Lecture:
Visual Bag of Words

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What we will learn today

- Visual bag of words (BoW)
- Spatial Pyramid Matching
- Naive Bayes

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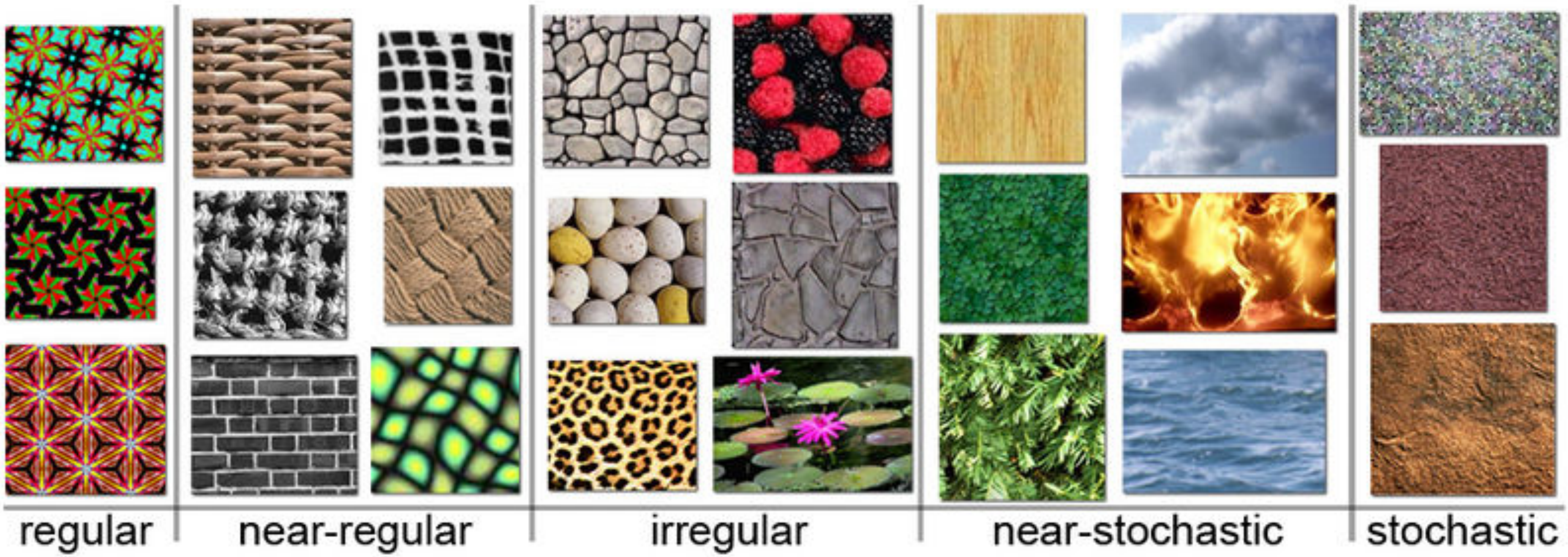
Bag of Words Models

Adapted from slides by Rob Fergus and Svetlana Lazebnik

Object → Bag of 'words'

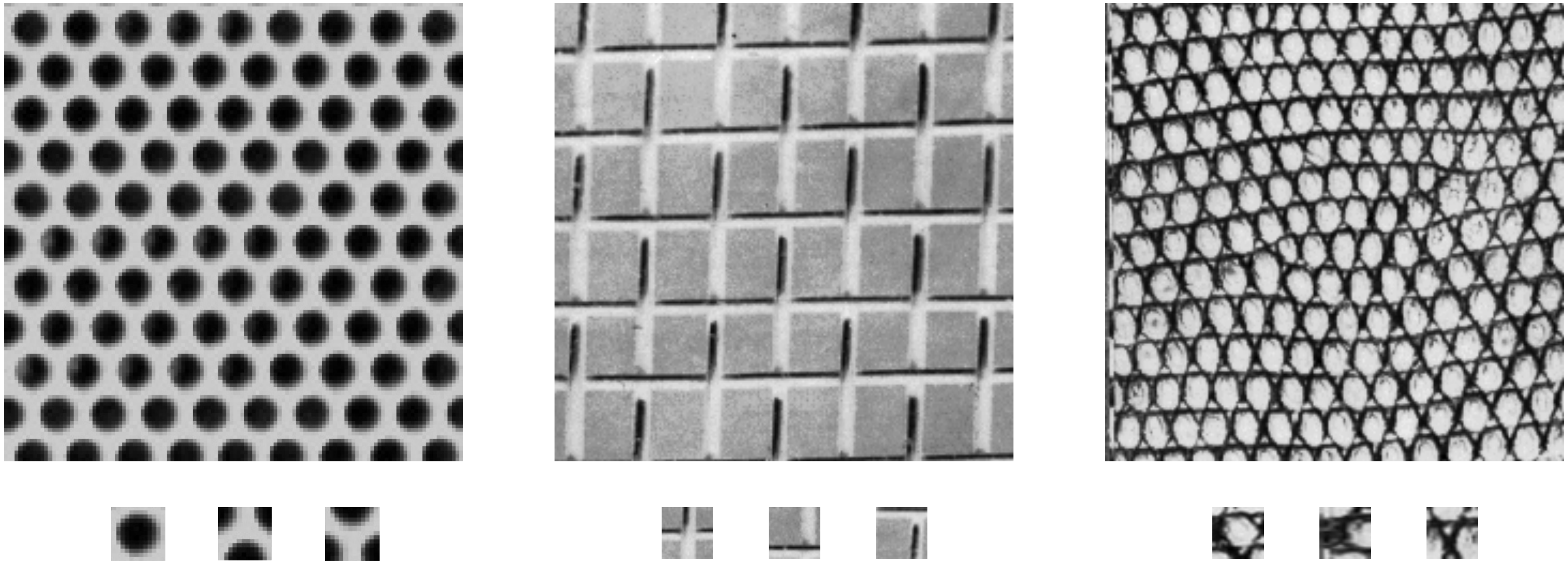
Origin 1: Texture Recognition



Example textures (from Wikipedia)

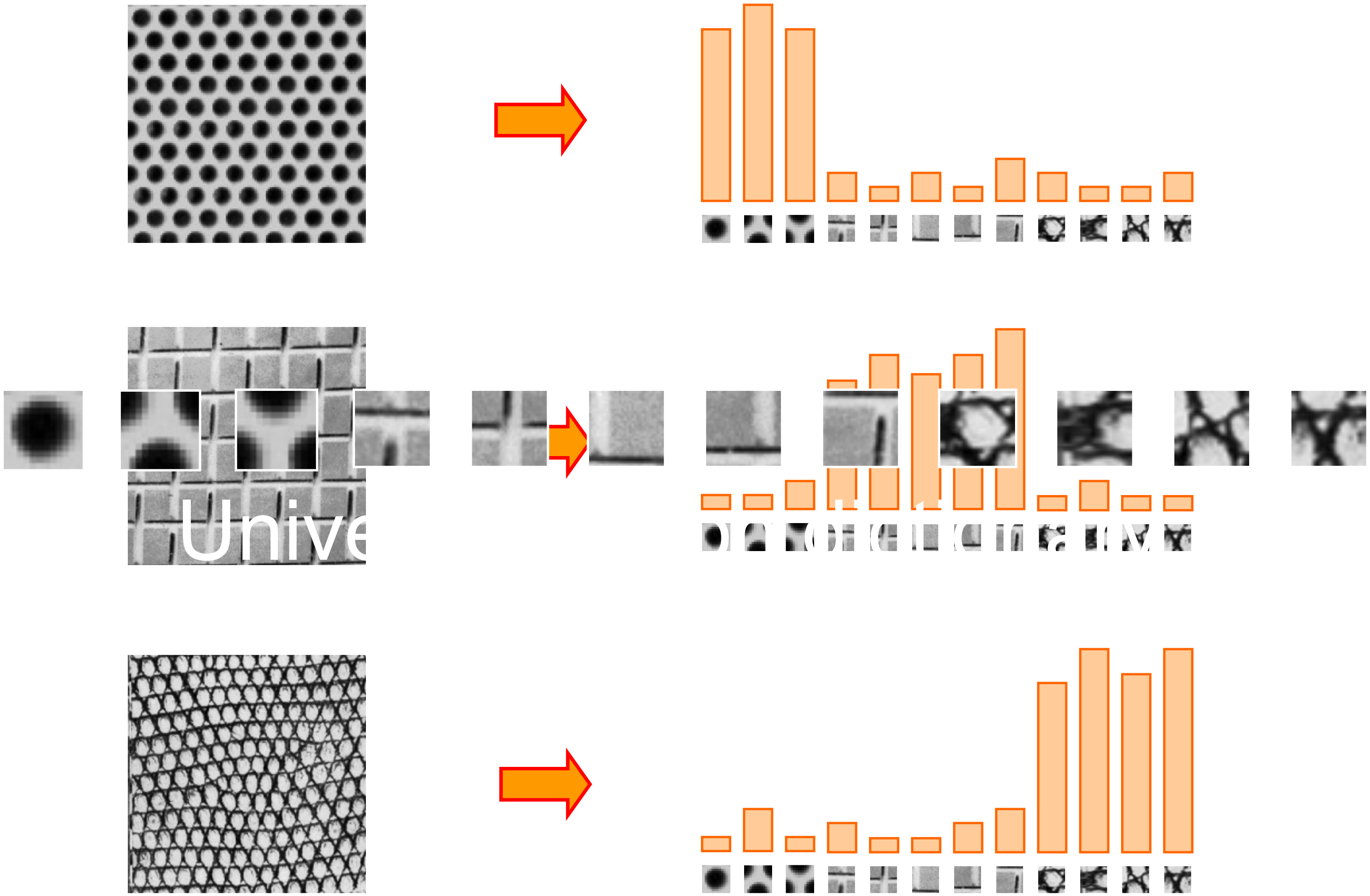
Origin 1: Texture recognition

- Texture is characterized by the repetition of basic elements or ***textons***



Julesz, 1981; Cula & Dana, 2001; Leung & Malik 2001; Mori, Belongie & Malik, 2001; Schmid 2001; Varma & Zisserman, 2002, 2003; Lazebnik, Schmid & Ponce, 2003

Origin 1: Texture recognition



Origin 2: Bag-of-words models

- Orderless document representation: frequencies of words from a dictionary Salton & McGill (1983)

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US Presidential Speeches Tag Cloud
`http://chir.ag/phernalia/preztags/`

