Lecture:
k-means & mean-shift clustering

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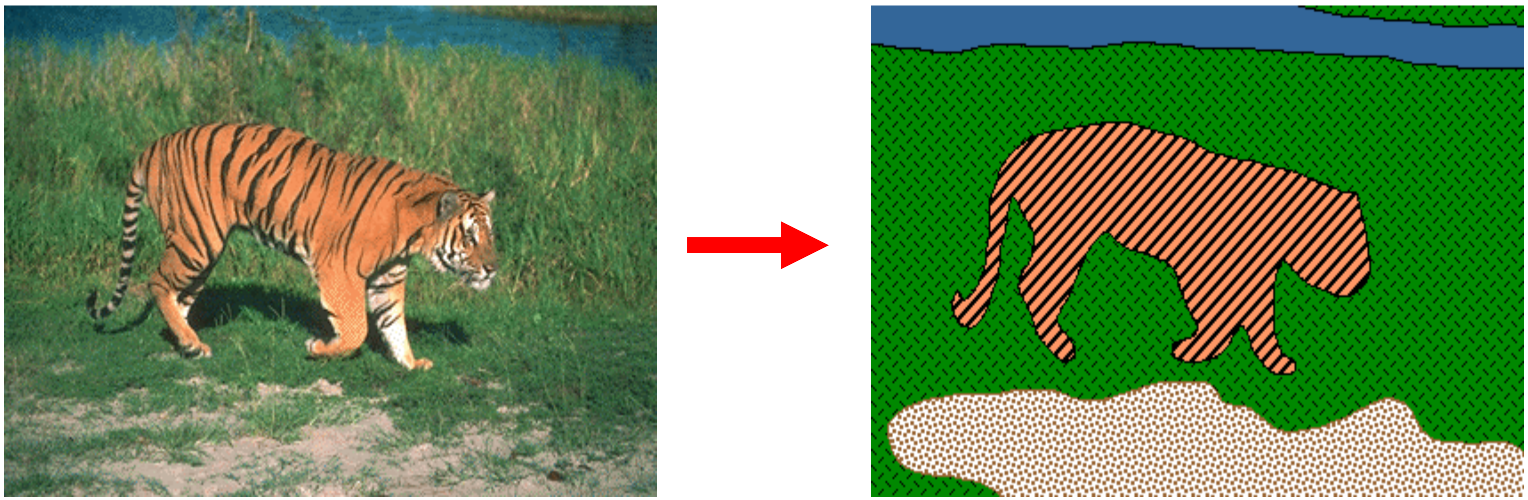
Recap: Image Segmentation

- Goal: identify groups of pixels that go together

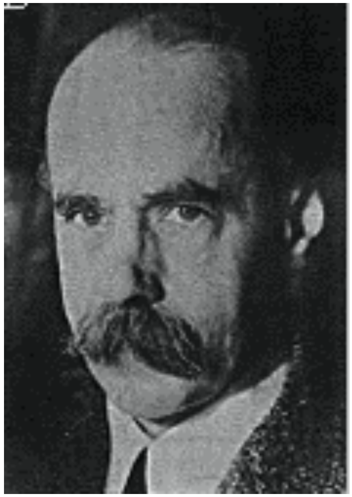


Recap: Gestalt Theory

- Gestalt: whole or group
  - Whole is greater than sum of its parts
  - Relationships among parts can yield new properties/features
- Psychologists identified series of factors that predispose set of elements to be grouped (by human visual system)

