# Lecture:
# Tracking

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Stanford Vision and Learning Lab

# What we will learn today?

- Feature Tracking
- Simple KLT tracker
- 2D transformations
- Iterative KLT tracker

**Reading:** [Szeliski] Chapters: 8.4, 8.5

[Fleet & Weiss, 2005]
http://www.cs.toronto.edu/pub/jepson/teaching/vision/2503/opticalFlow.pdf

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# Problem statement

Image sequence

Slide credit: Yonsei Univ.

# Problem statement

Feature point detection

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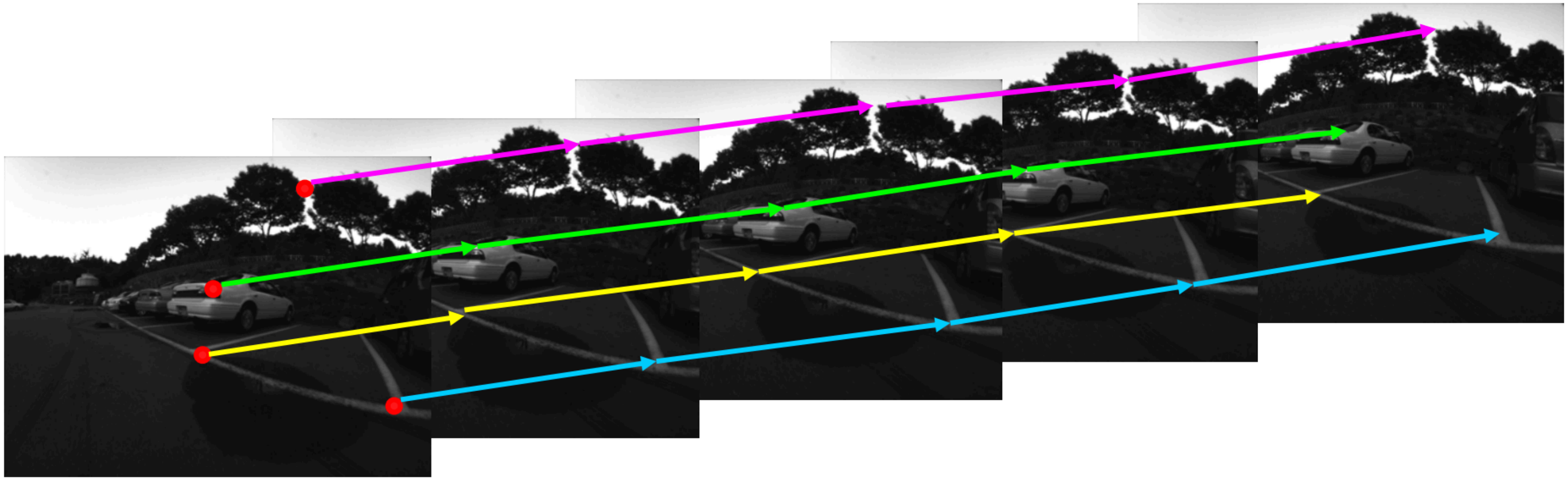
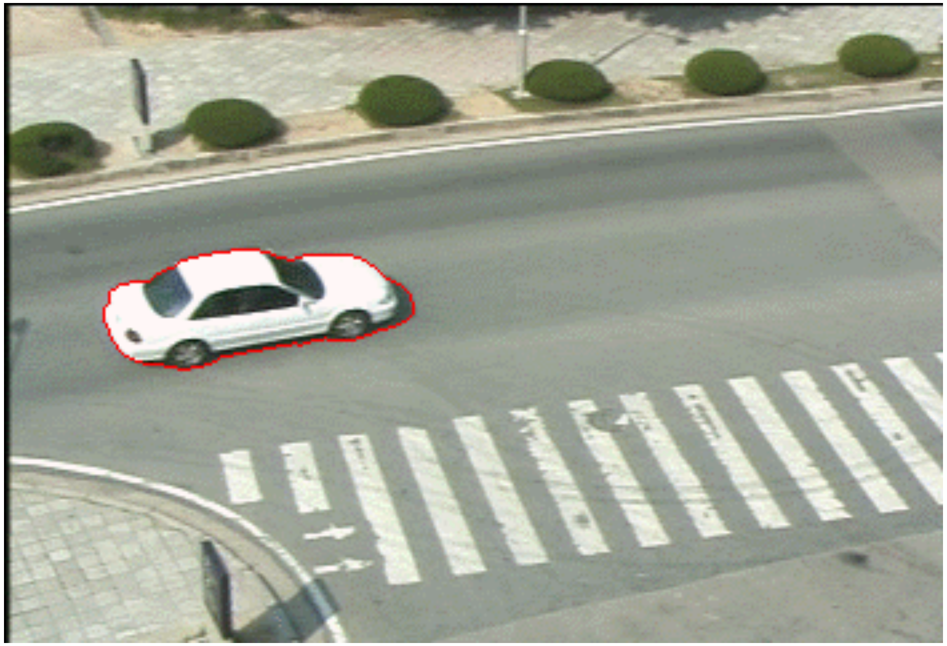
# Problem statement