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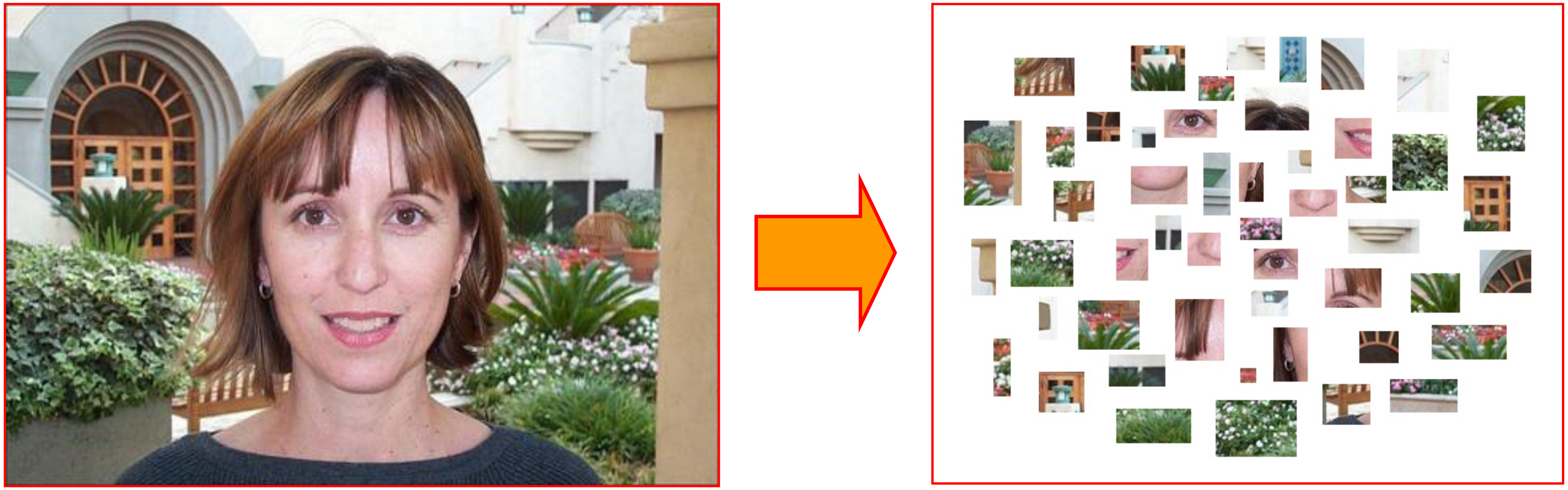
Bags of features for object recognition



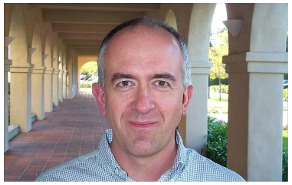
face, flowers, building

- Works pretty well for image-level classification and for recognizing object *instances*

Csurka et al. (2004), Willamowski et al. (2005), Grauman & Darrell (2005), Sivic et al. (2003, 2005)

Bags of features for object recognition



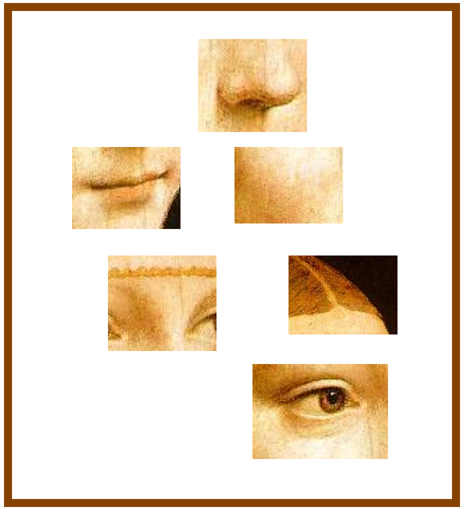
class	bag of features Zhang et al. (2005)	bag of features Willamowski et al. (2004)	Parts-and-shape model Fergus et al. (2003)
airplanes	**98.8**	97.1	90.2
cars (rear)	98.3	**98.6**	90.3
cars (side)	**95.0**	87.3	88.5
faces	**100**	99.3	96.4
motorbikes	**98.5**	98.0	92.5
spotted cats	**97.0**	—	90.0

Bag of features

- First, take a bunch of images, extract features, and build up a “dictionary” or “visual vocabulary” – a list of common features
- Given a new image, extract features and build a histogram – for each feature, find the closest visual word in the dictionary

Bag of features: outline

1. Extract features



Bag of features: outline

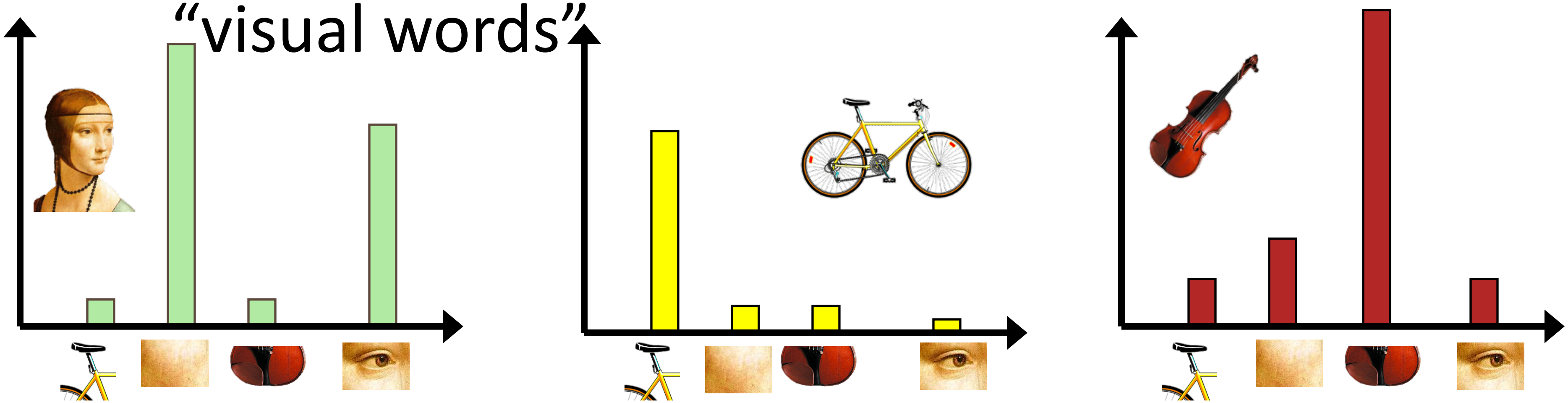
1. Extract features
2. Learn "visual vocabulary"

Bag of features: outline

1. Extract features
2. Learn “visual vocabulary”
3. Quantize features using visual vocabulary

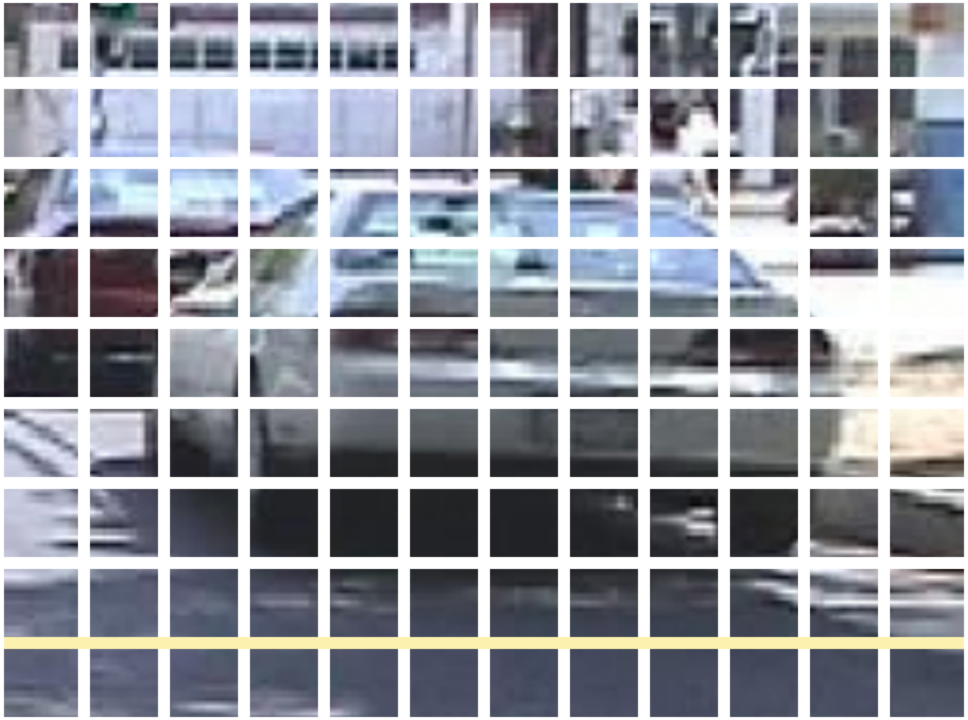
Bag of features: outline

1. Extract features
2. Learn "visual vocabulary"
3. Quantize features using visual vocabulary
4. Represent images by frequencies of "visual words"