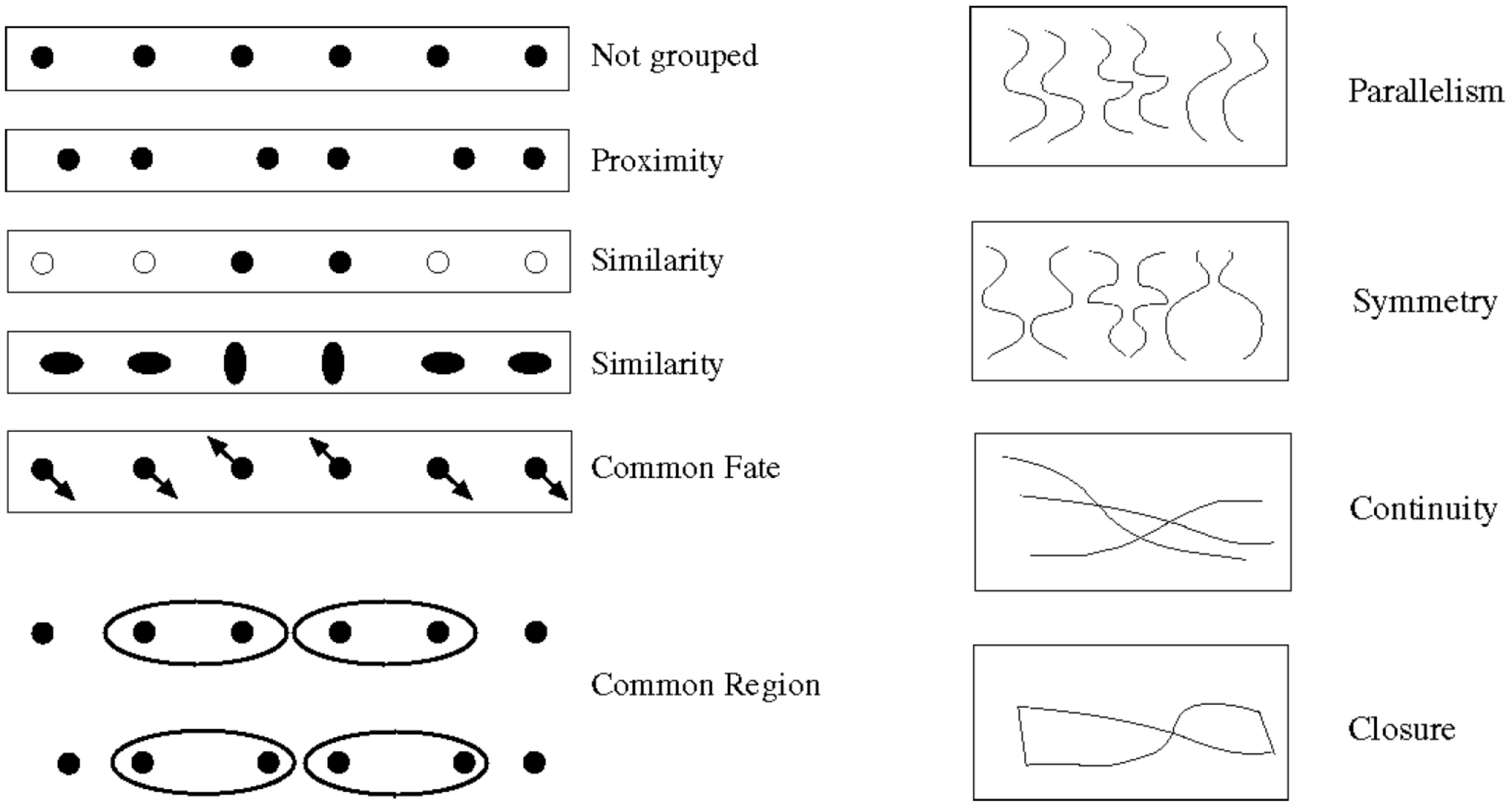
"I stand at the window and see a house, trees, sky. Theoretically I might say there were 327 brightnesses and nuances of colour. Do I have "327"? No. I have sky, house, and trees."

Max Wertheimer
(1880-1943)



Untersuchungen zur Lehre von der Gestalt,
***Psychologische Forschung*, Vol. 4, pp. 301-350, 1923**
http://psy.ed.asu.edu/~classics/Wertheimer/Forms/forms.htm

Recap: Gestalt Factors



- These factors make intuitive sense, but are very difficult to translate into algorithms.

What will we learn today?

- K-means clustering
- Mean-shift clustering

Reading: [FP] Chapters: 14.2, 14.4

D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

What will we learn today?