Feature point tracking



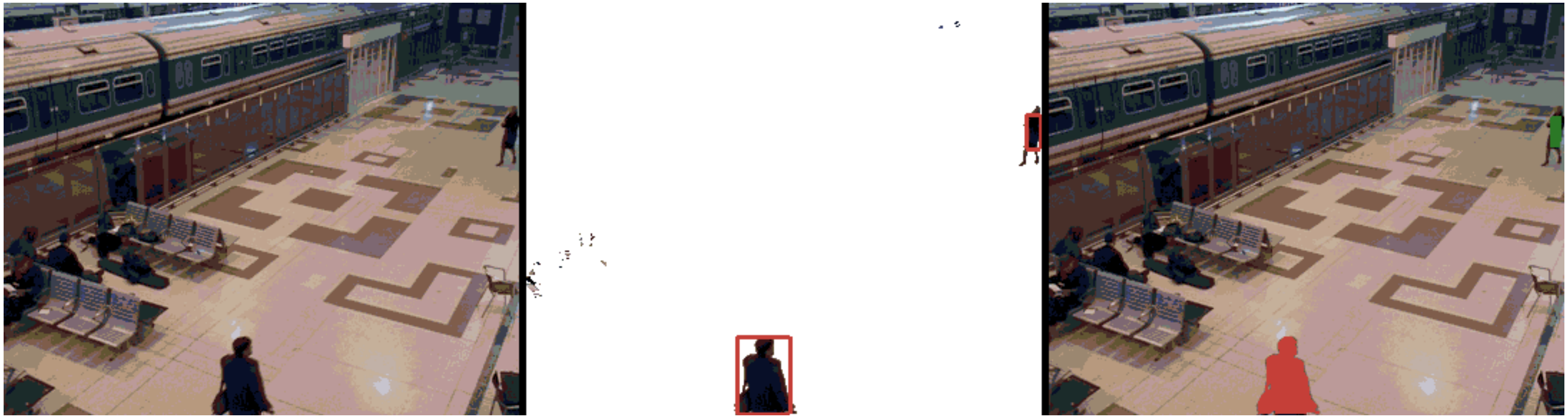
Slide credit: Yonsei Univ.

# Single object tracking



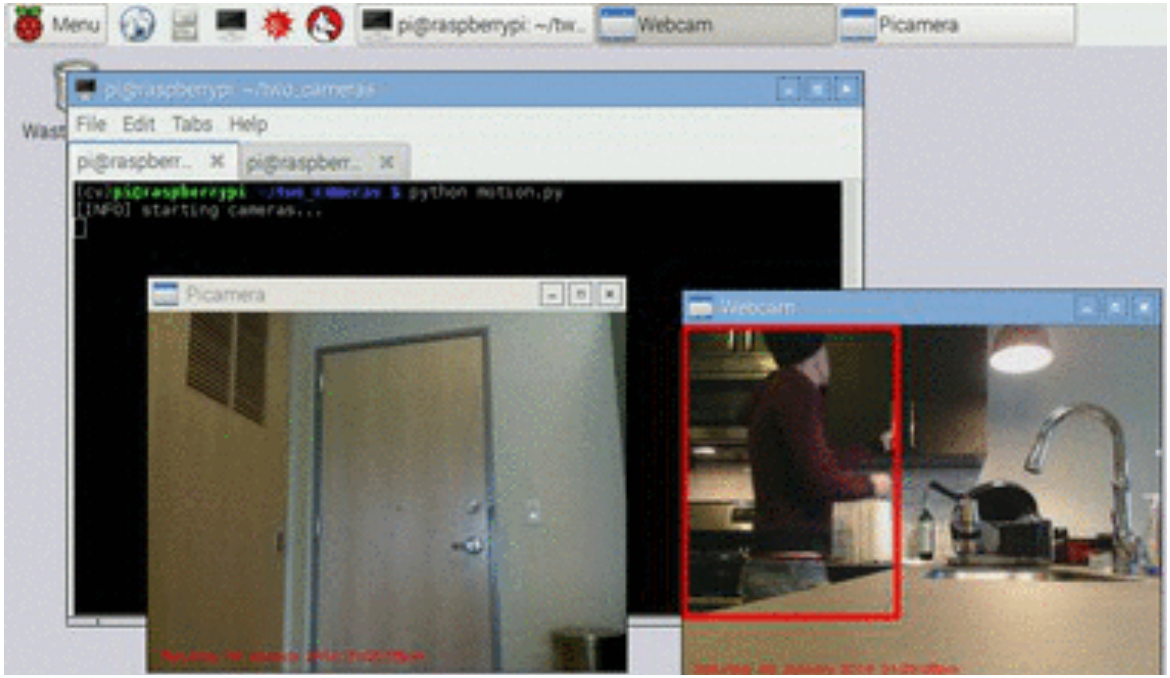
# Multiple object tracking

# Tracking with a fixed camera



# Tracking with a moving camera

# Tracking with multiple cameras



# Challenges in Feature tracking

- Figure out which features can be tracked
  - Efficiently track across frames
- Some points may change appearance over time
  - e.g., due to rotation, moving into shadows, etc.
- Drift: small errors can accumulate as appearance model is updated
- Points may appear or disappear.
  - need to be able to add/delete tracked points.