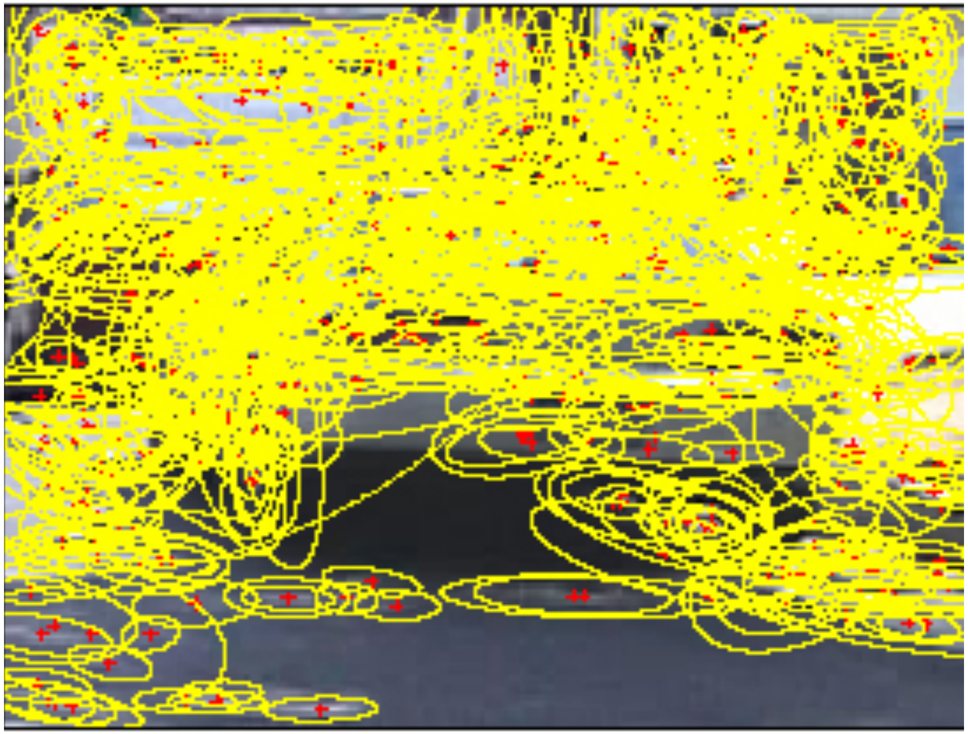
1. Feature extraction

- Regular grid
  - Vogel & Schiele, 2003
  - Fei-Fei & Perona, 2005



1. Feature extraction

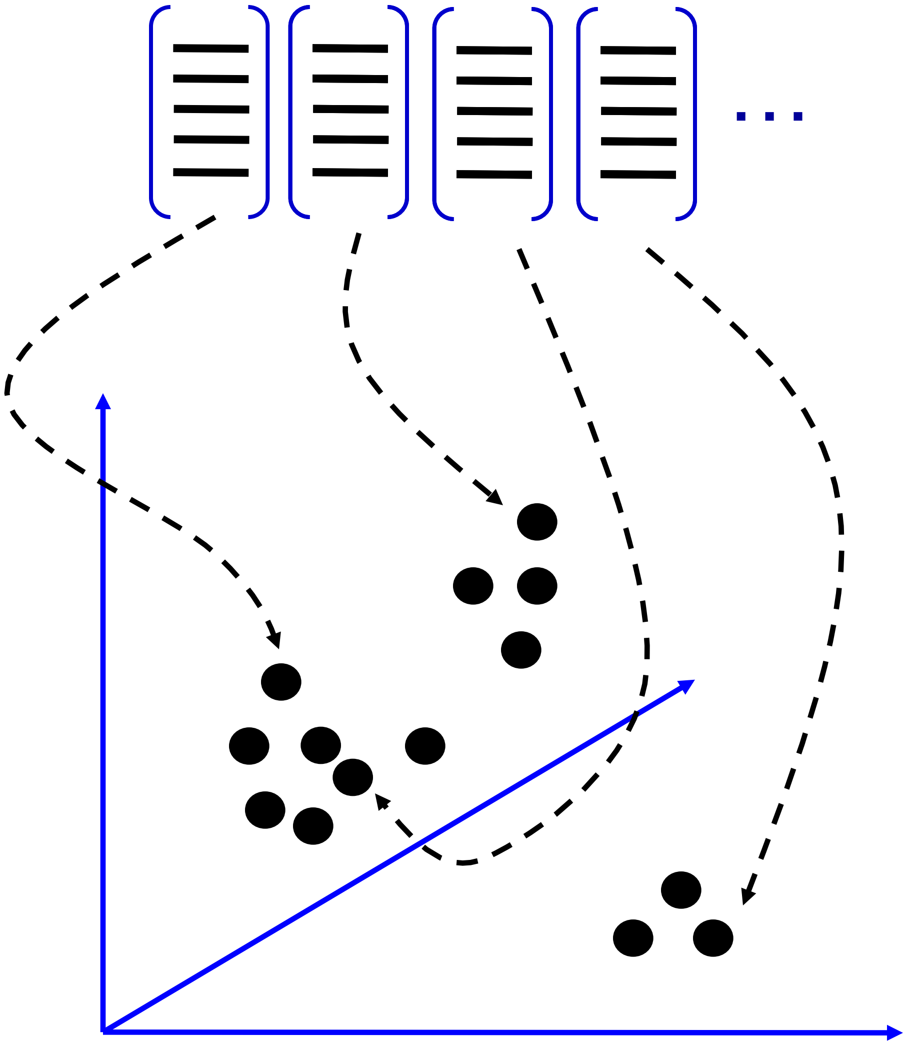
- Regular grid
  - Vogel & Schiele, 2003
  - Fei-Fei & Perona, 2005
- Interest point detector
  - Csurka et al. 2004
  - Fei-Fei & Perona, 2005
  - Sivic et al. 2005



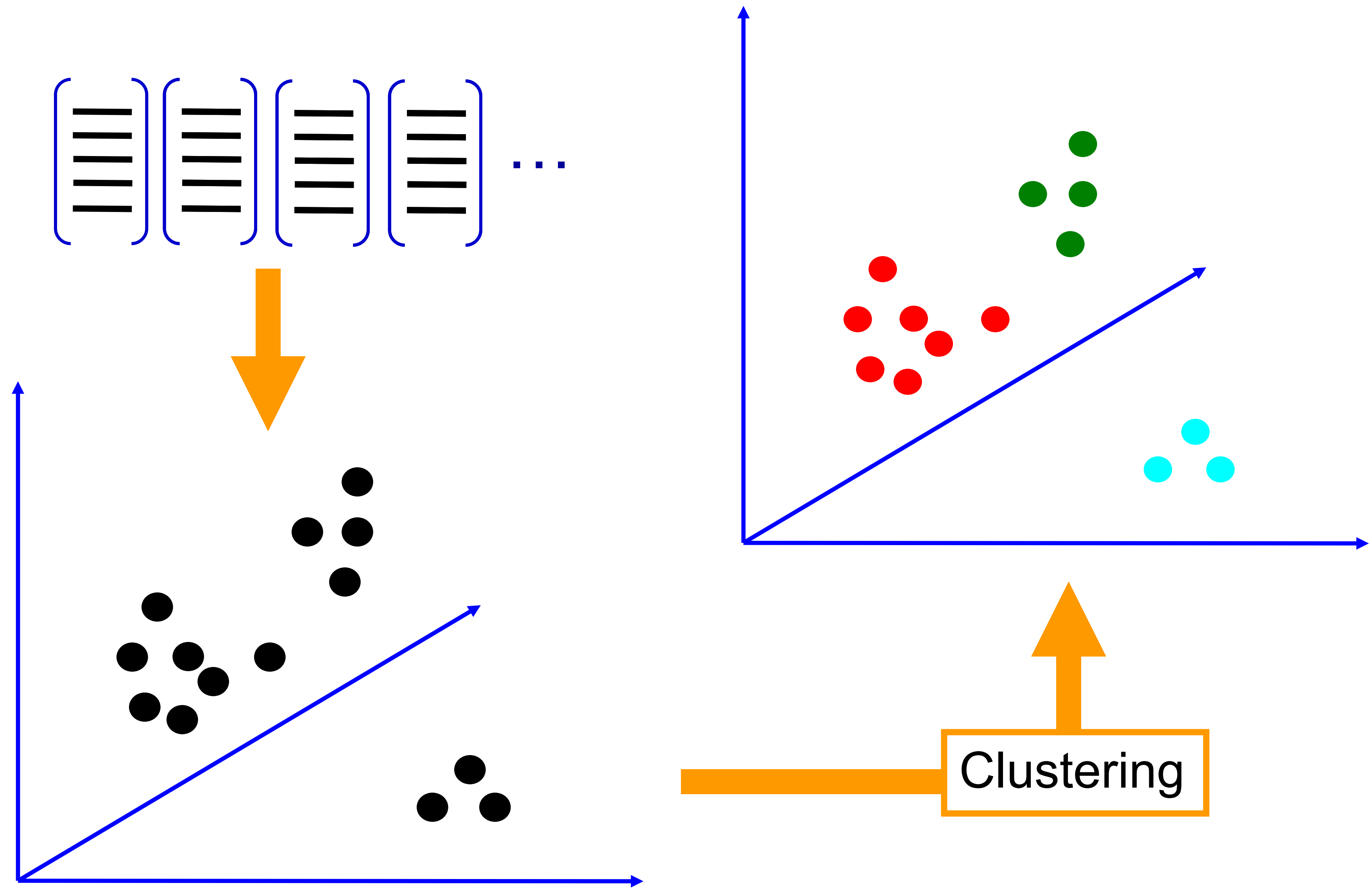
1. Feature extraction

- Regular grid
  - Vogel & Schiele, 2003
  - Fei-Fei & Perona, 2005
- Interest point detector
  - Csurka et al. 2004
  - Fei-Fei & Perona, 2005
  - Sivic et al. 2005
- Other methods
  - Random sampling (Vidal-Naquet & Ullman, 2002)
  - Segmentation-based patches (Barnard et al. 2003)

2. Learning the visual vocabulary

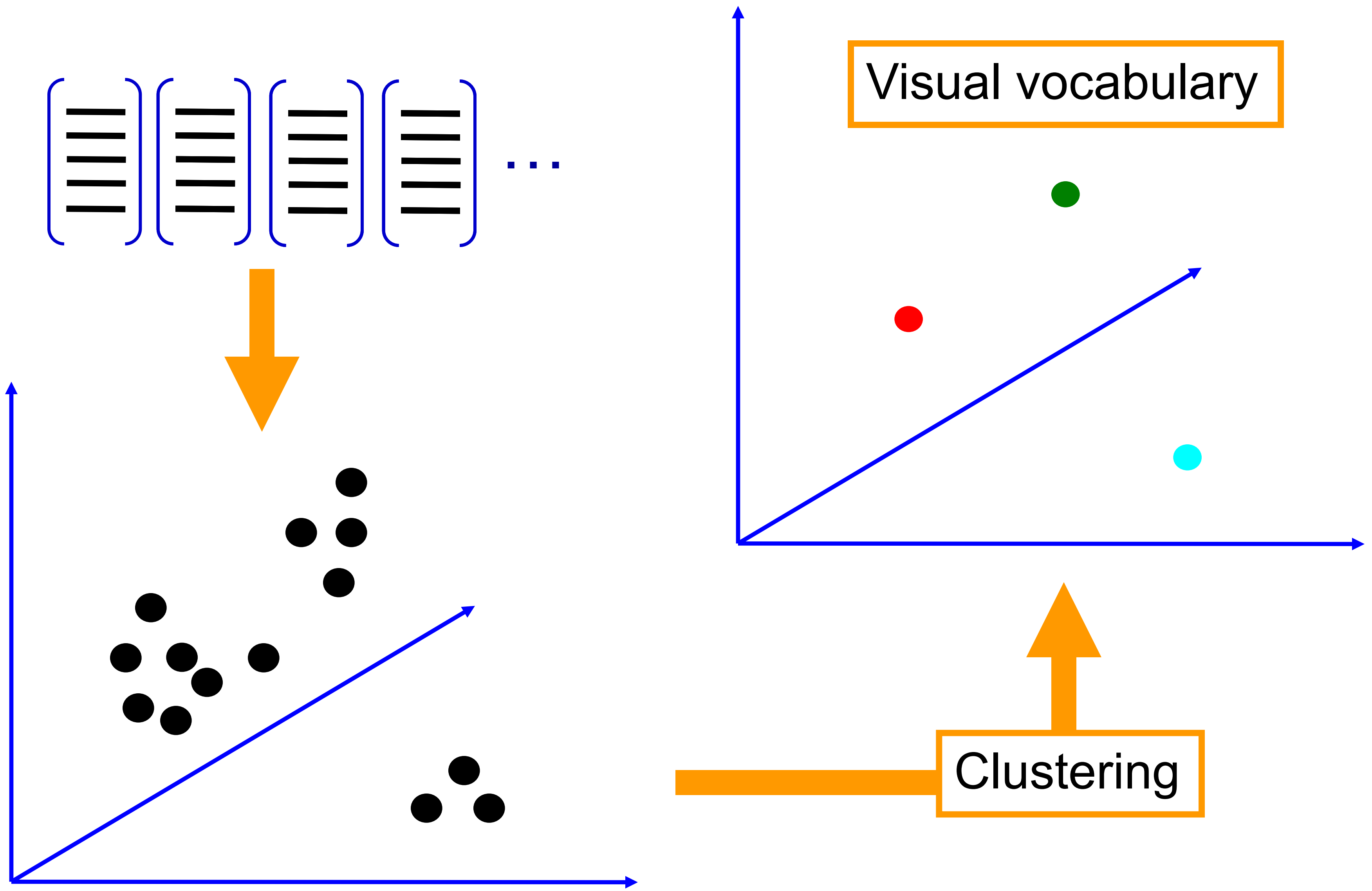


2. Learning the visual vocabulary



Slide credit: Josef Sivic

2. Learning the visual vocabulary



Slide credit: Josef Sivic

K-means clustering recap

- Want to minimize sum of squared Euclidean distances between points x_i and their nearest cluster centers m_k

$$D(X,M) = \sum_{\text{cluster } k} \sum_{\substack{\text{point } i \text{ in} \\ \text{cluster } k}} (x_i - m_k)^2$$

