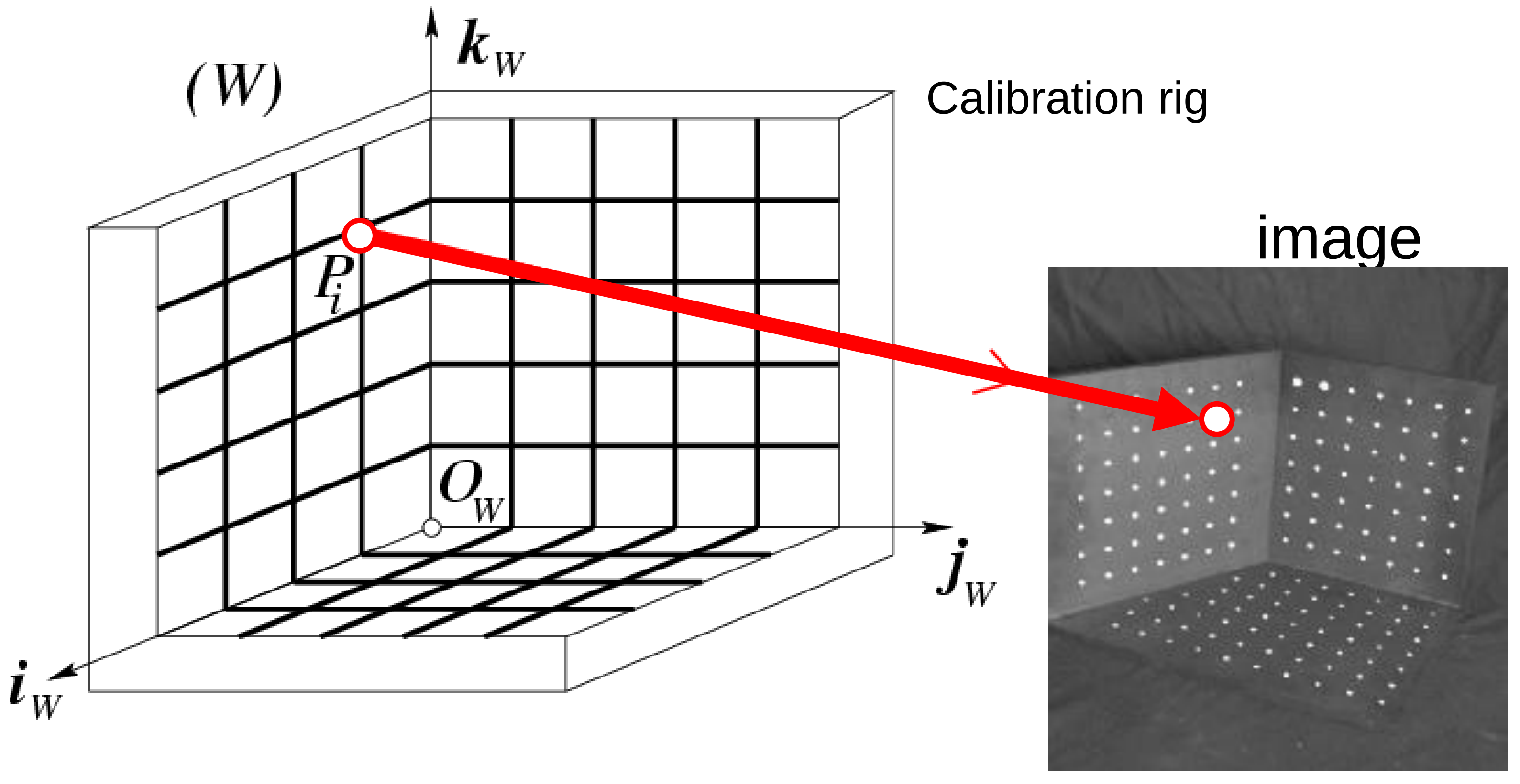
Calibration Problem



- $P_1 \ldots P_n$ with known positions in $[O_w, i_w, j_w, k_w]$
- $p_1, \ldots p_n$ known positions in the image

Goal: compute intrinsic and extrinsic parameters

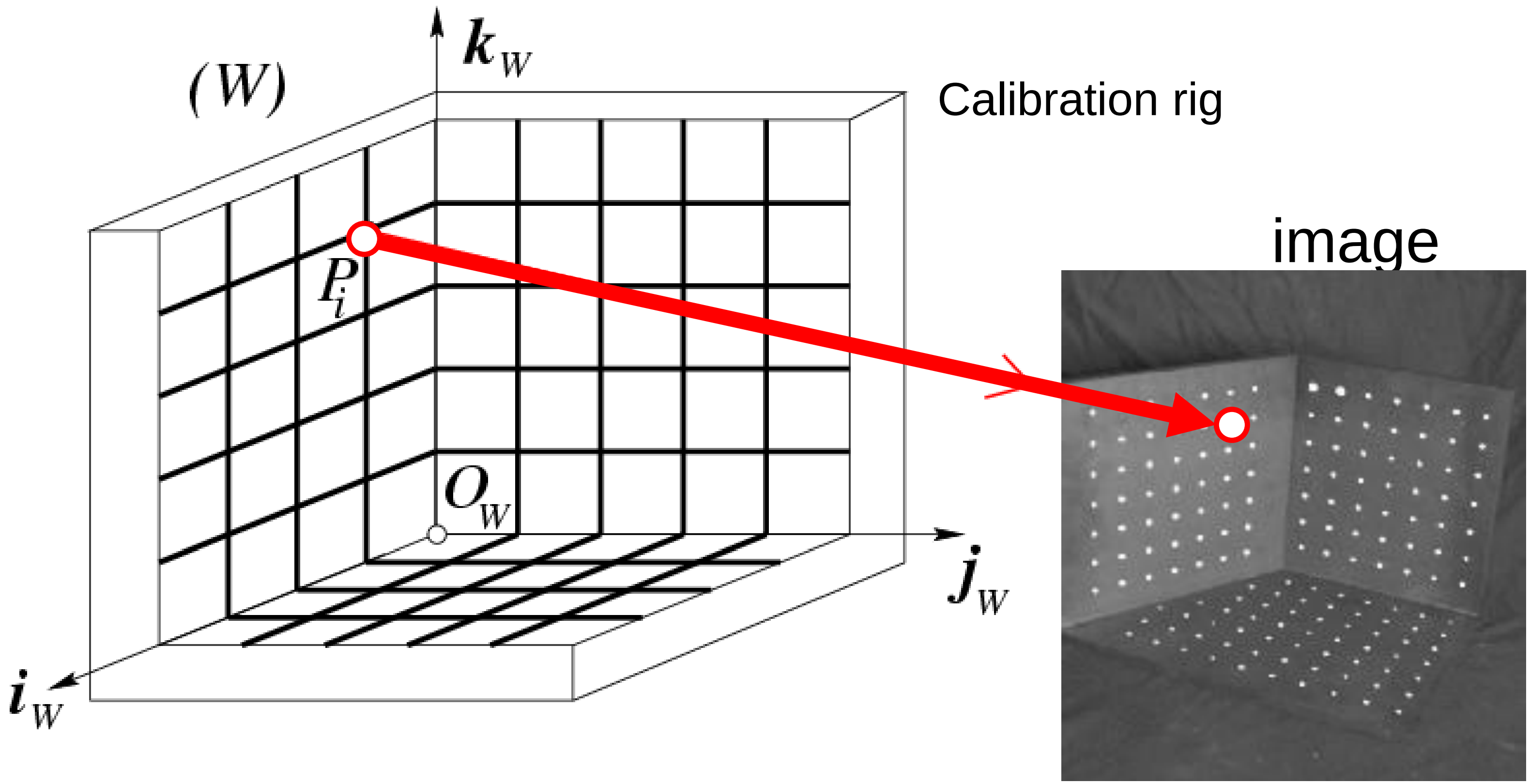
Calibration Problem



How many correspondences do we need?