# What are good features to track?

- Intuitively, we want to avoid smooth regions and edges. But is there a more is principled way to define good features?

- What kinds of image regions can we detect easily and consistently? Think about what you learnt earlier in the class.

# What are good features to track?

- Can measure "quality" of features from just a single image.
- Hence: tracking Harris corners (or equivalent) guarantees small error sensitivity!

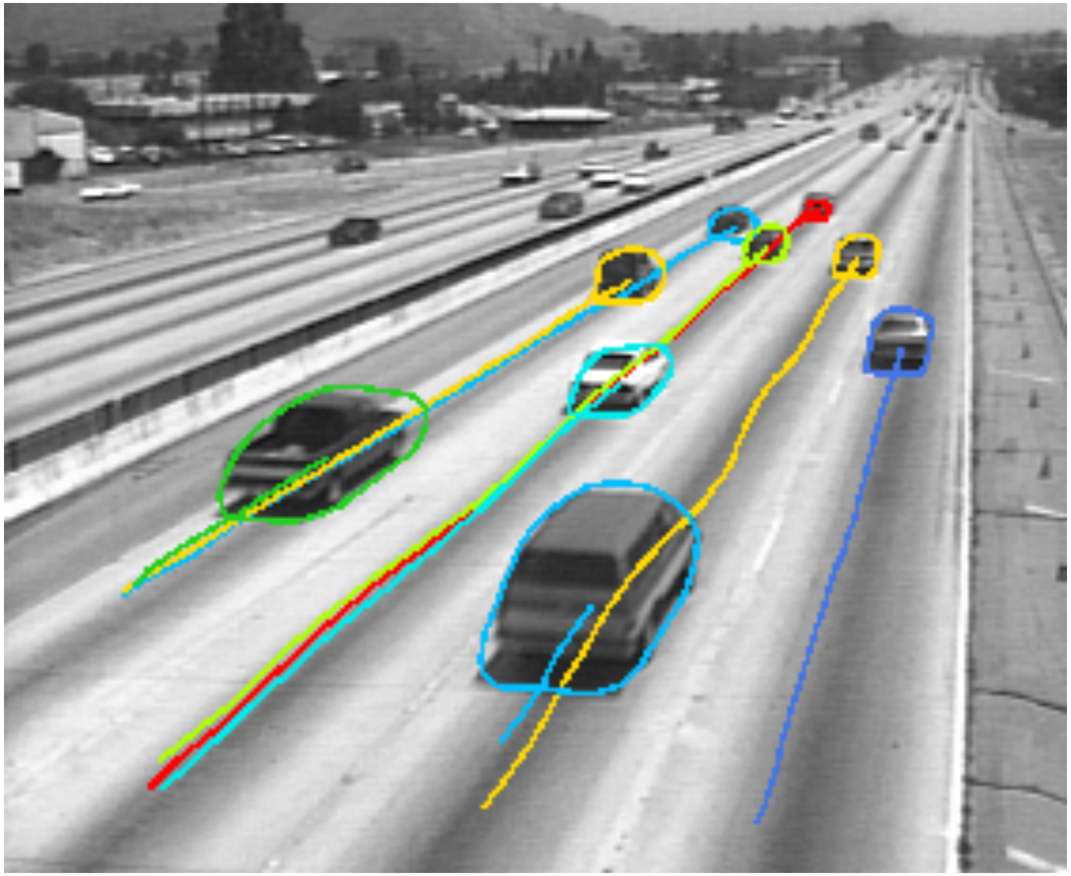
Source: Silvio Savarese

# Motion estimation techniques

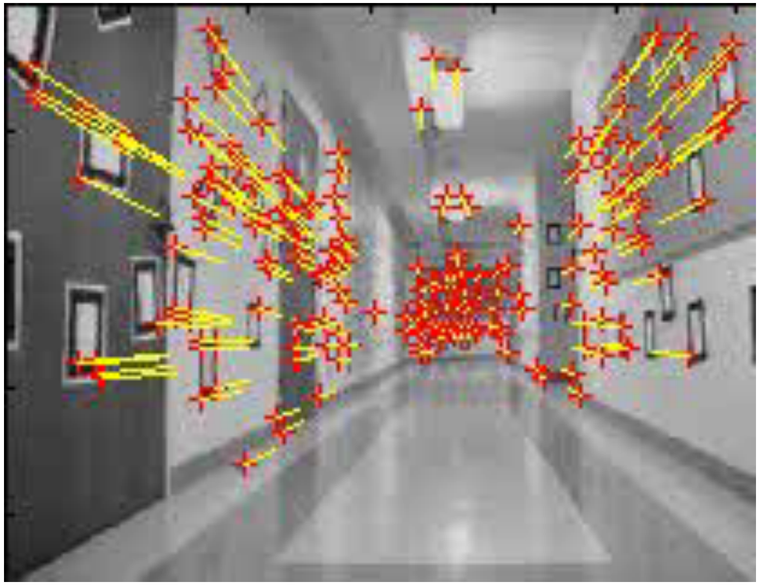
- Optical flow
  - Recover image motion at each pixel from spatio-temporal image brightness variations (optical flow)
- Feature-tracking
  - Extract visual features (corners, textured areas) and “track” them over multiple frames