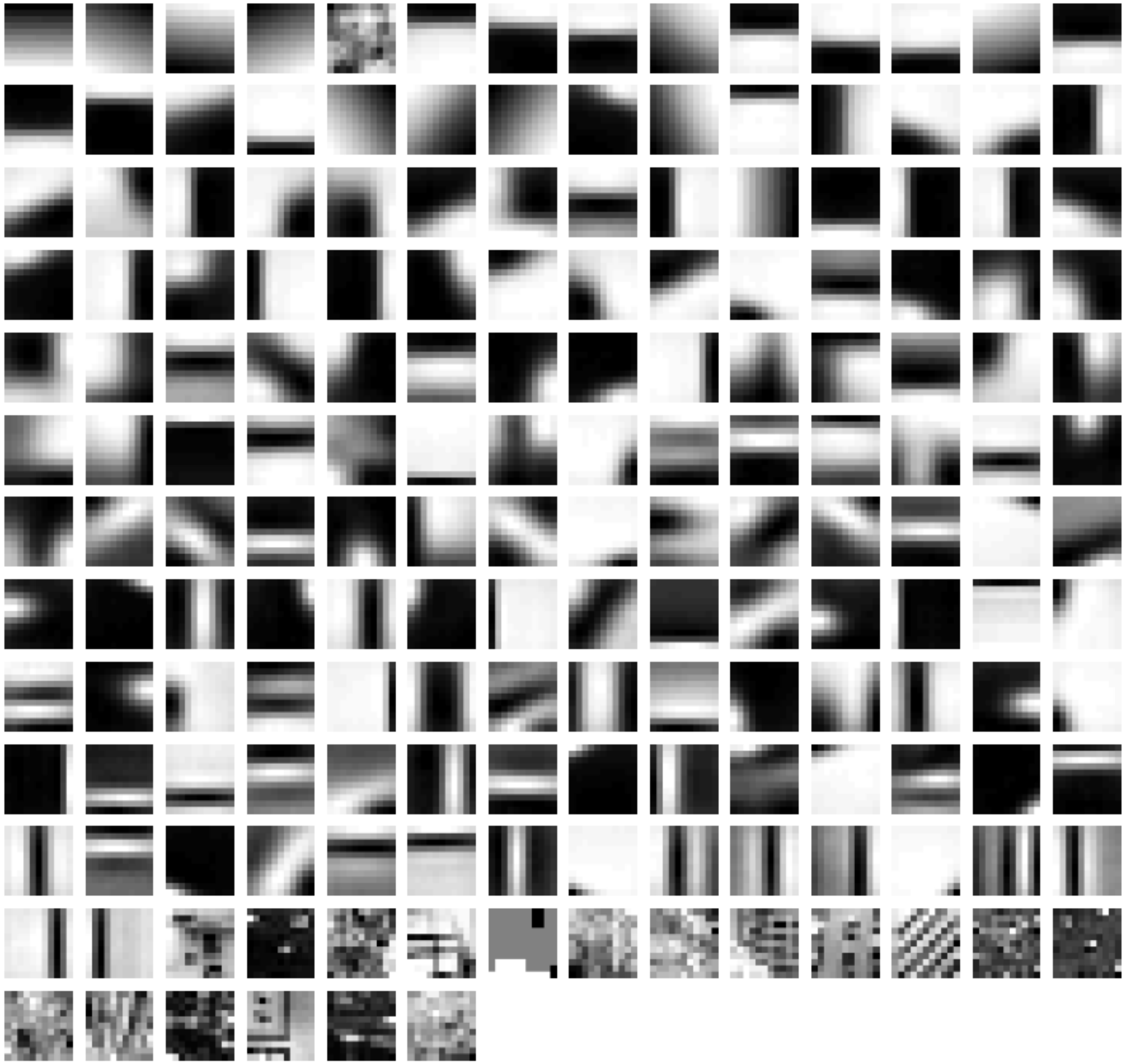
- Algorithm:
- Randomly initialize K cluster centers
- Iterate until convergence:
  - Assign each data point to the nearest center
  - Recompute each cluster center as the mean of all points assigned to it

From clustering to vector quantization

- Clustering is a common method for learning a visual vocabulary or codebook
  - Unsupervised learning process
  - Each cluster center produced by k-means becomes a codevector
  - Codebook can be learned on separate training set
  - Provided the training set is sufficiently representative, the codebook will be "universal"

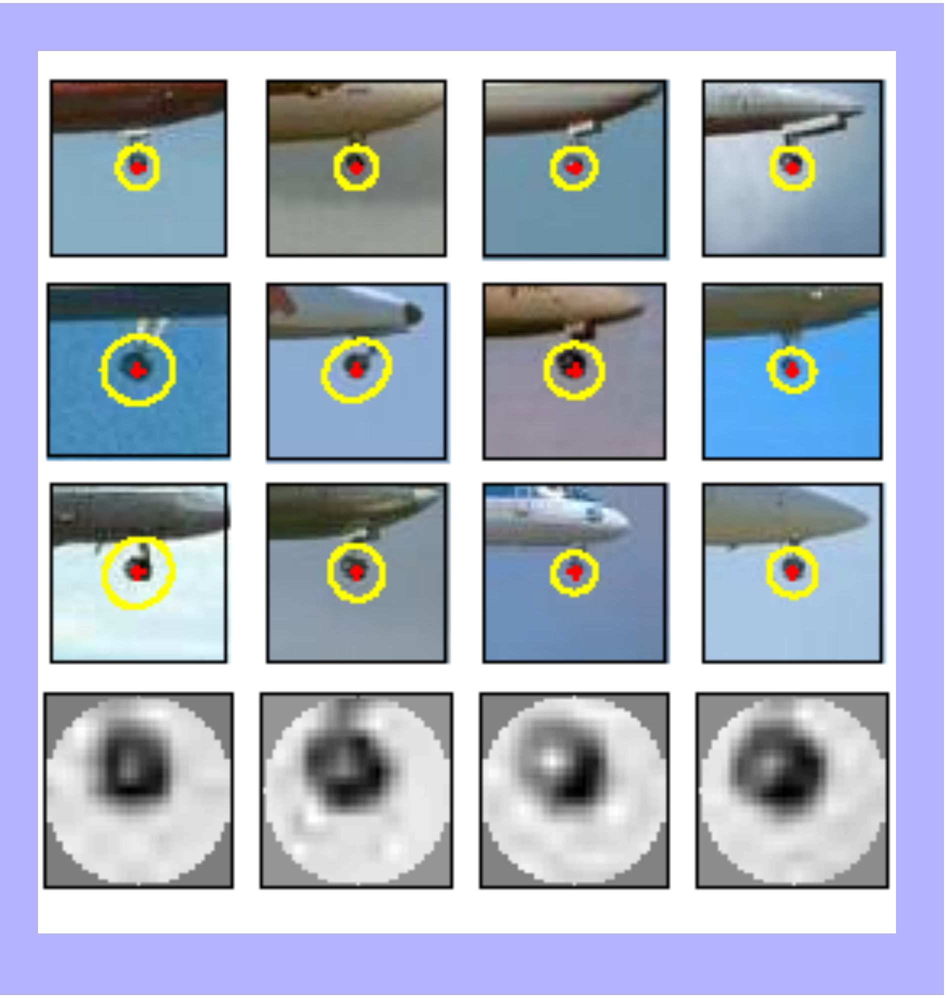
- The codebook is used for quantizing features
  - A *vector quantizer* takes a feature vector and maps it to the index of the nearest codevector in a codebook
  - Codebook = visual vocabulary
  - Codevector = visual word

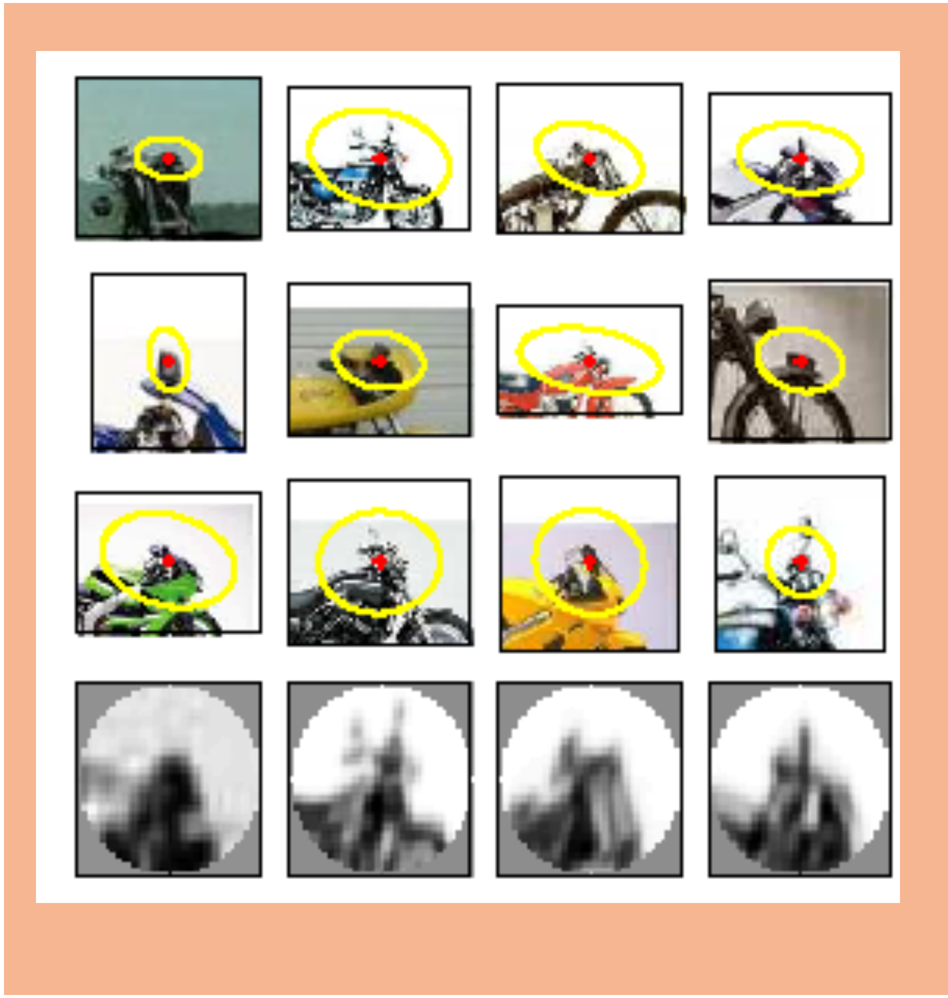
Example visual vocabulary



Fei-Fei et al. 2005

Image patch examples of visual words

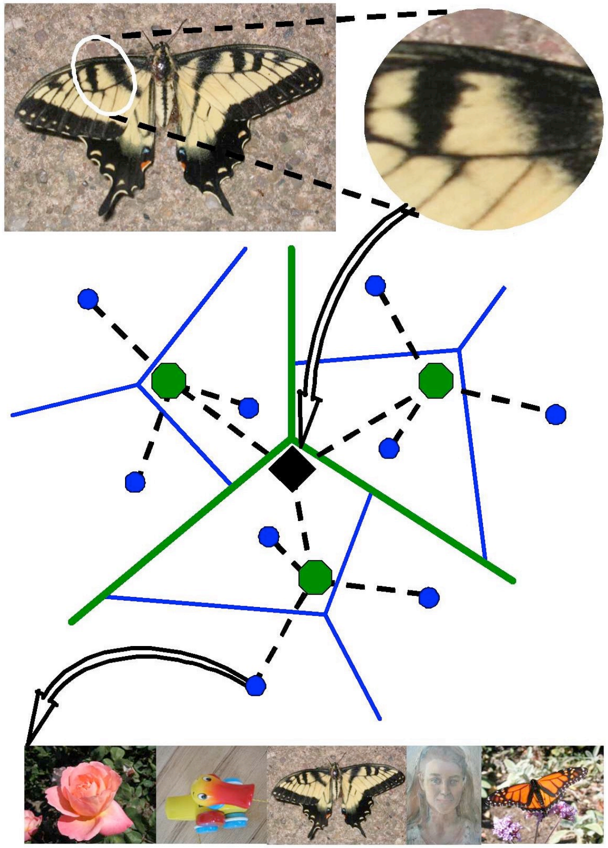




Sivic et al. 2005

Visual vocabularies: Issues

- How to choose vocabulary size?
  - Too small: visual words not representative of all patches
  - Too large: quantization artifacts, overfitting
- Computational efficiency
  - Vocabulary trees (Nister & Stewenius, 2006)