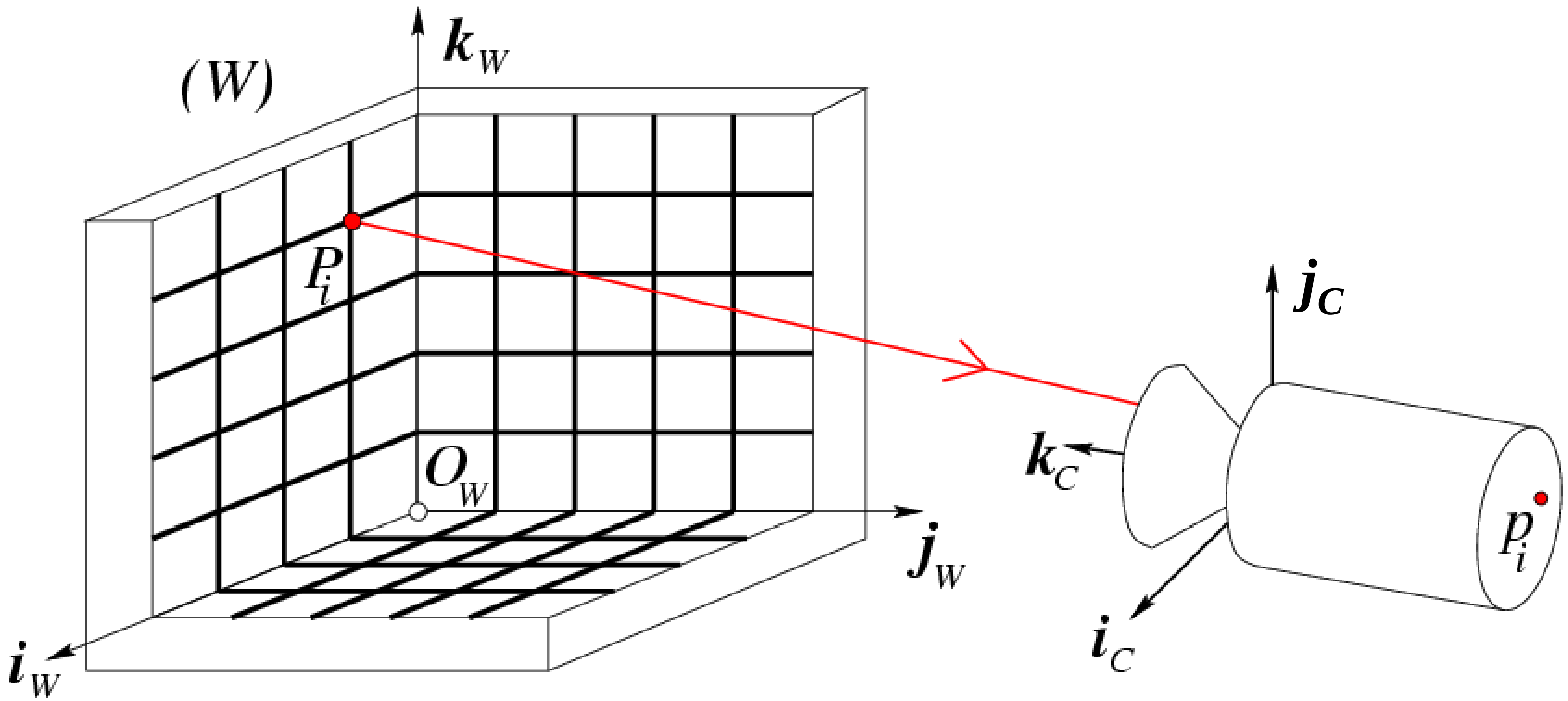
- M has 11 unknown
- We need 11 equations
- 6 correspondences would do it

Calibration Problem



In practice, using more than 6 correspondences enables more robust results

Calibration Problem



$$P_i \rightarrow M\,P_i \rightarrow p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{m}_1\,P_i}{\mathbf{m}_3\,P_i} \\ \dfrac{\mathbf{m}_2\,P_i}{\mathbf{m}_3\,P_i} \end{bmatrix} \qquad M = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

in pixels

Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{m}_1 \, P_i}{\mathbf{m}_3 \, P_i} \\ \dfrac{\mathbf{m}_2 \, P_i}{\mathbf{m}_3 \, P_i} \end{bmatrix}$$

$$u_i = \frac{\mathbf{m}_1 \, P_i}{\mathbf{m}_3 \, P_i} \rightarrow u_i(\mathbf{m}_3 \, P_i) = \mathbf{m}_1 \, P_i \rightarrow u_i(\mathbf{m}_3 \, P_i) - \mathbf{m}_1 \, P_i = 0$$

$$v_i = \frac{\mathbf{m}_2 \, P_i}{\mathbf{m}_3 \, P_i} \rightarrow v_i(\mathbf{m}_3 \, P_i) = \mathbf{m}_2 \, P_i \rightarrow v_i(\mathbf{m}_3 \, P_i) - \mathbf{m}_2 \, P_i = 0$$

Calibration Problem

$$
\begin{cases}
u_1(\mathbf{m}_3\ P_1) - \mathbf{m}_1\ P_1 = 0 \\
v_1(\mathbf{m}_3\ P_1) - \mathbf{m}_2\ P_1 = 0 \\
\vdots \\
u_i(\mathbf{m}_3\ P_i) - \mathbf{m}_1\ P_i = 0 \\
v_i(\mathbf{m}_3\ P_i) - \mathbf{m}_2\ P_i = 0 \\
\vdots \\
u_n(\mathbf{m}_3\ P_n) - \mathbf{m}_1\ P_n = 0 \\
v_n(\mathbf{m}_3\ P_n) - \mathbf{m}_2\ P_n = 0
\end{cases}
$$

Block Matrix Multiplication

$$\mathrm{A} = \begin{bmatrix} \mathrm{A}_{11} & \mathrm{A}_{12} \\ \mathrm{A}_{21} & \mathrm{A}_{22} \end{bmatrix} \qquad \mathrm{B} = \begin{bmatrix} \mathrm{B}_{11} & \mathrm{B}_{12} \\ \mathrm{B}_{21} & \mathrm{B}_{22} \end{bmatrix}$$

What is *AB* ?

$$AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

Calibration Problem

$$
\begin{cases}
-u_1(\mathbf{m}_3 P_1) + \mathbf{m}_1 P_1 = 0 \\
-v_1(\mathbf{m}_3 P_1) + \mathbf{m}_2 P_1 = 0 \\
\vdots \\
-u_n(\mathbf{m}_3 P_n) + \mathbf{m}_1 P_n = 0 \\
-v_n(\mathbf{m}_3 P_n) + \mathbf{m}_2 P_n = 0
\end{cases}
\longrightarrow
\mathbf{P}\,\mathbf{m} = 0
$$

known: $\mathbf{P}$

unknown: $\mathbf{m}$

Homogenous linear system

1x4

$$
\mathbf{P} \stackrel{\text{def}}{=} \begin{pmatrix}
P_1^T & \mathbf{0}^T & -u_1 P_1^T \\
\mathbf{0}^T & P_1^T & -v_1 P_1^T \\
 & \vdots & \\
P_n^T & \mathbf{0}^T & -u_n P_n^T \\
\mathbf{0}^T & P_n^T & -u_n P_n^T
\end{pmatrix}
$$

2n x 12

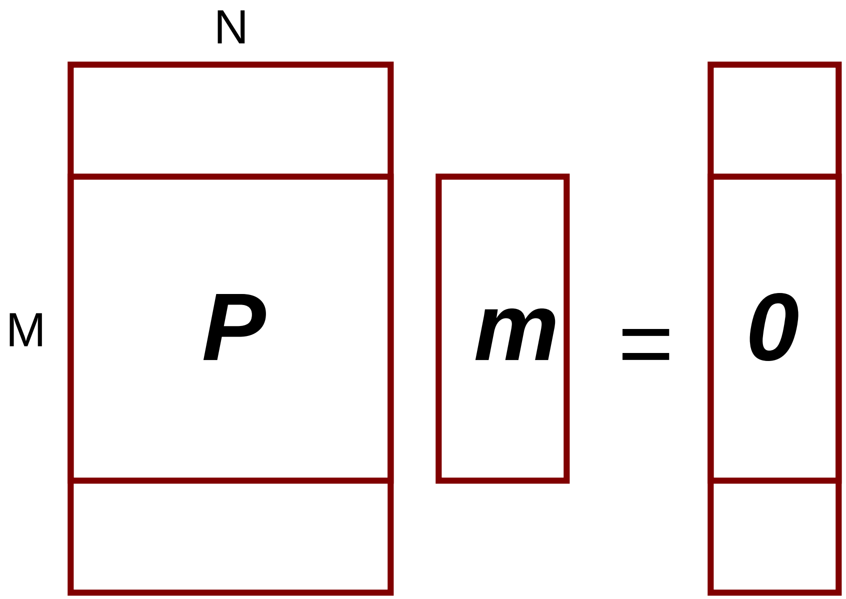
4x1

$$
\boldsymbol{m} \stackrel{\text{def}}{=} \begin{pmatrix}
\mathbf{m}_1^T \\
\mathbf{m}_2^T \\
\mathbf{m}_3^T
\end{pmatrix}
$$