# Optical flow can help track features

Once we have the features we want to track, lucas-kanade or other optical flow algorithm can help track those features

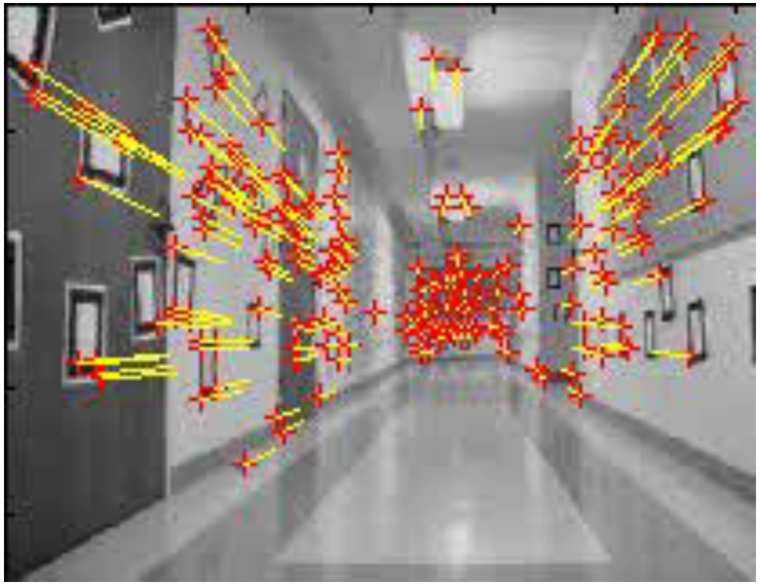


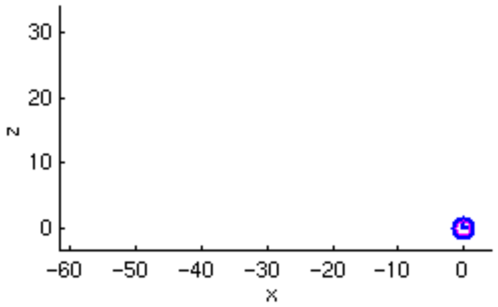
# Feature-tracking



Courtesy of Jean-Yves Bouguet – Vision Lab, California Institute of Technology

# Feature-tracking



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# Simple KLT tracker

1. Find a good point to track (harris corner)
2. For each Harris corner compute motion (translation or affine) between consecutive frames.
3. Link motion vectors in successive frames to get a track for each Harris point
4. Introduce new Harris points by applying Harris detector at every m (10 or 15) frames
5. Track new and old Harris points using steps 1-3