- K-means clustering
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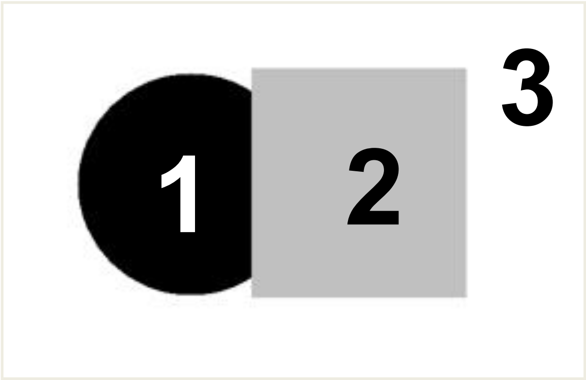
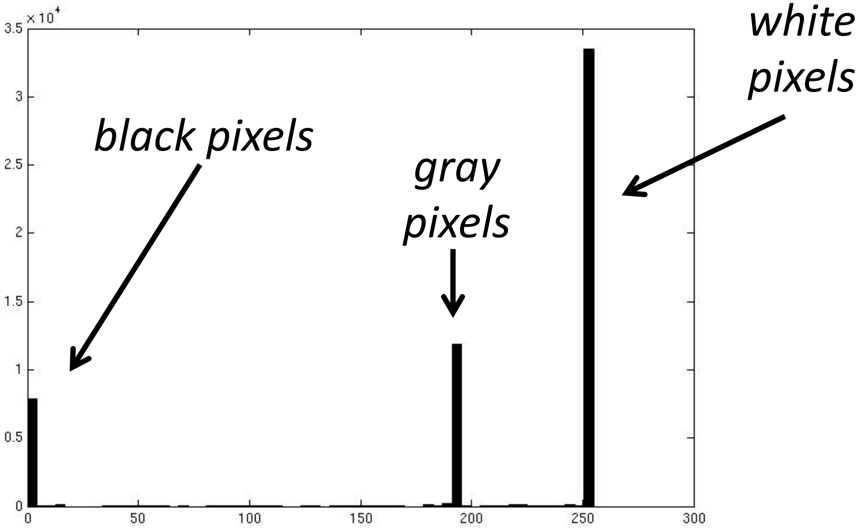
Image Segmentation: Toy Example



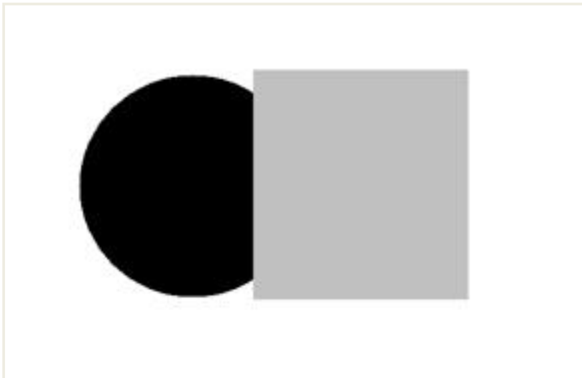
input image



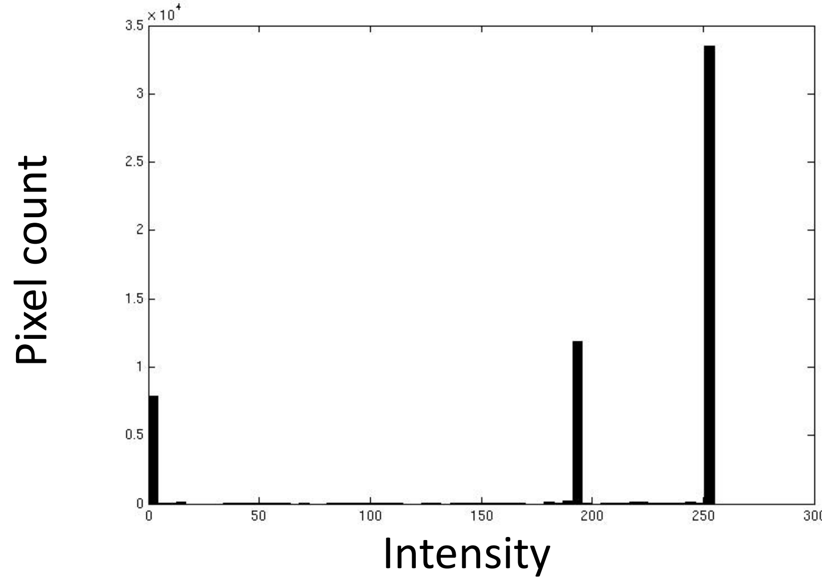
intensity

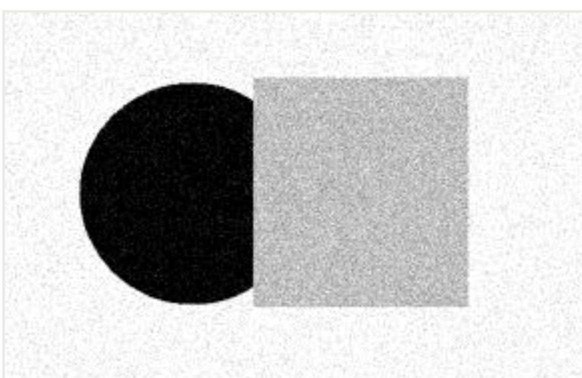
- These intensities define the three groups.
- We could label every pixel in the image according to which of these primary intensities it is.
  - i.e., segment the image based on the intensity feature.
- What if the image isn't quite so simple?