12x1

Homogeneous M x N Linear Systems

M=number of equations = 2n
N=number of unknown = 11



Rectangular system (M>N)

- 0 is always a solution
- To find non-zero solution

Minimize $|\mathbf{P}\ \mathbf{m}|^2$

under the constraint $|\mathbf{m}|^2 = 1$

Calibration Problem

$$\mathbf{P}\ \mathbf{m} = 0$$

- How do we solve this homogenous linear system?
- Using DLT (Direct Linear Transformation) algorithm via SVD decomposition

Calibration Problem

$$\mathbf{P}\,\mathbf{m} = 0$$

SVD decomposition of P

$$U_{2n\times 12}\; D_{12\times 12}\; V^{T}_{12\times 12}$$

Last column of V gives $\boldsymbol{m}$

Why? See pag 592 of HZ

$$\boldsymbol{m} \stackrel{\text{def}}{=} \begin{pmatrix} \mathbf{m}_1^{T} \\ \mathbf{m}_2^{T} \\ \mathbf{m}_3^{T} \end{pmatrix}$$

$\downarrow$

M

$$M\, P_i \rightarrow p_i$$

Extracting camera parameters

$$\frac{\mathcal{M}}{\rho} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta}\boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix} = K\begin{bmatrix} R & T \end{bmatrix}$$

A $\mathbf{b}$

$$K = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_o \\ 0 & \dfrac{\beta}{\sin\theta} & v_o \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\rho = \frac{\pm 1}{|\mathbf{a}_3|} \qquad u_o = \rho^2(\mathbf{a}_1 \cdot \mathbf{a}_3)$$

$$v_o = \rho^2(\mathbf{a}_2 \cdot \mathbf{a}_3)$$

$$\cos\theta = \frac{(\mathbf{a}_1 \times \mathbf{a}_3)\cdot(\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_1 \times \mathbf{a}_3| \cdot |\mathbf{a}_2 \times \mathbf{a}_3|}$$

Theorem (Faugeras, 1993)

Let $\mathcal{M} = (\mathcal{A} \quad \boldsymbol{b})$ be a 3×4 matrix and let $\boldsymbol{a}_i^T$ $(i = 1, 2, 3)$ denote the rows of the matrix $\mathcal{A}$ formed by the three leftmost columns of $\mathcal{M}$.

- A necessary and sufficient condition for $\mathcal{M}$ to be a perspective projection matrix is that $\text{Det}(\mathcal{A}) \neq 0$.
- A necessary and sufficient condition for $\mathcal{M}$ to be a zero-skew perspective projection matrix is that $\text{Det}(\mathcal{A}) \neq 0$ and

$$(\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3) = 0.$$

- A necessary and sufficient condition for $\mathcal{M}$ to be a perspective projection matrix with zero skew and unit aspect-ratio is that $\text{Det}(\mathcal{A}) \neq 0$ and

$$\begin{cases} (\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3) = 0, \\ (\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_1 \times \boldsymbol{a}_3) = (\boldsymbol{a}_2 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3). \end{cases}$$

Extracting camera parameters

$$\frac{\mathcal{M}}{\rho} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \frac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \frac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix} = K[R \quad T]$$

A (left block) **b** (right block)

$$A = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\alpha = \rho^2 |\mathbf{a}_1 \times \mathbf{a}_3| \sin\theta$$

$$\beta = \rho^2 |\mathbf{a}_2 \times \mathbf{a}_3| \sin\theta$$

→ f

Extracting camera parameters

$$\frac{\mathcal{M}}{\rho} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \frac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \frac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix} = \mathrm{K}[\mathrm{R} \quad \mathrm{T}]$$

$$\mathrm{A} \qquad \qquad \mathbf{b}$$

$$\mathrm{A} = \begin{bmatrix} \mathbf{a}_1^{\mathrm{T}} \\ \mathbf{a}_2^{\mathrm{T}} \\ \mathbf{a}_3^{\mathrm{T}} \end{bmatrix} \qquad \mathrm{b} = \begin{bmatrix} \mathrm{b}_1 \\ \mathrm{b}_2 \\ \mathrm{b}_3 \end{bmatrix}$$

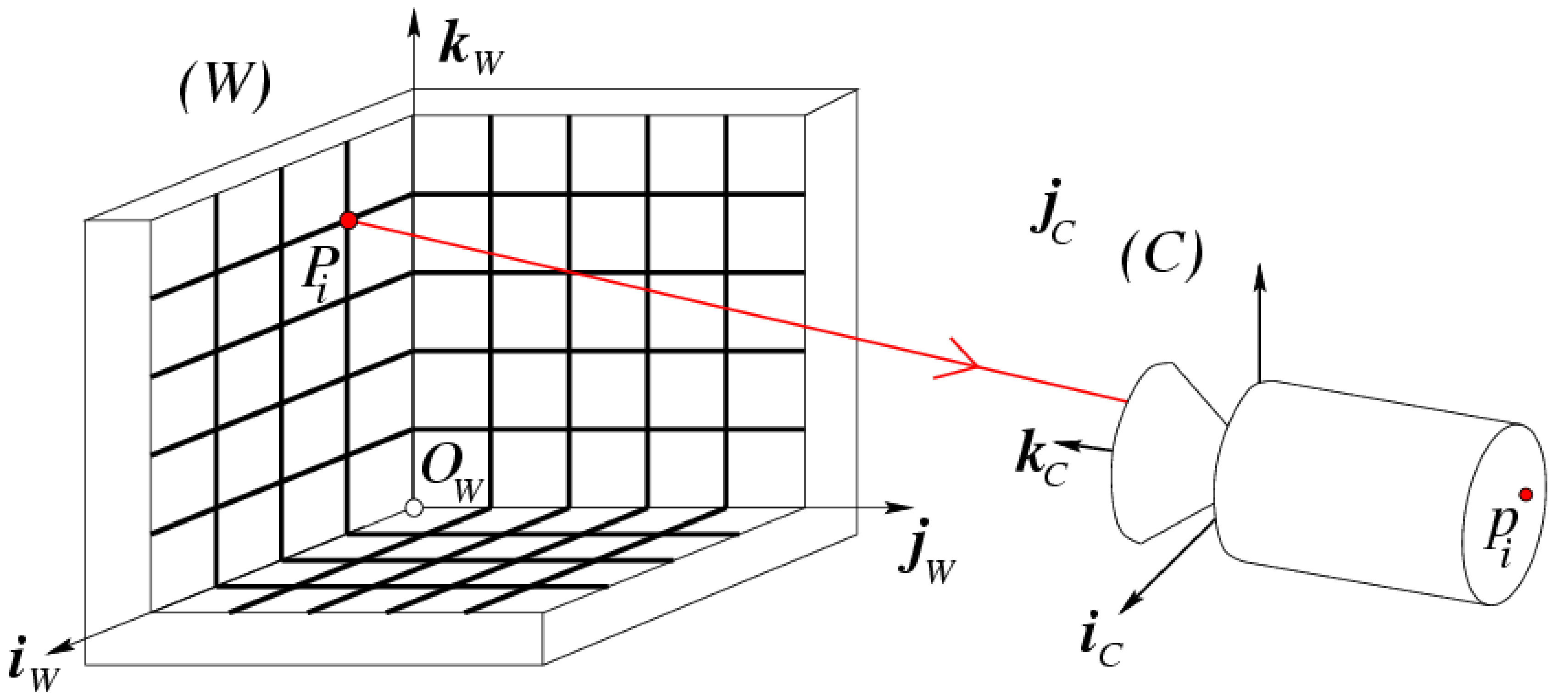
Estimated values

Extrinsic

$$\mathbf{r}_1 = \frac{(\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_2 \times \mathbf{a}_3|} \qquad \mathbf{r}_3 = \frac{\pm \mathbf{a}_3}{|\mathbf{a}_3|}$$

$$\mathbf{r}_2 = \mathbf{r}_3 \times \mathbf{r}_1 \qquad \mathrm{T} = \rho\, \mathrm{K}^{-1} \mathbf{b}$$