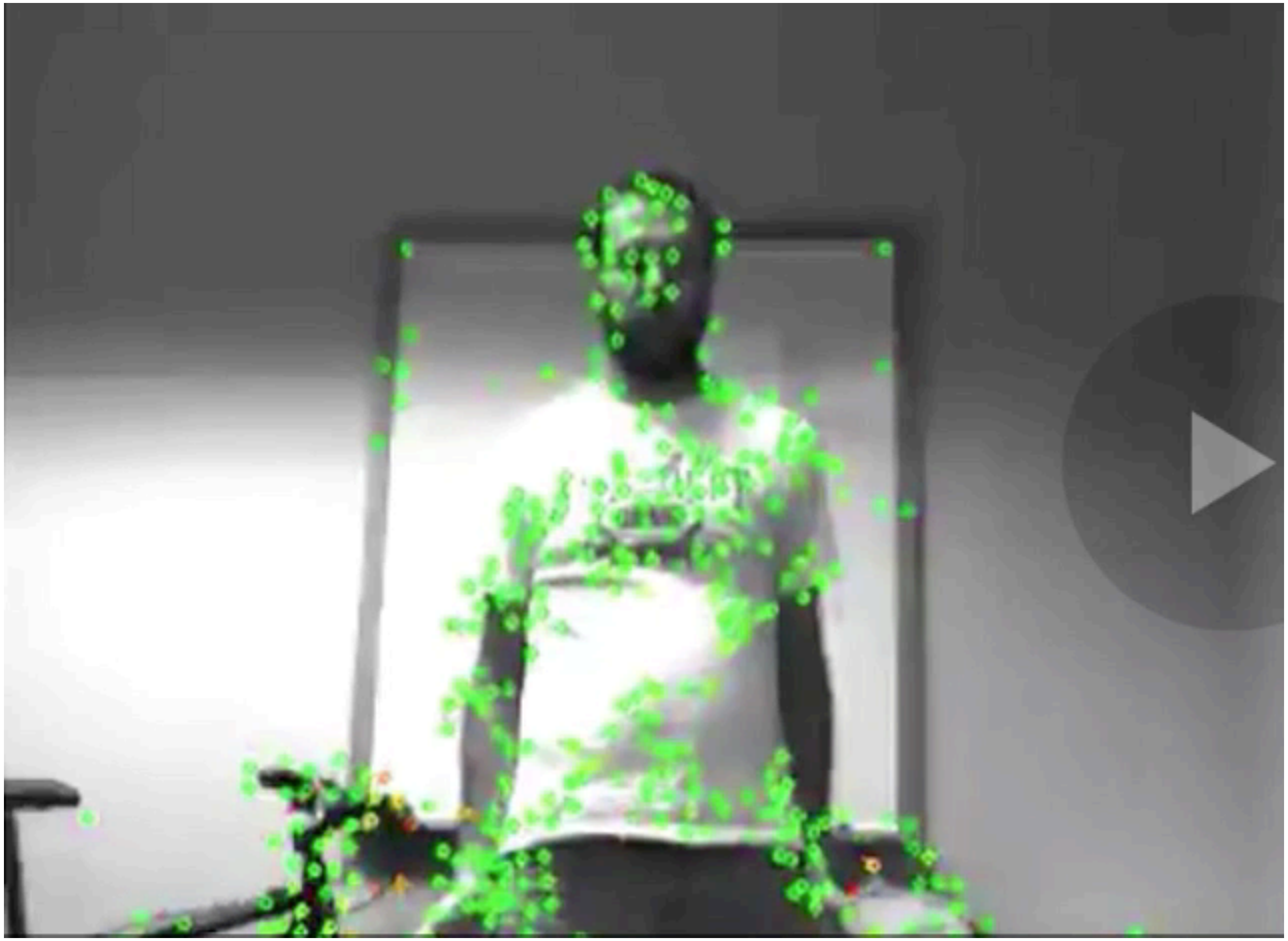
# KLT tracker for fish

Video credit: Kanade

# Tracking cars

Video credit: Kanade

# Tracking movement



Video credit: Kanade

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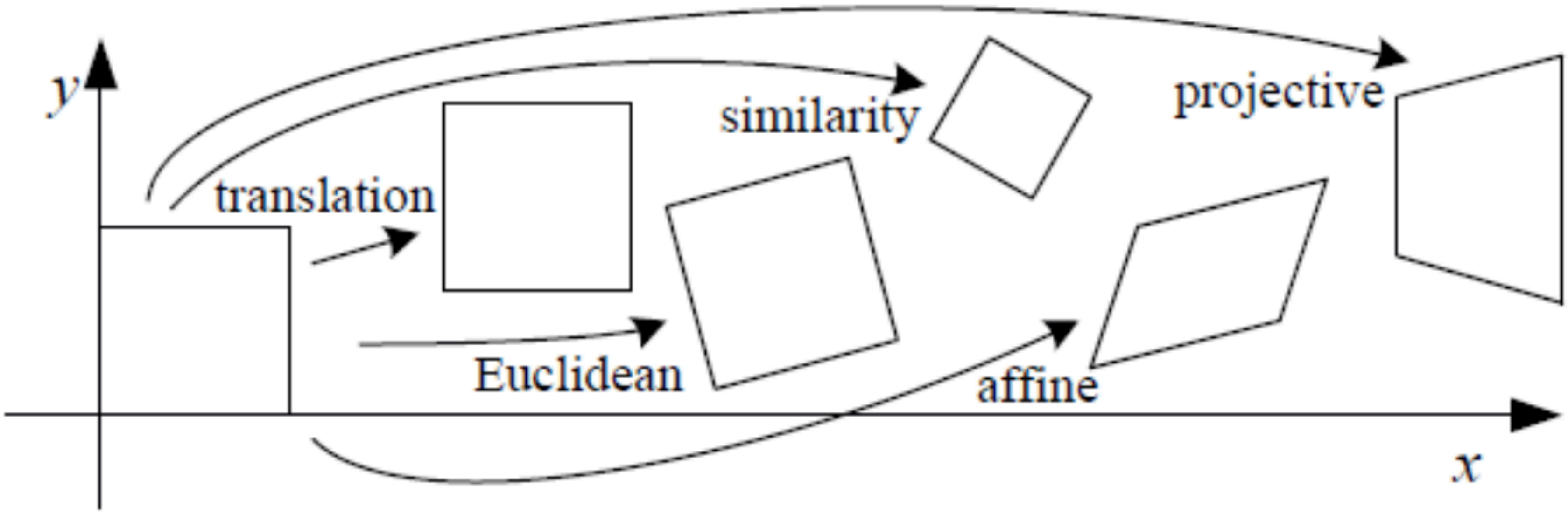
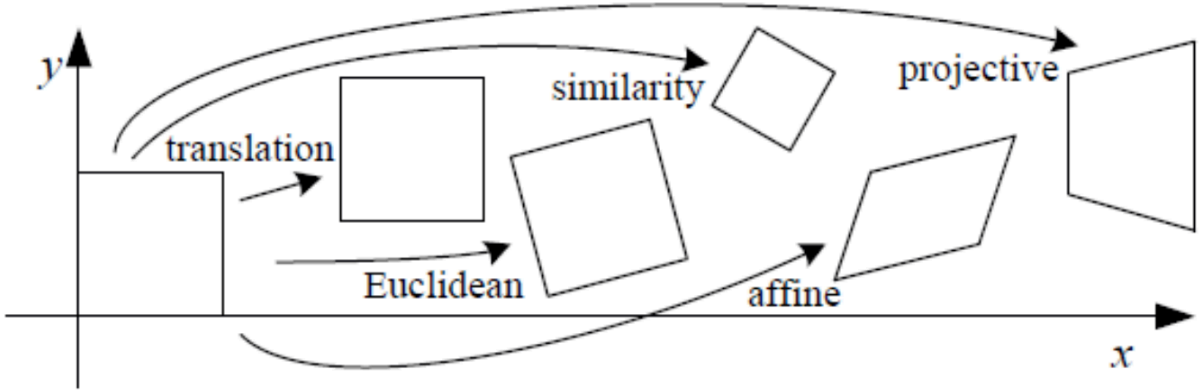
# Types of 2D transformations