3. Image representation

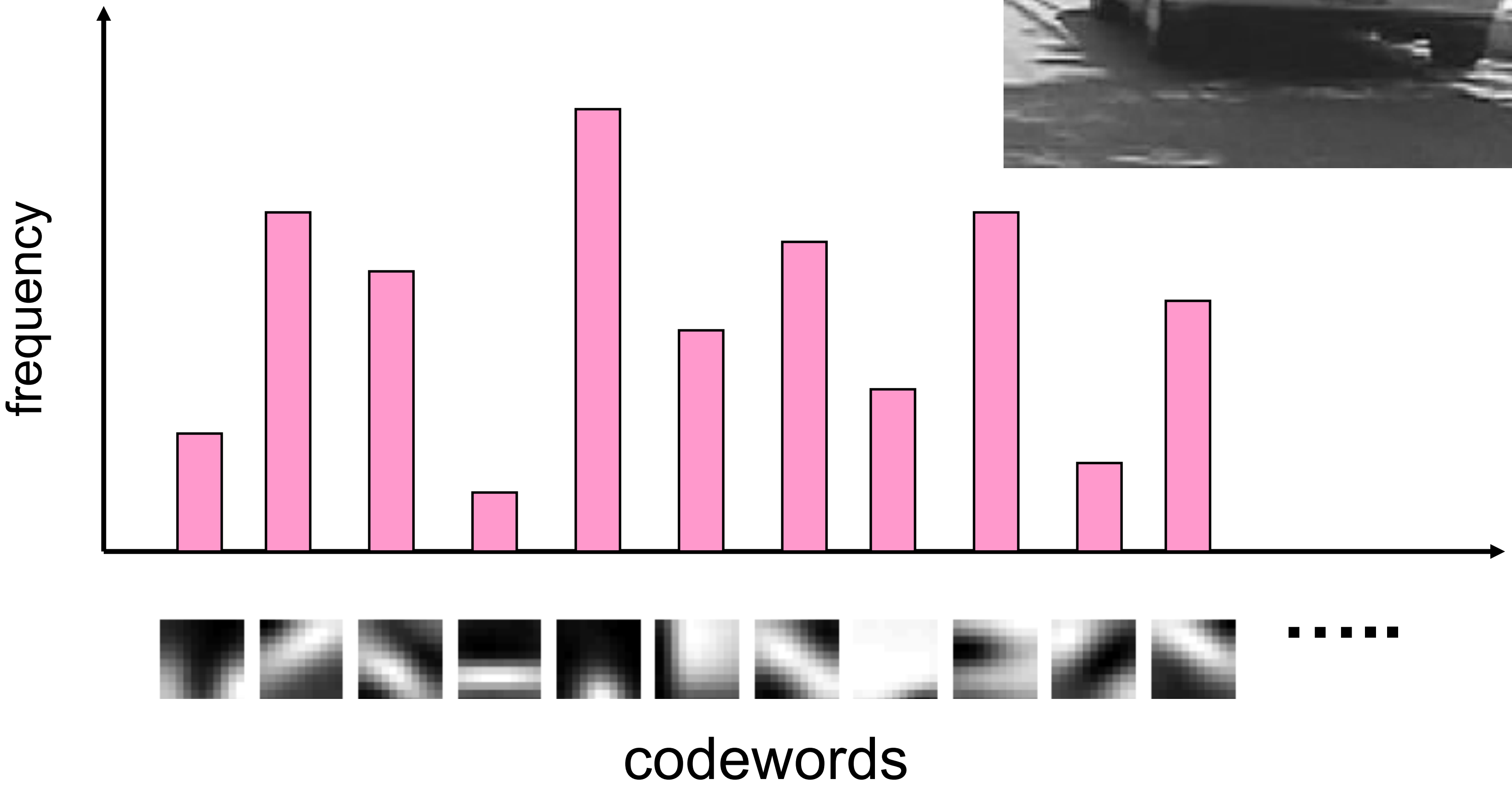
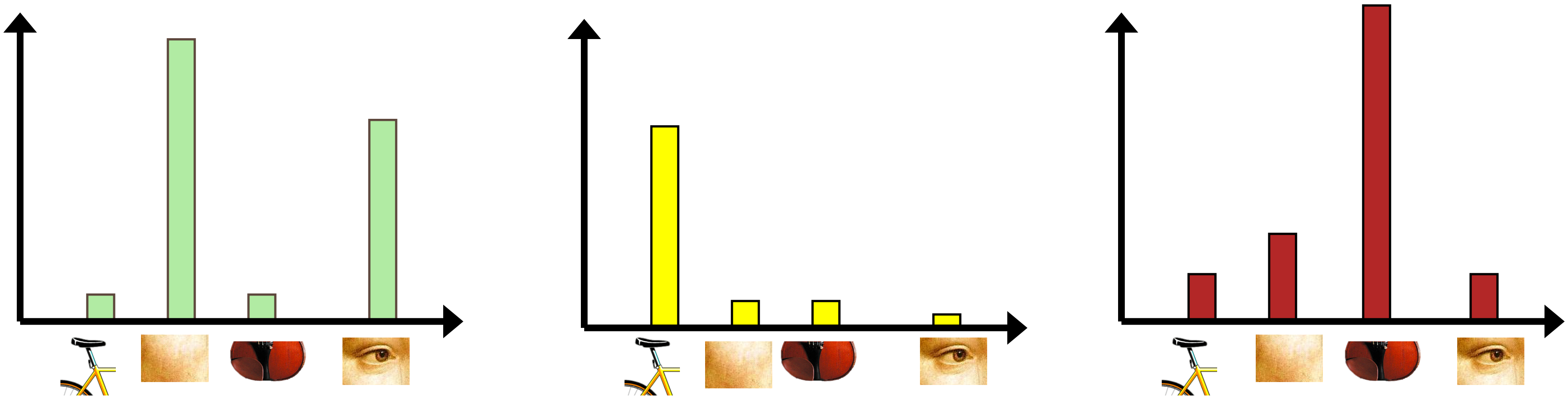


Image classification

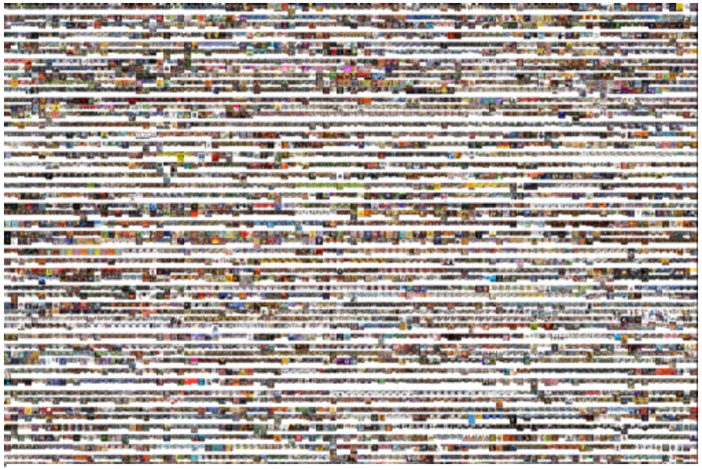
- Given the bag-of-features representations of images from different classes, how do we learn a model for distinguishing them?



Uses of BoW representation

- Treat as feature vector for standard classifier
  - e.g k-nearest neighbors, support vector machine
- Cluster BoW vectors over image collection
  - Discover visual themes

Large-scale image matching



11,400 images of game covers
(Caltech games dataset)

- Bag-of-words models have been useful in matching an image to a large database of object *instances*

how do I find this image in the database?

Large-scale image search

Build the database:

- Extract features from the database images
- Learn a vocabulary using k-means (typical k: 100,000)
- Compute *weights* for each word
- Create an inverted file mapping words → images

Weighting the words

- Just as with text, some visual words are more discriminative than others

the, and, or vs. ***cow, AT&T, Cher***

- the bigger fraction of the documents a word appears in, the less useful it is for matching
  - e.g., a word that appears in *all* documents is not helping us

TF-IDF weighting

- Instead of computing a regular histogram distance, we'll weight each word by it's *inverse document frequency*

- inverse document frequency (IDF) of word *j* =

$$\log \frac{\text{number of documents}}{\text{number of documents in which } j \text{ appears}}$$

TF-IDF weighting

- To compute the value of bin *j* in image *I*:

term frequency of *j* in *I* **x** *inverse document frequency* of *j*

Inverted file

- Each image has ~1,000 features
- We have ~100,000 visual words
  →each histogram is extremely sparse (mostly zeros)

- Inverted file
  – mapping from words to documents

```
"a":      {2}
"banana": {2}
"is":     {0, 1, 2}
"it":     {0, 1, 2}
"what":   {0, 1}
```

Inverted file

- Can quickly use the inverted file to compute similarity between a new image and all the images in the database
  - Only consider database images whose bins overlap the query image

Large-scale image search

query image top 6 results

- Cons:
  - performance degrades as the database grows

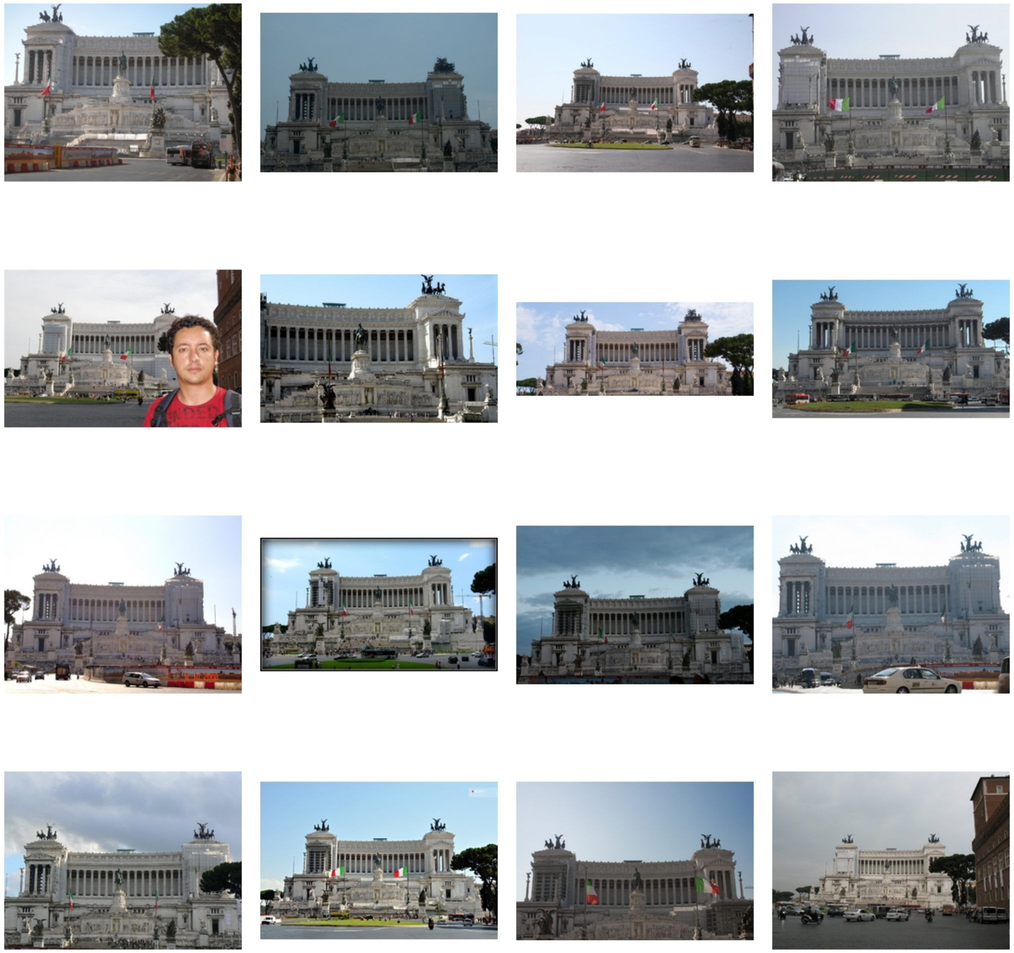
Large-scale image search

- Pros:
  - Works well for CD covers, movie posters
  - Real-time performance possible