Degenerate cases



- P_i's cannot lie on the same plane!
- Points cannot lie on the intersection curve of two quadric surfaces

Lecture 3
Camera Calibration

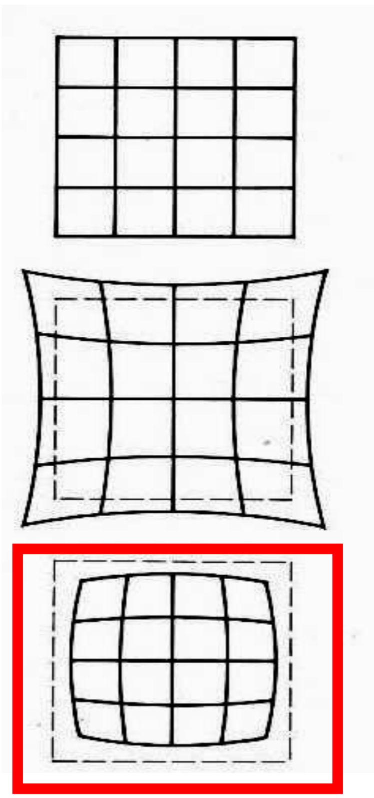
- Recap of projective cameras
- Camera calibration problem
- Camera calibration with radial distortion
- Example

Reading: [FP] Chapter 3
[HZ] Chapter 7

Some slides in this lecture are courtesy to Profs. J. Ponce, F-F Li

Radial Distortion

- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens



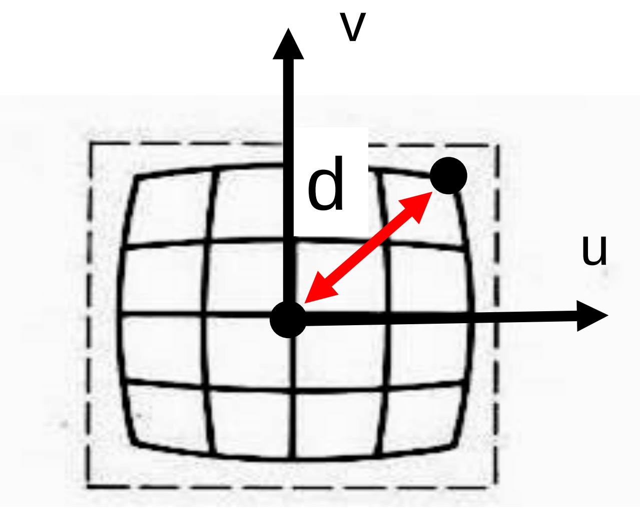
No distortion

Pin cushion

Barrel

Radial Distortion

Image magnification in(de)creases with distance from the optical center



$$\begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix} M \, P_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = p_i$$

$$d^2 = a\,u^2 + b\,v^2 + c\,u\,v$$

To model radial behavior

Distortion coefficient

$$\lambda = 1 \pm \sum_{p=1}^{3} \kappa_p d^{2p}$$

Polynomial function

Radial Distortion

$$\begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix} M\, P_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = p_i \qquad Q = \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \mathbf{q}_3 \end{bmatrix}$$

Q

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{q}_1\, P_i}{\mathbf{q}_3\, P_i} \\ \frac{\mathbf{q}_2\, P_i}{\mathbf{q}_3\, P_i} \end{bmatrix} \rightarrow \begin{cases} u_i \mathbf{q}_3\, P_i = \mathbf{q}_1\, P_i \\ v_i \mathbf{q}_3\, P_i = \mathbf{q}_2\, P \end{cases}$$

Is this a linear system of equations?

No! why?

General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 \, \mathrm{P_i}}{\mathbf{q}_3 \, \mathrm{P_i}} \\ \dfrac{\mathbf{q}_2 \, \mathrm{P_i}}{\mathbf{q}_3 \, \mathrm{P_i}} \end{bmatrix} \longrightarrow X = f(P)$$

measurement (X) parameter (P)

f( ) is nonlinear

-Newton Method

-Levenberg-Marquardt Algorithm

- Iterative, starts from initial solution
- May be slow if initial solution far from real solution
- Estimated solution may be function of the initial solution
- Newton requires the computation of J, H
- Levenberg-Marquardt doesn't require the computation of H

General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 \, \mathrm{P_i}}{\mathbf{q}_3 \, \mathrm{P_i}} \\ \dfrac{\mathbf{q}_2 \, \mathrm{P_i}}{\mathbf{q}_3 \, \mathrm{P_i}} \end{bmatrix} \longrightarrow X = f(P)$$

measurement (X)

parameter (P)

f( ) is nonlinear

A possible algorithm

1. Solve linear part of the system to find approximated solution
2. Use this solution as initial condition for the full system
3. Solve full system using Newton or L.M.

General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 \mathrm{P_i}}{\mathbf{q}_3 \mathrm{P_i}} \\ \dfrac{\mathbf{q}_2 \mathrm{P_i}}{\mathbf{q}_3 \mathrm{P_i}} \end{bmatrix} \longrightarrow X = f(P)$$

measurement (X)

parameter (P)

f( ) is nonlinear

Typical assumptions:

- zero-skew, square pixel
- u_o, v_o = known center of the image
- no distortion

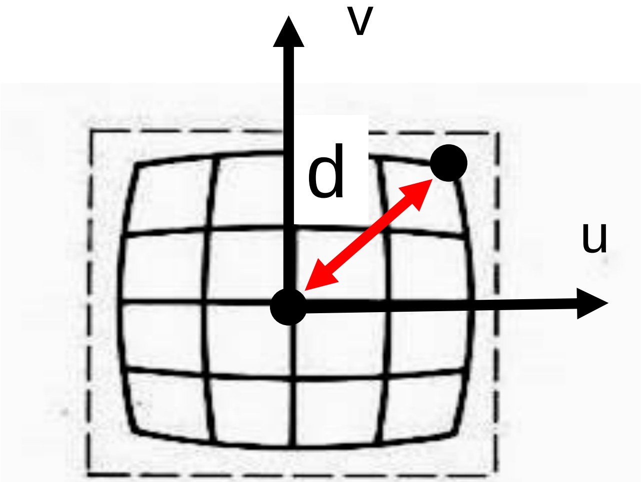
→ Just estimate f and R, T

Radial Distortion

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 P_i}{\mathbf{q}_3 P_i} \\ \dfrac{\mathbf{q}_2 P_i}{\mathbf{q}_3 P_i} \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \dfrac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \dfrac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix}$$

Can we estimate m_1 and m_2 and ignore the radial distortion?