Slide credit: Kristen Grauman

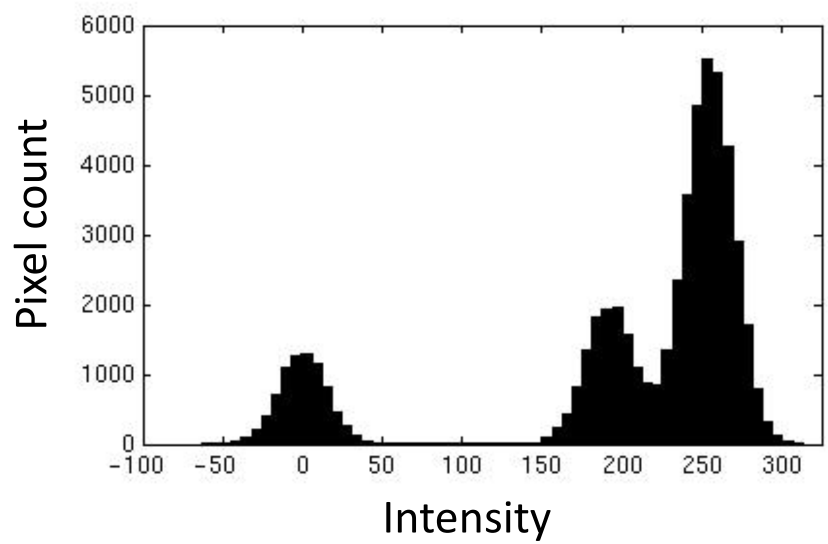


Input image

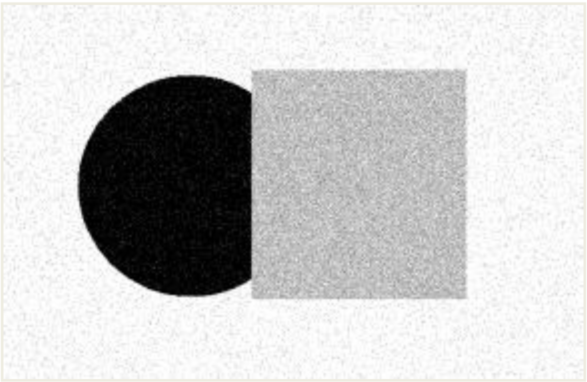




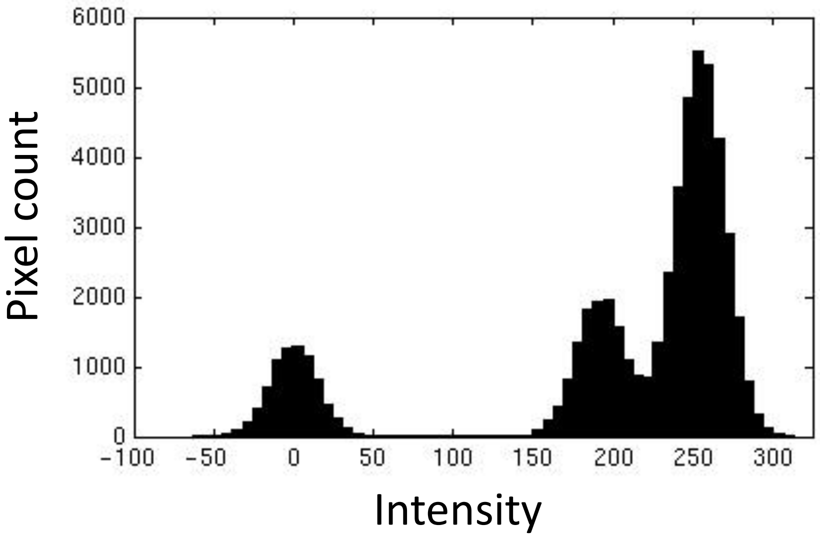
Input image



Slide credit: Kristen Grauman

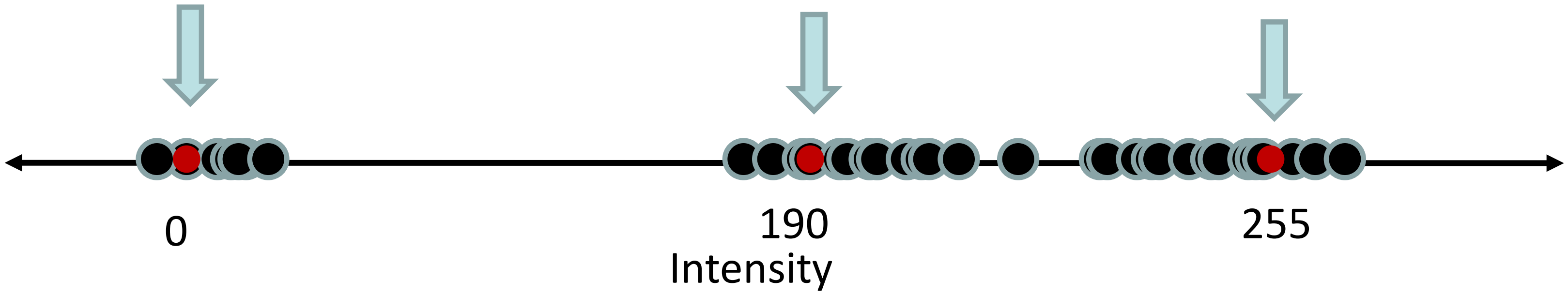


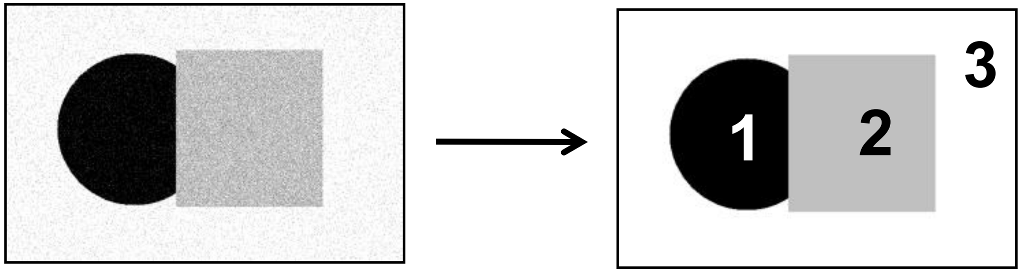
Input image



- Now how to determine the three main intensities that define our groups?
- We need to cluster.

Slide credit: Kristen Grauman





- Goal: choose three “centers” as the representative intensities, and label every pixel according to which of these centers it is nearest to.
- Best cluster centers are those that minimize Sum of Square Distance (SSD) between all points and their nearest cluster center c_i:

$$SSD = \sum_{cluster\, i} \sum_{x \in cluster\, i} (x - c_i)^2$$

Slide credit: Kristen Grauman

Clustering for Summarization

Goal: cluster to minimize variance in data given clusters

– Preserve information

$$c^*,\ \delta^* = \arg\min_{c,\delta} \frac{1}{N} \sum_{j}^{N} \sum_{i}^{K} \delta_{ij} \left(c_i - x_j\right)^2$$