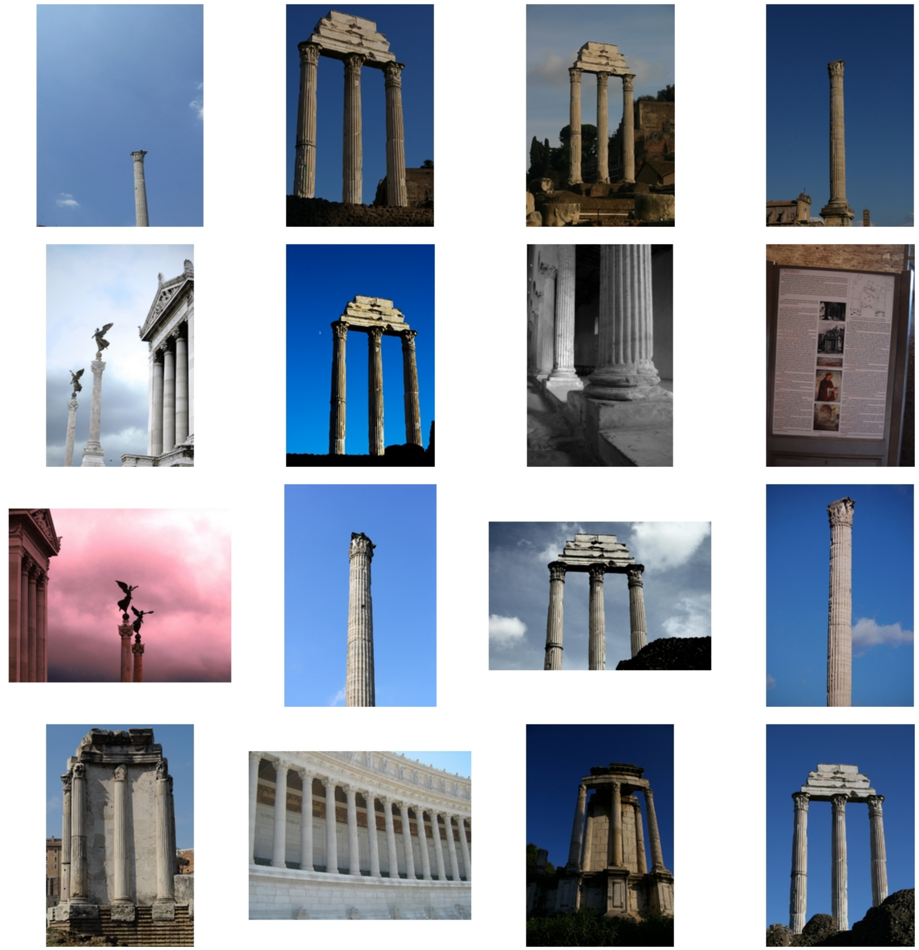
real-time retrieval from a database of 40,000 CD covers

Nister & Stewenius, **Scalable Recognition with a Vocabulary Tree**

Example bag-of-words matches



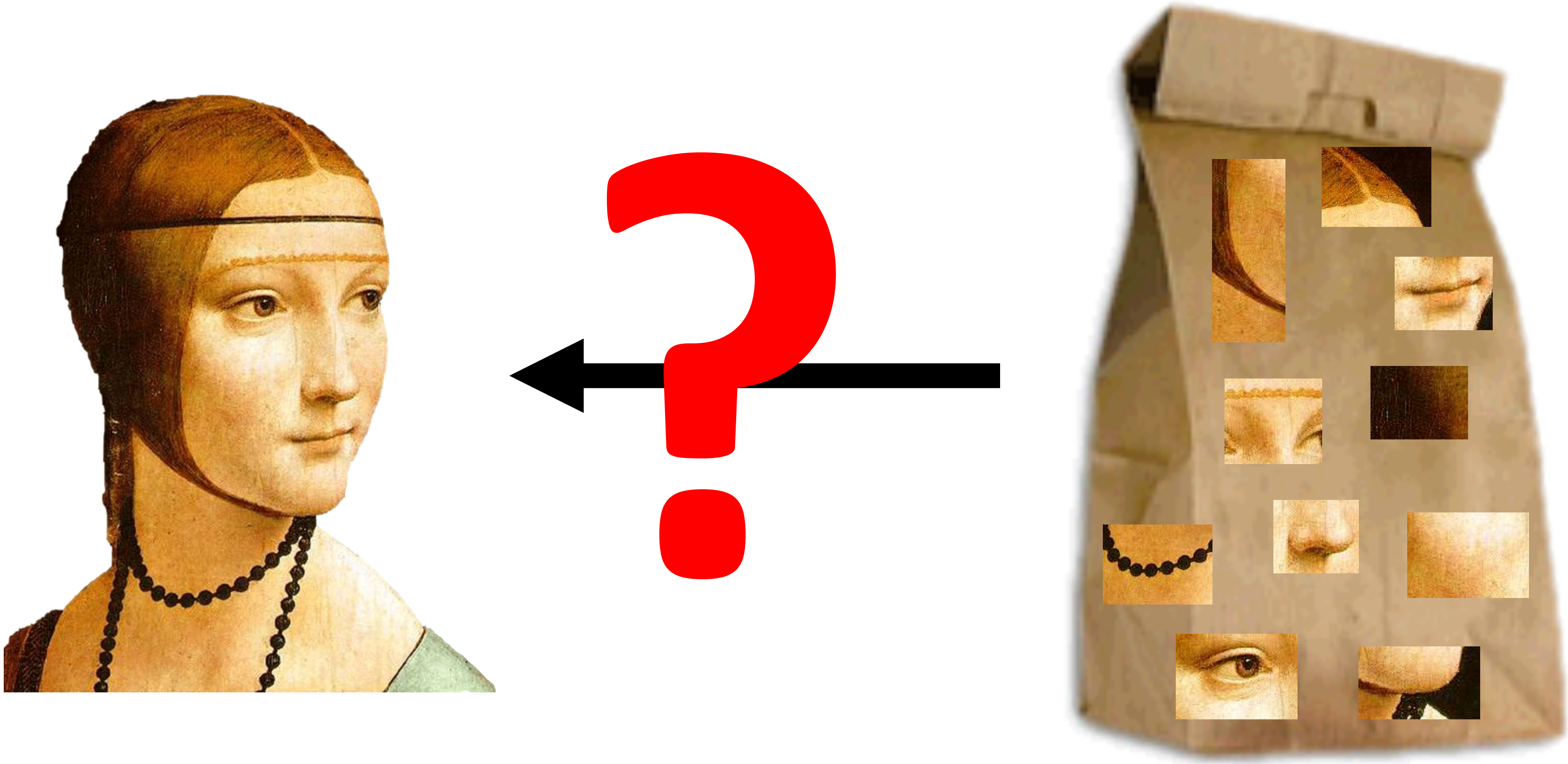
Example bag-of-words matches



Matching Statistics

Dataset	Size	Matches possible	Matches Tried	Matches Found	Time
Dubrovnik	58K	1.6 Billion	2.6M	0.5M	5 hrs
Rome	150K	11.2 Billion	8.8M	2.7M	13 hrs
Venice	250K	31.2 Billion	35.5M	6.2M	27 hrs

What about spatial info?



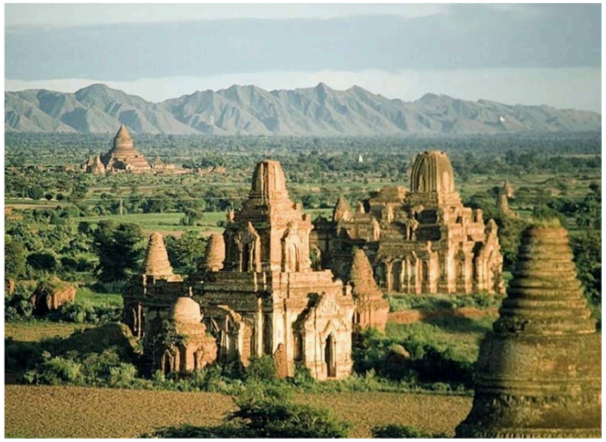
What we will learn today

- Visual bag of words (BoW)
- Spatial Pyramid Matching
- Naïve Bayes

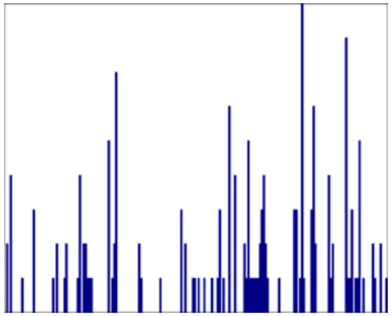
Pyramids

- Very useful for representing images.
- Pyramid is built by using multiple copies of image.
- Each level in the pyramid is 1/4 of the size of previous level.
- The lowest level is of the highest resolution.
- The highest level is of the lowest resolution.