Hint:



$$\frac{u_i}{v_i} = \text{slope}$$

Radial Distortion

Estimating m_1 and m_2…

$$\mathrm{p_i} = \begin{bmatrix} \mathrm{u_i} \\ \mathrm{v_i} \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \dfrac{\mathbf{m}_1 \mathrm{P_i}}{\mathbf{m}_3 \mathrm{P_i}} \\ \dfrac{\mathbf{m}_2 \mathrm{P_i}}{\mathbf{m}_3 \mathrm{P_i}} \end{bmatrix} \rightarrow \frac{u_i}{v_i} = \frac{\dfrac{(\mathbf{m}_1 P_i)}{(\mathbf{m}_3 P_i)}}{\dfrac{(\mathbf{m}_2 P_i)}{(\mathbf{m}_3 P_i)}} = \frac{\mathbf{m}_1 P_i}{\mathbf{m}_2 P_i}$$

$$\begin{cases} v_1(\mathbf{m}_1 P_1) - u_1(\mathbf{m}_2 P_1) = 0 \\ v_i(\mathbf{m}_1 P_i) - u_i(\mathbf{m}_2 P_i) = 0 \\ \vdots \\ v_n(\mathbf{m}_1 P_n) - u_n(\mathbf{m}_2 P_n) = 0 \end{cases}$$

$$\mathrm{Q}\,\mathbf{n} = 0 \qquad \mathbf{n} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \end{bmatrix}$$

Get $\mathbf{m}_1$ and $\mathbf{m}_2$ by SVD

Tsai [87]

Radial Distortion

Once that $\mathbf{m}_1$ and $\mathbf{m}_2$ are estimated…

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \dfrac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \dfrac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix}$$

$\mathbf{m}_3$ is non linear function of $\mathbf{m}_1, \mathbf{m}_2, \lambda$

There are some degenerate configurations for which m_1 and m_2 cannot be computed

Lecture 2

Camera Calibration

- Recap of projective cameras
- Camera calibration problem
- Camera calibration with radial distortion
- Example

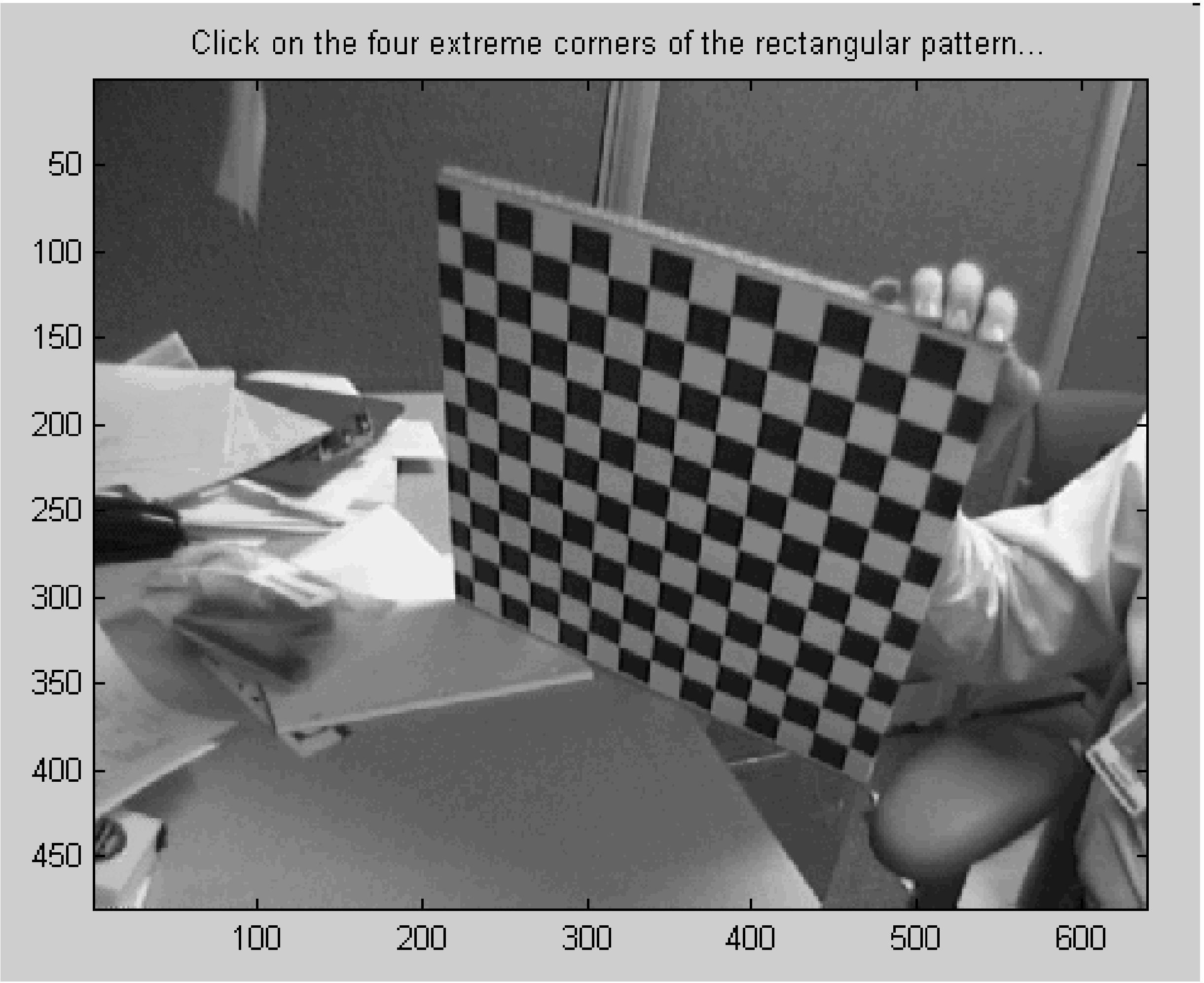
Reading: [FP] Chapter 3
[HZ] Chapter 7

Some slides in this lecture are courtesy to Profs. J. Ponce, F-F Li

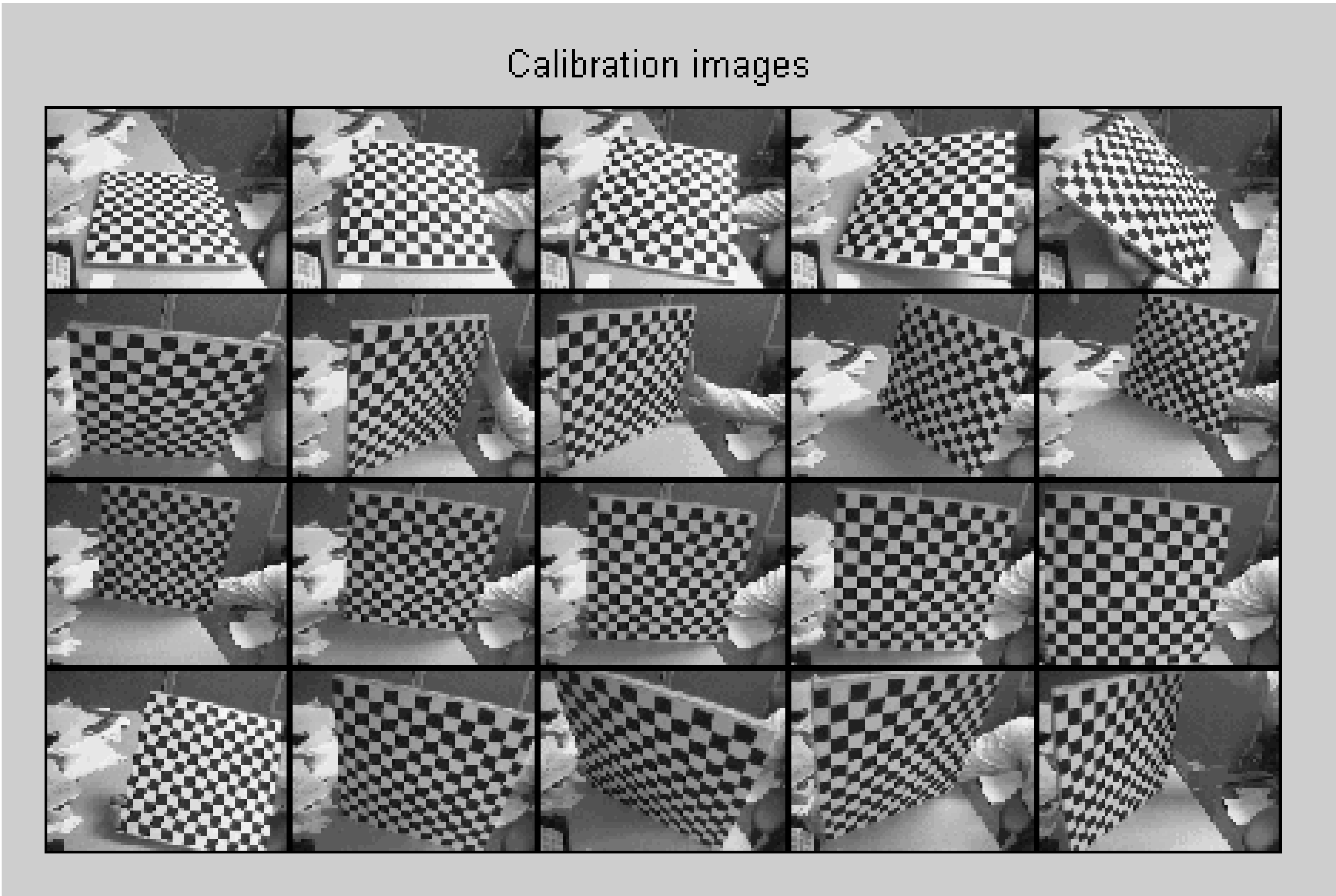
Calibration Procedure

Camera Calibration Toolbox for Matlab
J. Bouguet – [1998-2000]