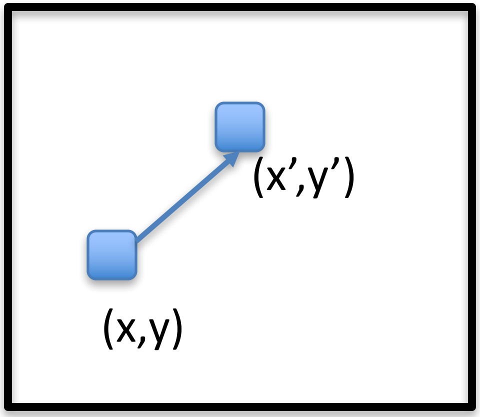


# Depending on camera and objects, choose the right transformations



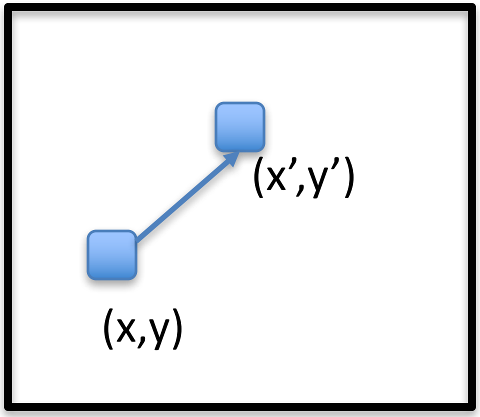

- Fixed overhead cameras will see only **translation** transformations.
- Fixed cameras of a basketball game will see **similarity** transformations.
- People in pedestrian detections can see **affine** transformations.
- And moving cameras can see **projective** transformations.

# Translation



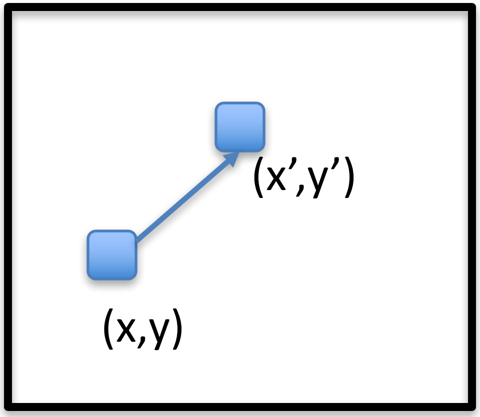

- Let the initial feature be located by (x, y).
- In the next frame, it has translated to (x', y').
- We can write the transformation as:
  $x' = x + b_1$
  $y' = y + b_2$

# Translation




- $x' = x + b_1$
  $y' = y + b_2$
- We can write this as a matrix transformation using homogeneous coordinates:
- $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 & b_1 \\ 0 & 1 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

# Translation

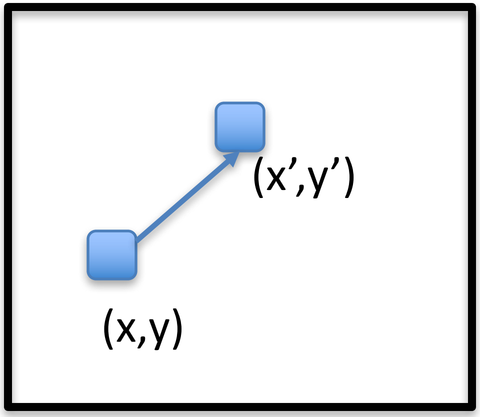



- $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 & b_1 \\ 0 & 1 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- We will write the above transformation:

- $$W = \begin{bmatrix} 1 & 0 & b_1 \\ 0 & 1 & b_2 \end{bmatrix}$$

# Displacement Model for Translation




- $W(\boldsymbol{x};\boldsymbol{p}) = \begin{bmatrix} 1 & 0 & b_1 \\ 0 & 1 & b_2 \end{bmatrix}$
- There are only two parameters:
$$\boldsymbol{p} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$
- The derivative of the transformation w.r.t. **p**:
- $\dfrac{\partial W}{\partial \boldsymbol{p}}(\boldsymbol{x};\boldsymbol{p}) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- This is called the Jacobian.

# Similarity motion

- Rigid motion includes scaling + translation.
- We can write the transformations as:
  $x' = ax + b_1$
  $y' = ay + b_2$
- $W = \begin{bmatrix} \mathrm{a} & 0 & b_1 \\ 0 & \mathrm{a} & b_2 \end{bmatrix}$
- $\boldsymbol{p} = \begin{bmatrix} a & \mathrm{b}_1 & \mathrm{b}_2 \end{bmatrix}^T$
- $\frac{\partial W}{\partial \boldsymbol{p}}(\boldsymbol{x};\boldsymbol{p}) = \begin{bmatrix} x & 1 & 0 \\ y & 0 & 1 \end{bmatrix}$

# Affine motion

- Affine motion includes scaling + rotation + translation.
- $x' = a_1x + a_2y + b1$
  $y' = a_3x + a_4y + b_2$
- $W = \begin{bmatrix} a_1 & a_2 & b_1 \\ a_3 & a_4 & b_2 \end{bmatrix}$
- $\boldsymbol{p} = \begin{bmatrix} a_1 & \mathrm{a}_2 & \mathrm{b}_1 & a_3 & a_4 & b_2 \end{bmatrix}^T$
- $\frac{\partial W}{\partial \boldsymbol{p}}(\boldsymbol{x};\boldsymbol{p}) = \begin{bmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{bmatrix}$

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# Problem formulation

- Given a video sequence, find all the features and track them across the video.
- First, use Harris corner detection to find the features.
- For each feature at location $\mathbf{x} = [x \;\; y]^T$:
  - Choose a descriptor create an initial template for that feature: $T(\boldsymbol{x})$.

# KLT objective

- Our aim is to minimize the difference between the template T(x) and the description of the new location of x after undergoing the transformation.

$$\sum_{\mathbf{x}}[I(W(\mathbf{x};\mathbf{p}))-T(\mathbf{x})]^2$$

- For all the features $\boldsymbol{x}$ in the image $\boldsymbol{I}$,
  - $(I\ W(\boldsymbol{x};\boldsymbol{p}))$ is the estimate of where the features move to in the next frame after the transformation defined by $W(\boldsymbol{x};\boldsymbol{p})$. Recall that $\boldsymbol{p}$ is our vector of parameters.

# KLT objective

- Instead of minimizing this function:

$$\sum_{\mathbf{x}} [I(W(\mathbf{x};\mathbf{p})) - T(\mathbf{x})]^2$$

- We will instead represent $\boldsymbol{p} = \boldsymbol{p_0} + \Delta\boldsymbol{p}$
  - Where $\boldsymbol{p_0}$ is going to be fixed and we will solve for $\Delta\boldsymbol{p}$, which is a small value.
- We can initialize $\boldsymbol{p_0}$ with our best guess of what the motion is and initialize $\Delta\boldsymbol{p}$ as zero.

# A little bit of math: Taylor series

- Taylor series is defined as:
- $f(x+\ \Delta x) = f(x) + \Delta x \frac{\partial f}{\partial x} + \Delta x^2 \frac{\partial^2 f}{\partial x^2} + \ldots$

- Assuming that $\Delta x$ is small.
- We can apply this expansion to the KLT tracker and only use the first two terms:

# Expanded KLT objective

$$\sum_x [I(W(\boldsymbol{x};\boldsymbol{p_0}+\Delta\boldsymbol{p})) - T(x)]^2$$

$$\approx \sum_x \left[ I(W(\boldsymbol{x};\boldsymbol{p_0})) + \nabla I \frac{\partial W}{\partial \boldsymbol{p}} \Delta\boldsymbol{p} - T(x) \right]^2$$

It's a good thing we have already calculated what $\frac{\partial W}{\partial \boldsymbol{p}}$ would look like for affine, translations and other transformations!