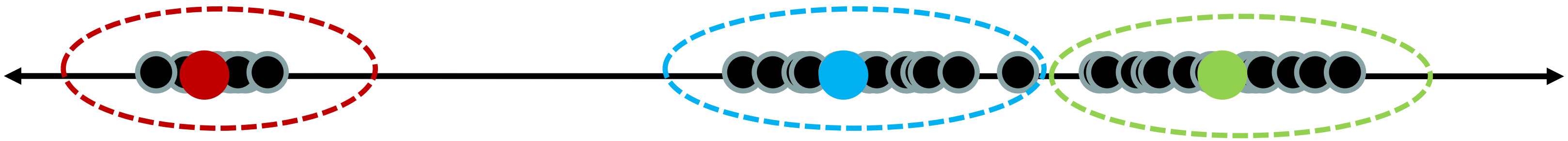
Cluster center: c_i

Data: x_j

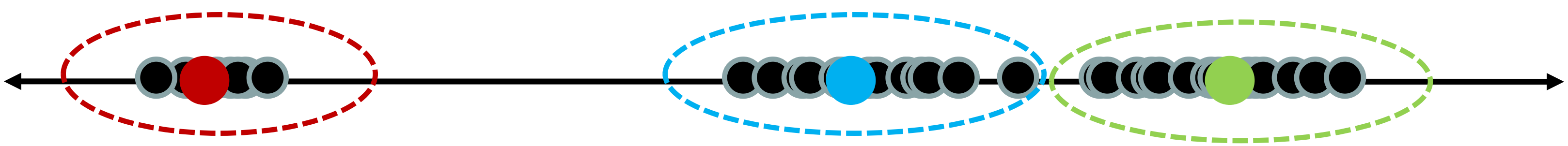
Whether x_j is assigned to c_i: δ_{ij}

Clustering

- With this objective, it is a “chicken and egg” problem:
  - If we knew the *cluster centers*, we could allocate points to groups by assigning each to its closest center.



  - If we knew the *group memberships*, we could get the centers by computing the mean per group.



Slide credit: Kristen Grauman

K-means clustering

1. Initialize ($t = 0$): cluster centers $c_1, \ldots, c_K$

2. Compute δ^t: assign each point to the closest center
   – δ^t denotes the set of assignment for each x_j to cluster c_i at iteration *t*

$$\delta^t = \underset{\delta}{\operatorname{argmin}} \frac{1}{N} \sum_j^N \sum_i^K \delta_{ij}^{t-1} \left(c_i^{t-1} - x_j \right)^2$$

1. Computer c^t: update cluster centers as the mean of the points

$$c^t = \underset{c}{\operatorname{argmin}} \frac{1}{N} \sum_j^N \sum_i^K \delta_{ij}^{t} \left(c_i^{t-1} - x_j \right)^2$$

1. Update $t = t + 1$, Repeat Step 2-3 till stopped

Slide: Derek Hoiem

K-means clustering

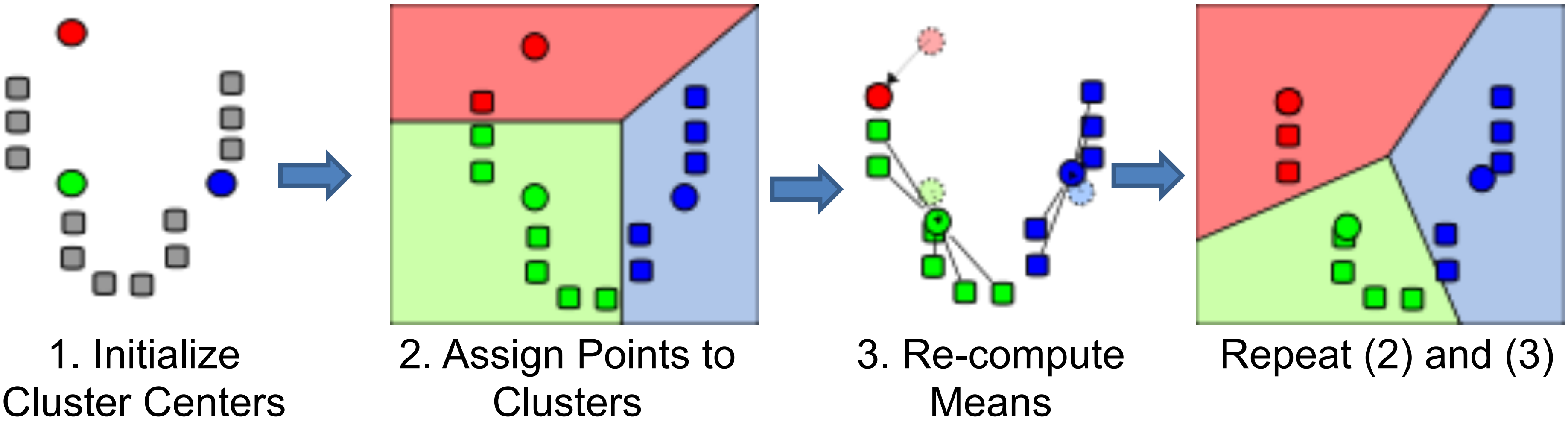
1. Initialize ($t = 0$): cluster centers $c_1, \ldots, c_K$
   - Commonly used: random initialization
   - Or greedily choose K to minimize residual
2. Compute δ^t: assign each point to the closest center
   - Typical distance measure:
     - Euclidean $sim(x, x') = x^T x'$
     - Cosine $sim(x, x') = x^T x' / (\|x\| \cdot \|x'\|)$
     - Others
1. Computer c^t: update cluster centers as the mean of the points

$$c^t = \underset{c}{\operatorname{argmin}} \frac{1}{N} \sum_j^N \sum_i^K \delta_{ij}^t \left(c_i^{t-1} - x_j \right)^2$$