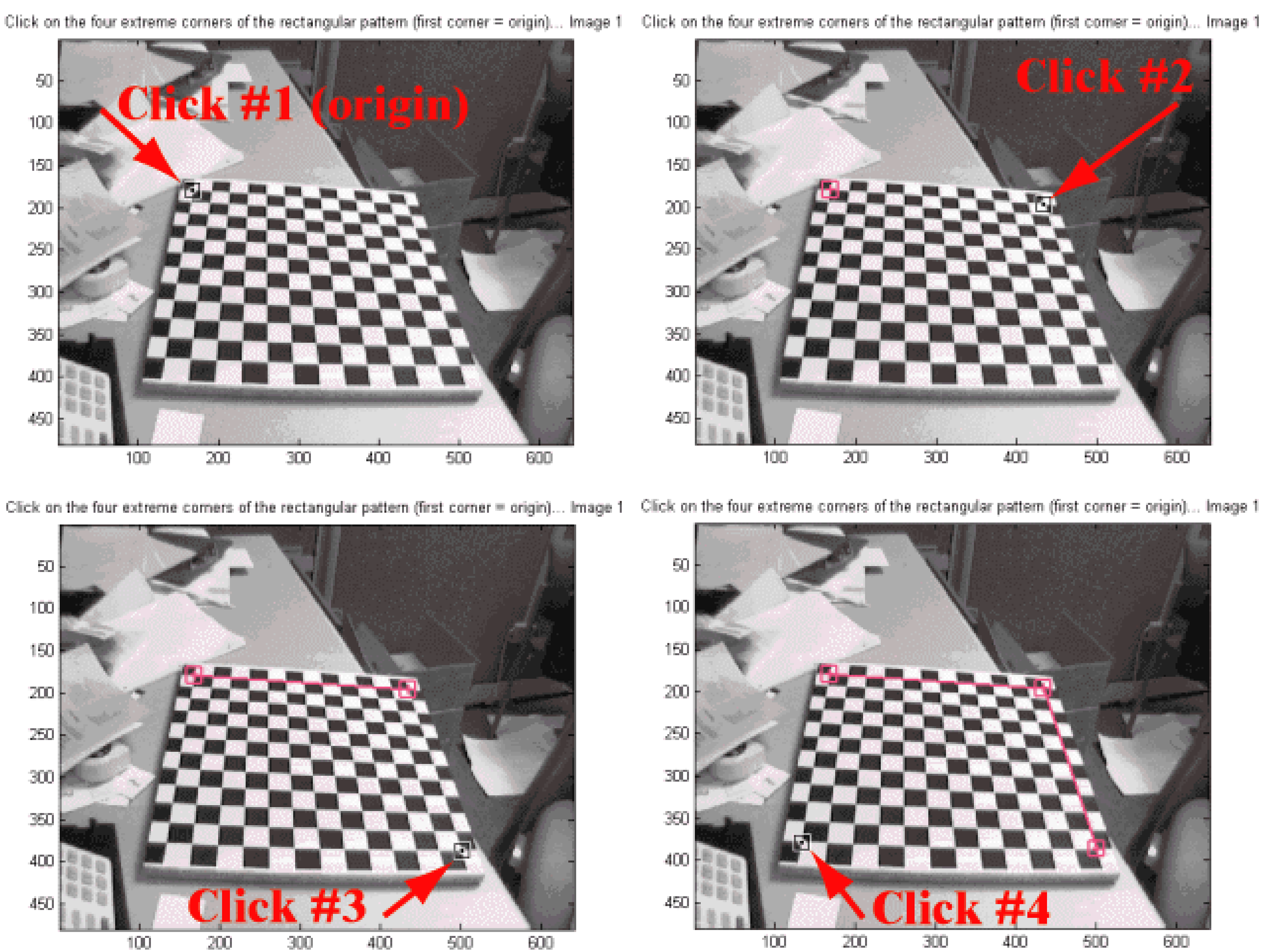
http://www.vision.caltech.edu/bouguetj/calib_doc/index.html#examples