# Expanded KLT objective

- So our aim is to find the $\Delta \boldsymbol{p}$ that minimizes the following:

$$\underset{\Delta \boldsymbol{p}}{\operatorname{argmin}} \sum_{x} \left[ I(W(\boldsymbol{x}; \boldsymbol{p_0})) + \nabla I \frac{\partial W}{\partial \boldsymbol{p}} \Delta \boldsymbol{p} - T(x) \right]^2$$

- Where $\nabla I = [I_x \quad I_y]$
- Differentiate wrt $\Delta \boldsymbol{p}$ and setting it to zero:

$$\sum_{x} \left[ \nabla I \frac{\partial W}{\partial \boldsymbol{p}} \right]^T \left[ I(W(\boldsymbol{x}; \boldsymbol{p_0})) + \nabla I \frac{\partial W}{\partial \boldsymbol{p}} \Delta \boldsymbol{p} - T(x) \right] = 0$$

# Solving for $\Delta \boldsymbol{p}$

- Solving for $\Delta \boldsymbol{p}$ in:

$$\sum_{x} \left[\nabla I \frac{\partial W}{\partial \boldsymbol{p}}\right]^{T} \left[I(W(\boldsymbol{x}; \boldsymbol{p_0})) + \nabla I \frac{\partial W}{\partial \boldsymbol{p}} \Delta \boldsymbol{p} - T(x)\right] = 0$$

- we get:

$$\Delta \boldsymbol{p} = H^{-1} \sum_{x} \left[\nabla I \frac{\partial W}{\partial \boldsymbol{p}}\right]^{T} [T(x) - I(W(\boldsymbol{x}; \boldsymbol{p_0}))]$$

where $H = \sum_x \left[\nabla I \frac{\partial W}{\partial \boldsymbol{p}}\right]^{T} \left[\nabla I \frac{\partial W}{\partial \boldsymbol{p}}\right]$

# Interpreting the H matrix for translation transformations

$$H = \sum_{x} \left[\nabla I \frac{\partial W}{\partial \boldsymbol{p}}\right]^T \left[\nabla I \frac{\partial W}{\partial \boldsymbol{p}}\right]$$

Recall that

1. $\nabla I = [I_x \quad I_y]$ and
2. for translation motion, $\frac{\partial W}{\partial \boldsymbol{p}}(\boldsymbol{x};\boldsymbol{p}) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Therefore,

$$H = \sum_{x} \left[[I_x \quad I_y]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right]^T \left[[I_x \quad I_y]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right]$$

$$= \boxed{\sum_{x} \begin{bmatrix} {I_x}^2 & I_x I_y \\ I_x I_y & {I_y}^2 \end{bmatrix}}$$

That's the Harris corner detector we learnt in class!!!

# Interpreting the H matrix for affine transformations

$$H=\sum_{\mathbf{x}}\begin{bmatrix} I_x^2 & I_xI_y & xI_x^2 & yI_xI_y & xI_xI_y & yI_xI_y \\ I_xI_y & I_y^2 & xI_xI_y & yI_y^2 & xI_y^2 & yI_y^2 \\ xI_x^2 & yI_xI_y & x^2I_x^2 & y^2I_xI_y & xyI_xI_y & y^2I_xI_y \\ yI_xI_y & yI_y^2 & xyI_xI_y & y^2I_y^2 & xyI_y^2 & y^2I_y^2 \\ xI_xI_y & xI_y^2 & x^2I_xI_y & xyI_y^2 & x^2I_y^2 & xyI_y^2 \\ yI_xI_y & yI_y^2 & xyI_xI_y & y^2I_y^2 & xyI_y^2 & y^2I_y^2 \end{bmatrix}$$

Can you derive this yourself similarly to how we derived the translation transformation?

# Overall KLT tracker algorithm

Given the features from Harris detector:

1. Initialize $\boldsymbol{p_0}$ and $\Delta\boldsymbol{p}$ .
2. Compute the initial templates $T(x)$ for each feature.
3. Transform the features in the image $I$ with $W(\boldsymbol{x};\boldsymbol{p_0})$.
4. Measure the error: $I(W(\boldsymbol{x};\boldsymbol{p_0})) - T(x)$.
5. Compute the image gradients $\nabla I = [I_x \quad I_y]$.
6. Evaluate the Jacobian $\frac{\partial W}{\partial \boldsymbol{p}}$.
7. Compute steepest descent $\nabla I \frac{\partial W}{\partial \boldsymbol{p}}$.
8. Compute Inverse Hessian $H^{-1}$
9. Calculate the change in parameters $\Delta\boldsymbol{p}$
10. Update parameters $\boldsymbol{p} = \boldsymbol{p_0} + \Delta\boldsymbol{p}$

# Iterative KLT

- Once you find a transformation for two frames, you will repeat this process for every couple of frames.
- Run Harris detector every 15-20 frames to find new features.

# Challenges to consider

- Implementation issues
- Window size
  - Small window more sensitive to noise and may miss larger motions (without pyramid)
  - Large window more likely to cross an occlusion boundary (and it's slower)
  - 15x15 to 31x31 seems typical
- Weighting the window
  - Common to apply weights so that center matters more (e.g., with Gaussian)

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