Bag of words + pyramids

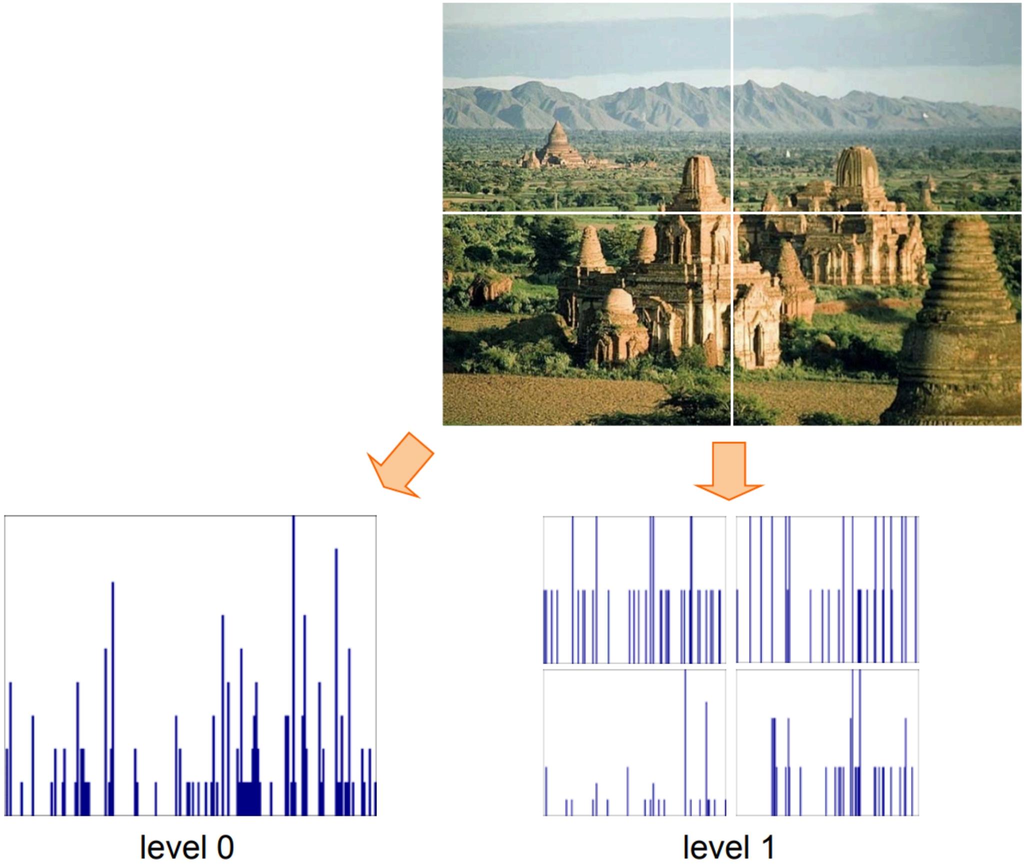


Locally orderless representation at several levels of spatial resolution



level 0

Bag of words + pyramids



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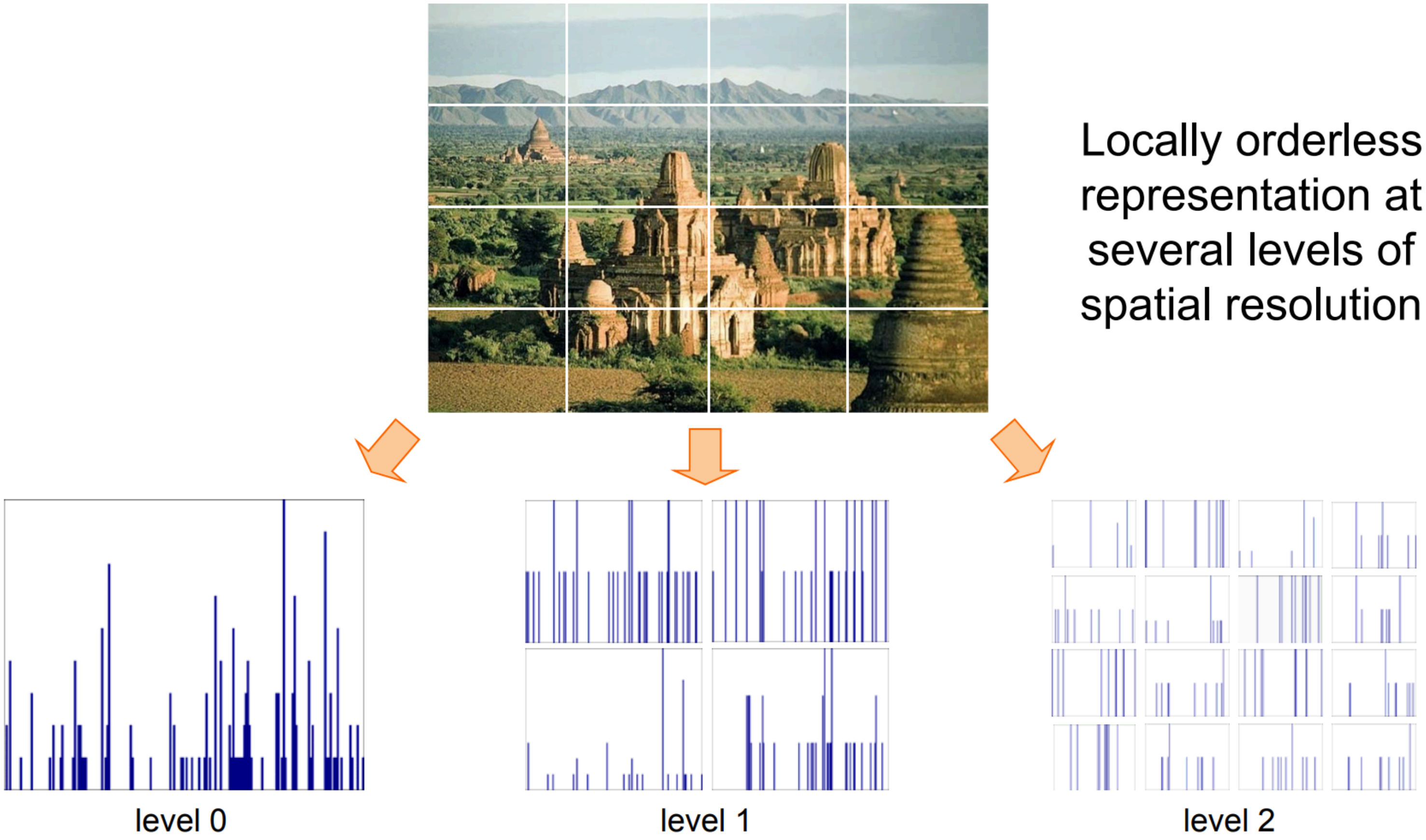
Bag of words + pyramids



Locally orderless representation at several levels of spatial resolution

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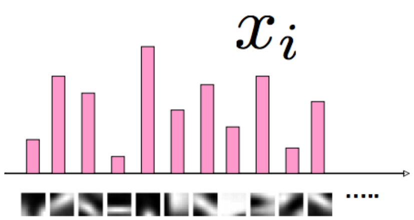
Naïve Bayes

- Classify image using histograms of occurrences on visual words:

$$\mathbf{x} =$$



- if only present/absence of a word is taken into account: $x_i \in \{0, 1\}$
- Naïve Bayes classifier assumes that visual words are conditionally independent given object class

Csurka Bray, Dance & Fan, 2004

Naïve Bayes - prior

- Model for each object class:

$$P(x \mid c) = \prod_{i=1}^{m} P(x_i \mid c)$$

- Class priors $P(c)$ encode how likely we are to see one class versus others.
- Note that:

$$\prod_{i=1}^{m} P(c) = 1$$

Csurka Bray, Dance & Fan, 2004

Naïve Bayes - posterior

- With the equations from the previous slides, we can now calculate the probability that an image represented by $\boldsymbol{x}$ belongs to class category c.

$$P(c \mid \boldsymbol{x}) = \frac{P(c)\, P(\boldsymbol{x} \mid c)}{\sum_{c'} P(c')\, P(\boldsymbol{x} \mid c)}$$

Bayes Theorem

Naïve Bayes – posterior

- With the equations from the previous slides, we can now calculate the probability that an image represented by $\boldsymbol{x}$ belongs to class category c.

$$P(c \mid \boldsymbol{x}) = \frac{P(c)\, P(\boldsymbol{x} \mid c)}{\sum_{c'} P(c')\, P(\boldsymbol{x} \mid c')}$$

$$P(c \mid \boldsymbol{x}) = \frac{P(c) \prod_{i=1}^{m} P(x_i \mid c)}{\sum_{c'} P(c') \prod_{i=1}^{m} P(x_i \mid c')}$$

Naïve Bayes - classification

- We can now classify that the image represented by $\boldsymbol{x}$ is belongs class that has the highest probability:

$$c^* = arg\max_c P(c \mid \boldsymbol{x})$$

$$c^* = arg\max_c \log P(c \mid \boldsymbol{x})$$

Let's break down the posterior

The probability that $\boldsymbol{x}$ belongs to class c_1:

$$P(c_1 \mid \boldsymbol{x}) = \frac{P(c_1) \prod_{i=1}^{m} P(x_i \mid c_1)}{\sum_{c'} P(c') \prod_{i=1}^{m} P(x_i \mid c')}$$

And the probability that $\boldsymbol{x}$ belongs to class c_2:

$$P(c_2 \mid \boldsymbol{x}) = \frac{P(c_2) \prod_{i=1}^{m} P(x_i \mid c_2)}{\sum_{c'} P(c') \prod_{i=1}^{m} P(x_i \mid c')}$$

Both their denominators are the same

The probability that $\boldsymbol{x}$ belongs to class c_1:

$$P(c_1 \mid \boldsymbol{x}) = \frac{P(c_1)\prod_{i=1}^{m} P(x_i \mid c_1)}{\sum_{c'} P(c')\prod_{i=1}^{m} P(x_i \mid c')}$$

And the probability that $\boldsymbol{x}$ belongs to class c_2:

$$P(c_2 \mid \boldsymbol{x}) = \frac{P(c_2)\prod_{i=1}^{m} P(x_i \mid c_2)}{\sum_{c'} P(c')\prod_{i=1}^{m} P(x_i \mid c')}$$

Both their denominators are the same

- Since we only want the max, we can ignore the denominator:

$$P(c_1 \mid \boldsymbol{x}) \propto P(c_1) \prod_{i=1}^{m} P(x_i \mid c_1)$$

$$P(c_2 \mid \boldsymbol{x}) \propto P(c_2) \prod_{i=1}^{m} P(x_i \mid c_2)$$

For the general class c,

$$P(c \mid \boldsymbol{x}) \propto P(c) \prod_{i=1}^{m} P(x_i \mid c)$$

For the general class c,

$$P(c \mid \boldsymbol{x}) \propto P(c) \prod_{i=1}^{m} P(x_i \mid c)$$

We can take the log:

$$\log P(c \mid \boldsymbol{x}) \propto logP(c) + \sum_{i=1}^{m} logP(x_i \mid c)$$

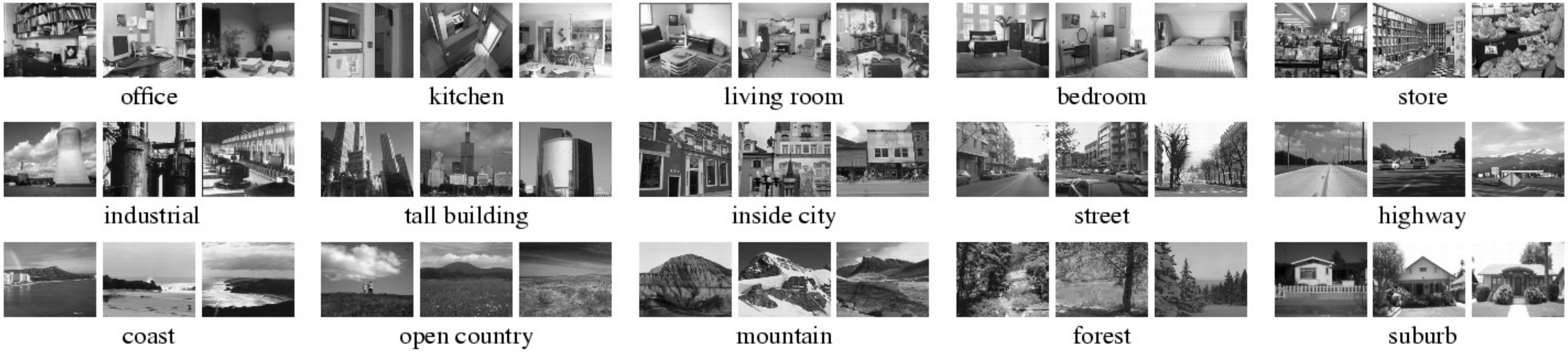
Naïve Bayes - classification

- So, the following classification becomes:

$$c^* = arg\max_c P(c \mid \boldsymbol{x})$$
$$c^* = arg\max_c \log P(c \mid \boldsymbol{x})$$