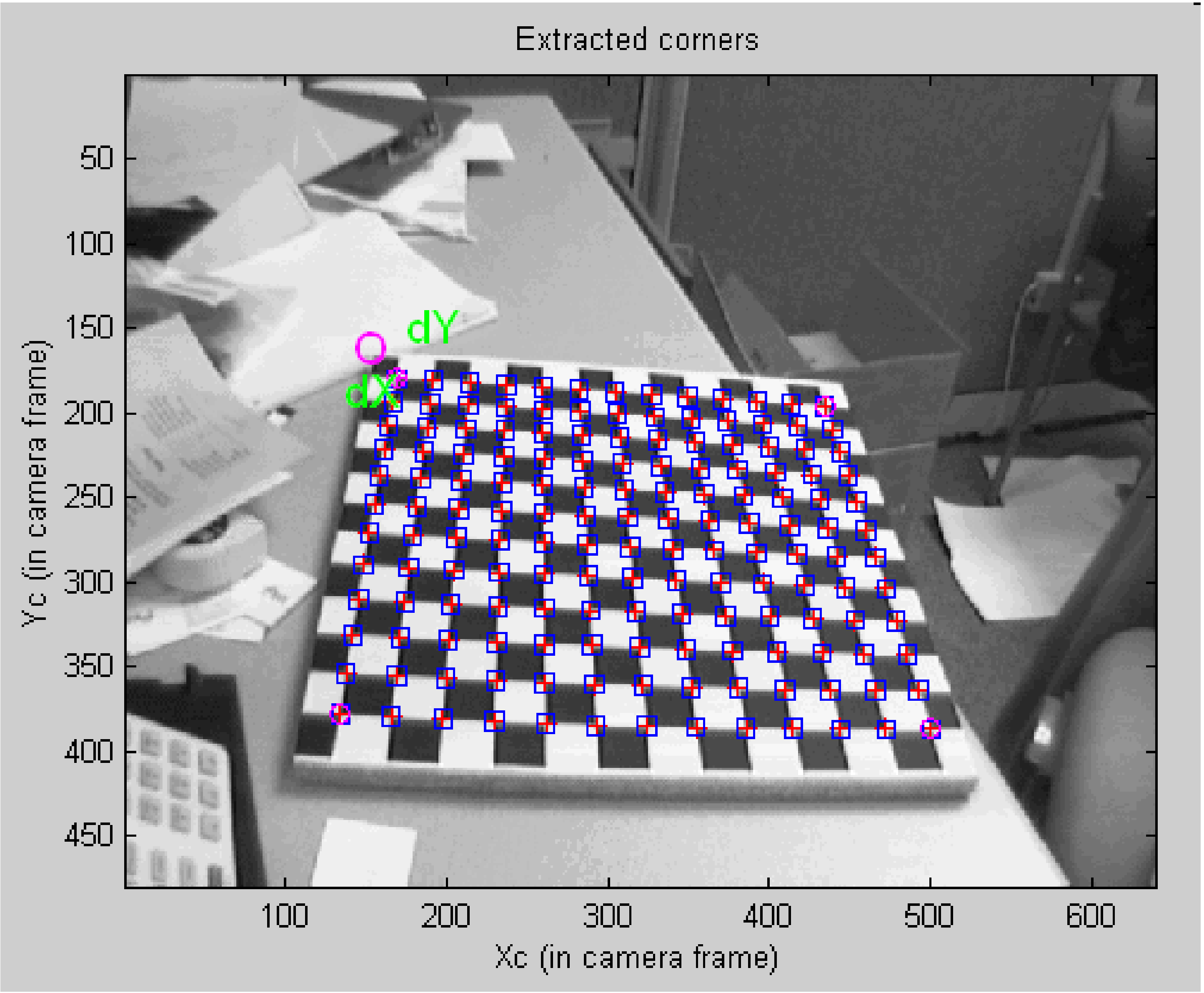
Calibration Procedure



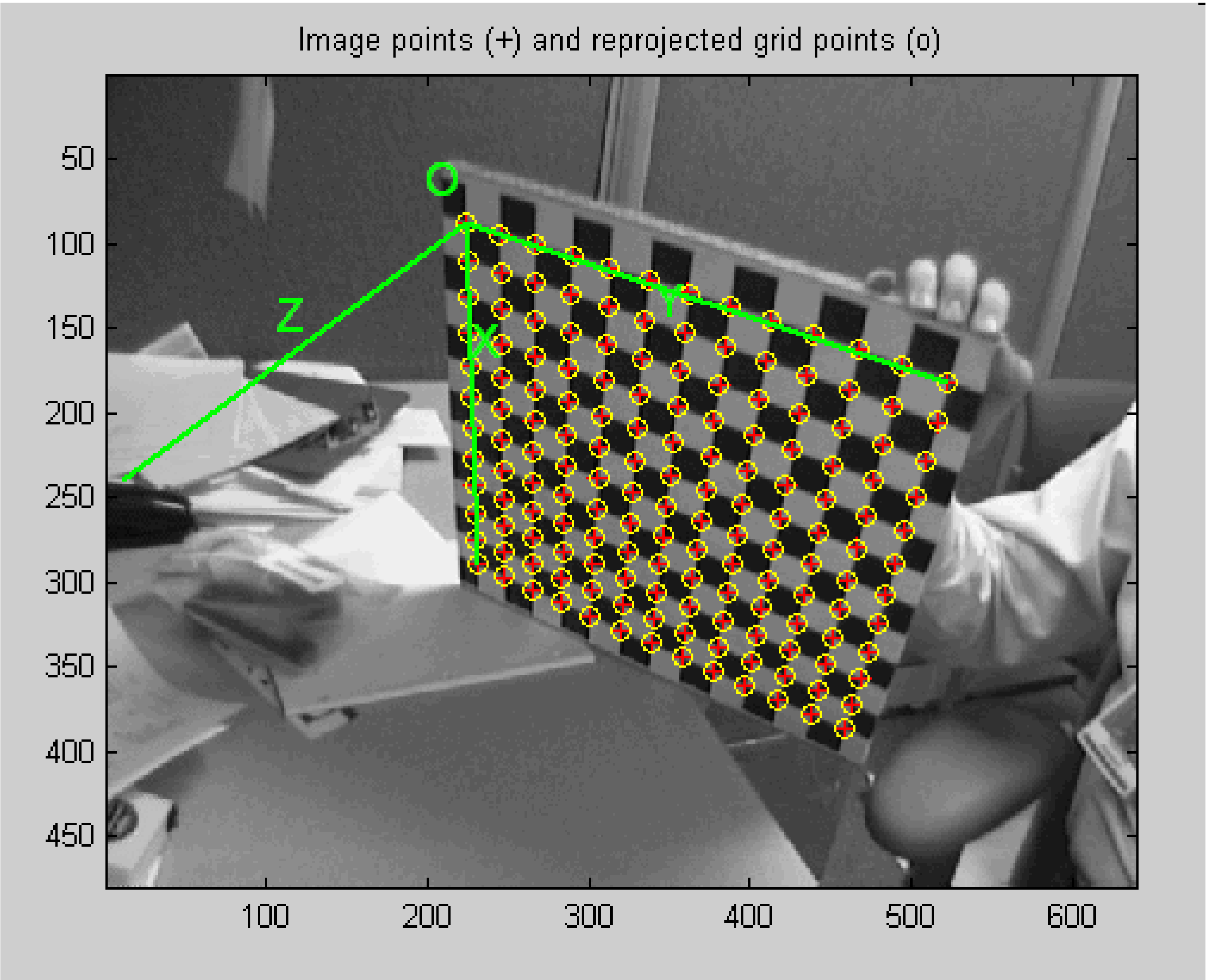
Calibration Procedure



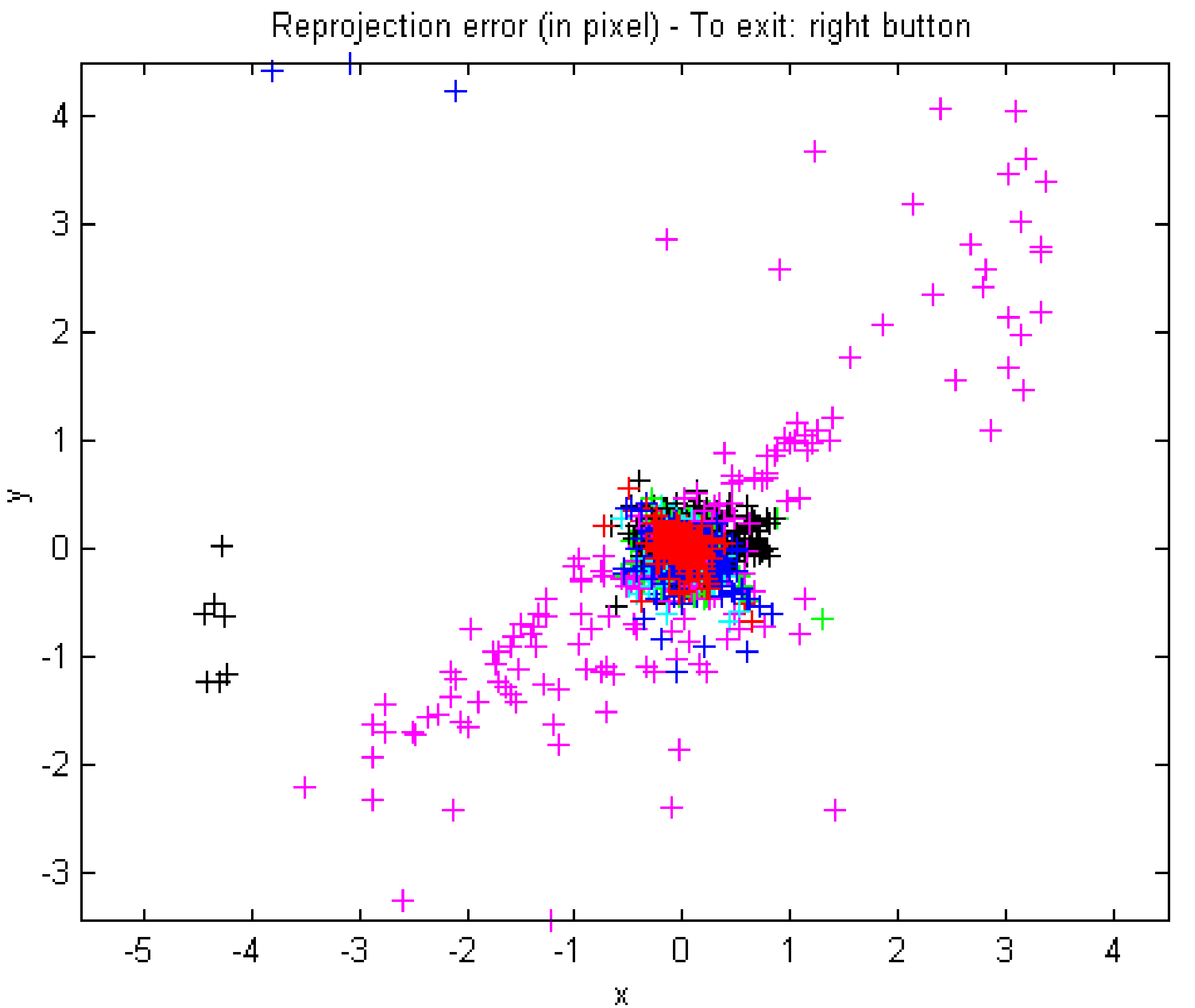
Calibration Procedure



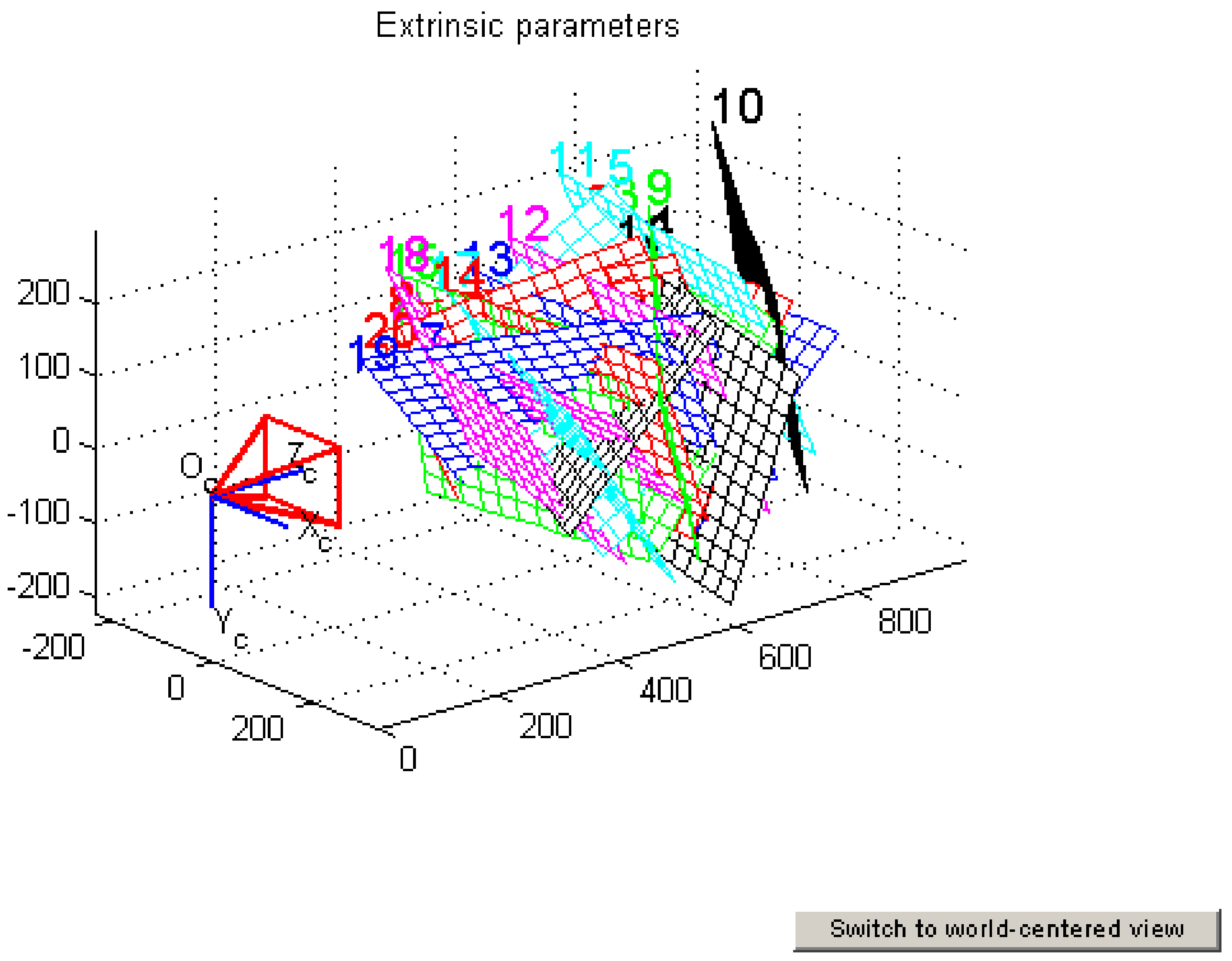
Calibration Procedure



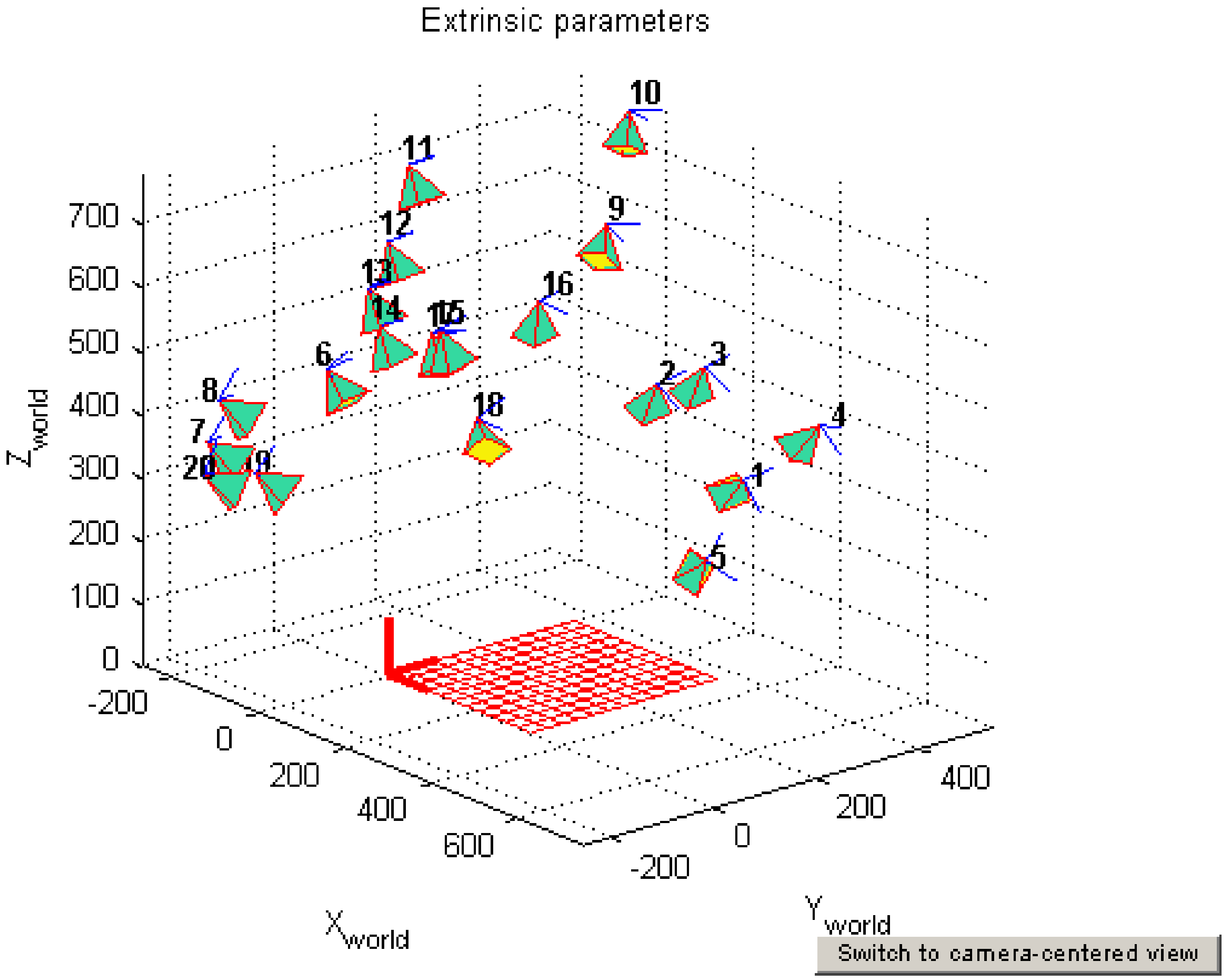
Calibration Procedure



Calibration Procedure



Calibration Procedure



Next lecture

- Single view reconstruction