$$c^* = arg\max_c logP(c) + \sum_{i=1}^{m} logP(x_i \mid c)$$

Scene category dataset

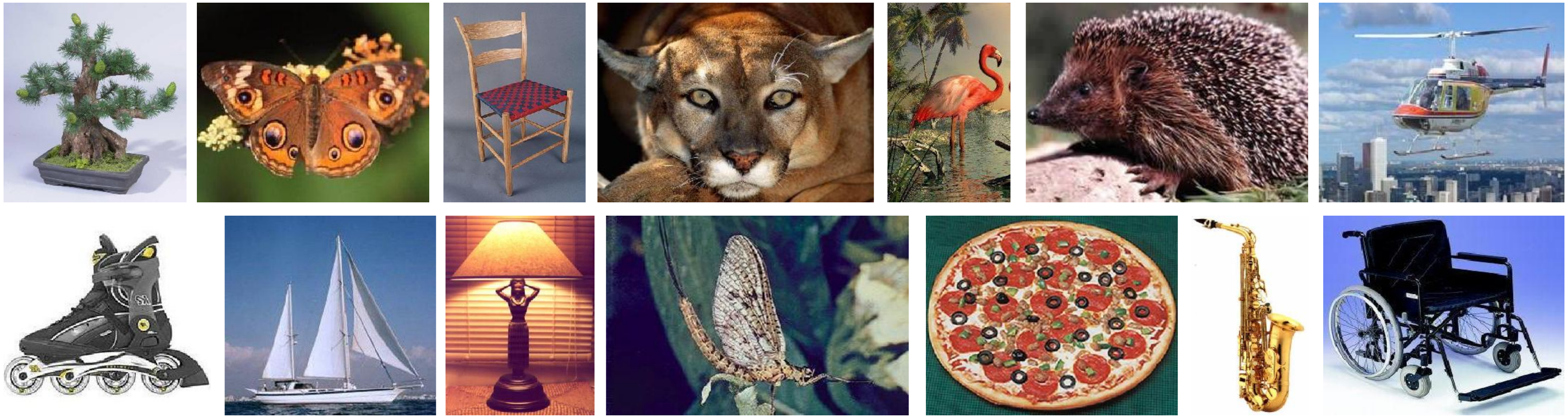


Multi-class classification results
(100 training images per class)

	Weak features (vocabulary size: 16)		Strong features (vocabulary size: 200)	
Level	Single-level	Pyramid	Single-level	Pyramid
0 (1×1)	45.3 ±0.5		72.2 ±0.6	
1 (2×2)	53.6 ±0.3	56.2 ±0.6	77.9 ±0.6	79.0 ±0.5
2 (4×4)	61.7 ±0.6	64.7 ±0.7	79.4 ±0.3	**81.1** ±0.3
3 (8×8)	63.3 ±0.8	**66.8** ±0.6	77.2 ±0.4	80.7 ±0.3

Caltech101 dataset

http://www.vision.caltech.edu/Image_Datasets/Caltech101/Caltech101.html



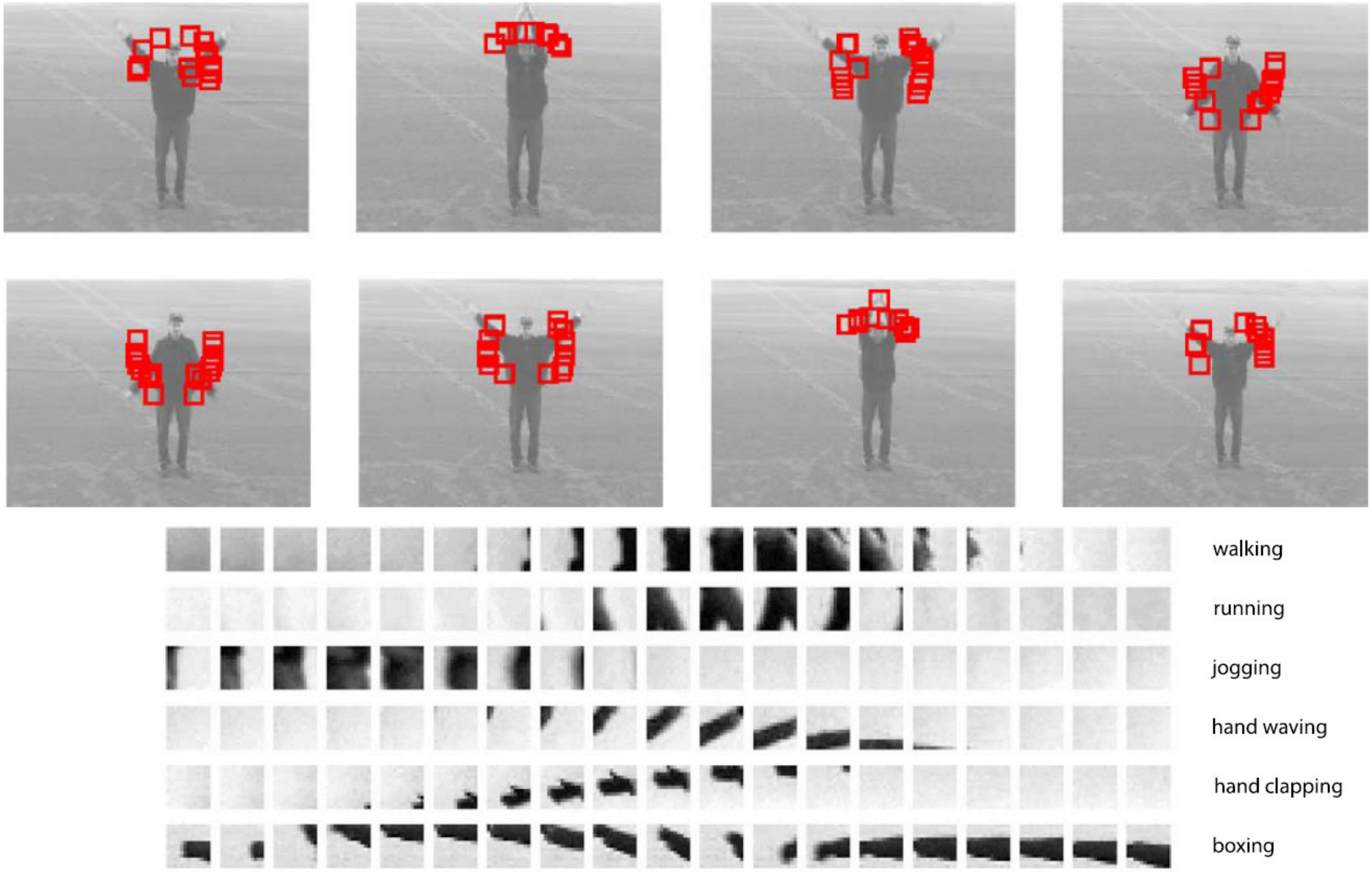
Multi-class classification results (30 training images per class)

	Weak features (16)		Strong features (200)	
Level	Single-level	Pyramid	Single-level	Pyramid
0	15.5 $\pm$0.9		41.2 $\pm$1.2	
1	31.4 $\pm$1.2	32.8 $\pm$1.3	55.9 $\pm$0.9	57.0 $\pm$0.8
2	47.2 $\pm$1.1	49.3 $\pm$1.4	63.6 $\pm$0.9	**64.6** $\pm$0.8
3	52.2 $\pm$0.8	**54.0** $\pm$1.1	60.3 $\pm$0.9	64.6 $\pm$0.7

Slide credit: Svetlana Lazebnik

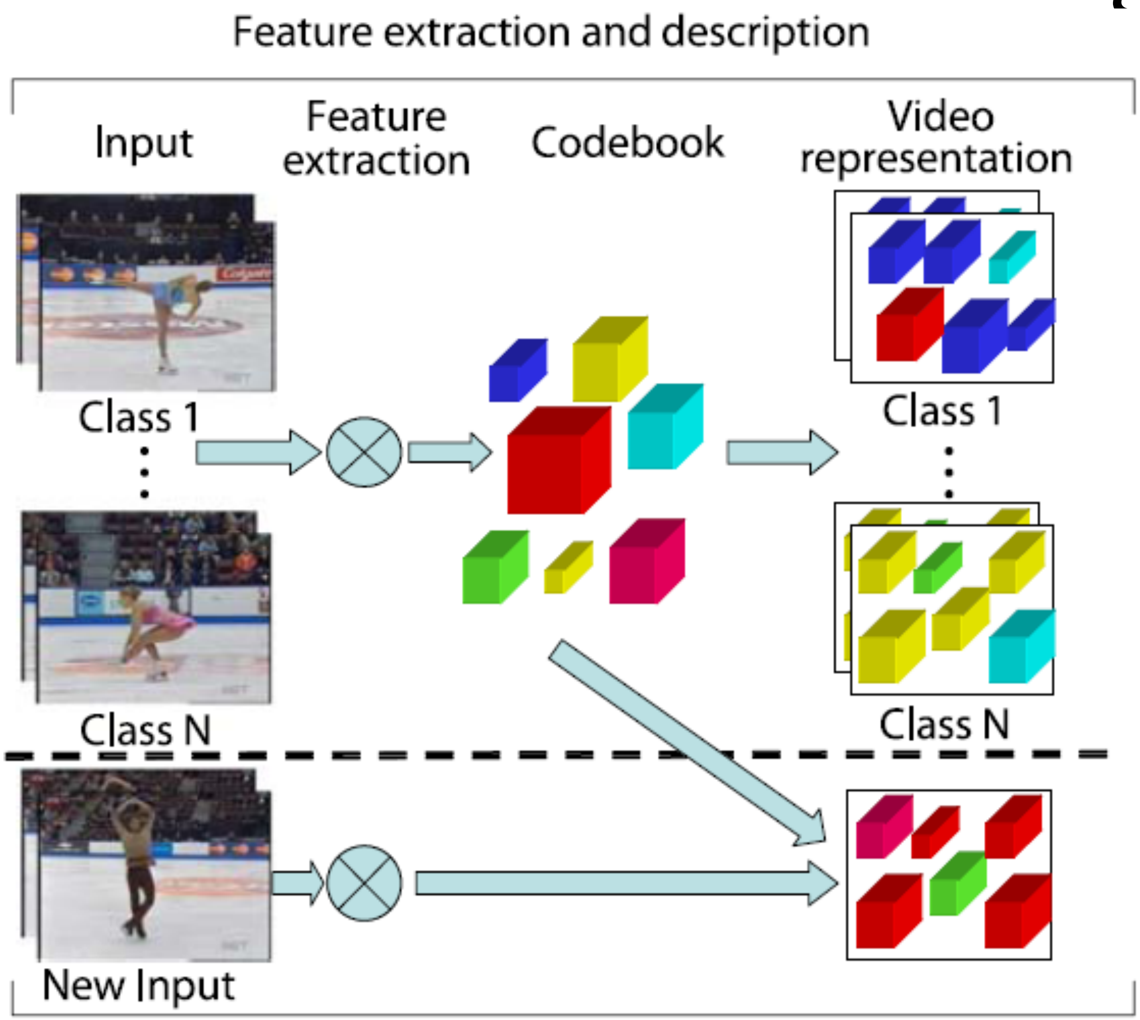
Bags of features for action recognition

Space-time interest points



Juan Carlos Niebles, Hongcheng Wang and Li Fei-Fei, **Unsupervised Learning of Human Action Categories Using Spatial-Temporal Words**, IJCV 2008.

Bags of features for action recognition



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