2. Update $t = t + 1$, Repeat Step 2-3 till stopped
   - c^t doesn't change anymore.

Slide: Derek Hoiem

K-means clustering



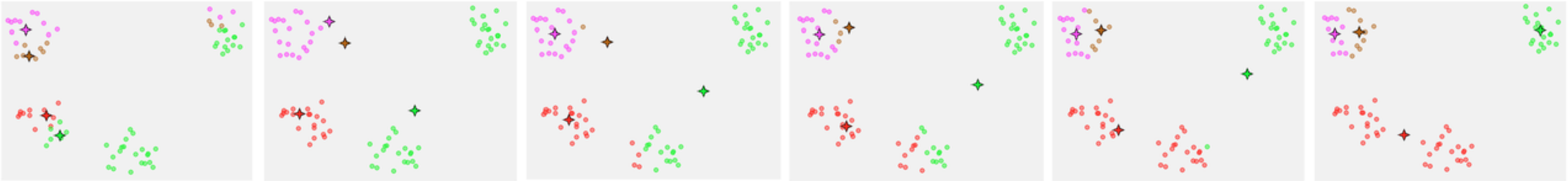
- Java demo:

http://home.dei.polimi.it/matteucc/Clustering/tutorial_html/AppletKM.html

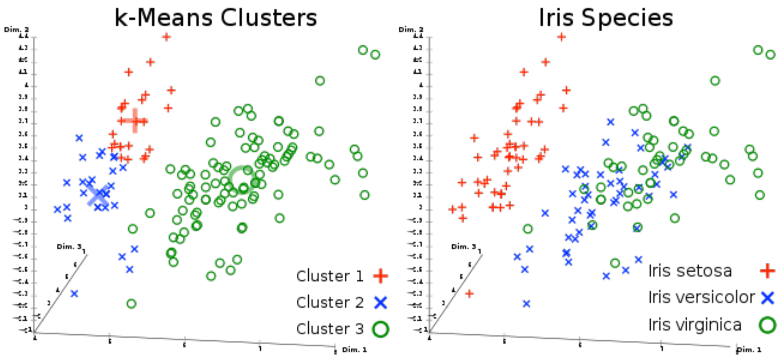
Illustration Source: wikipedia

K-means clustering

- Converges to a *local minimum* solution
  - Initialize multiple runs



- Better fit for spherical data

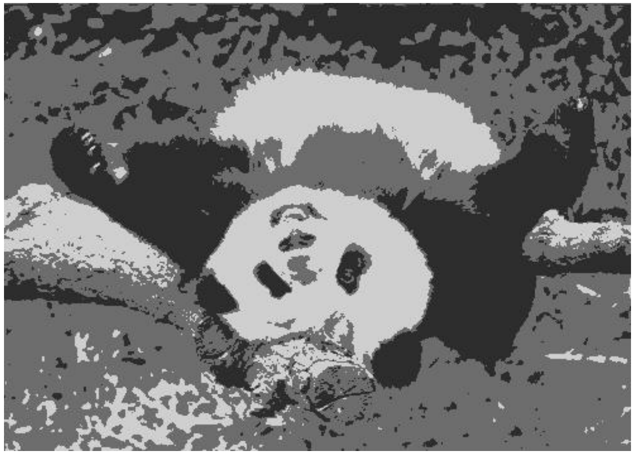


- Need to pick K (# of clusters)

Segmentation as Clustering

Original image

2 clusters



3 clusters

K-Means++

- Can we prevent arbitrarily bad local minima?

1. Randomly choose first center.
2. Pick new center with prob. proportional to $(x - c_i)^2$
   - (Contribution of *x* to total error)
3. Repeat until *K* centers.

- Expected error $= O(\log K)$* optimal

[Arthur & Vassilvitskii 2007]

Slide credit: Steve Seitz

Feature Space

- Depending on what we choose as the *feature space*, we can group pixels in different ways.

- Grouping pixels based on intensity similarity

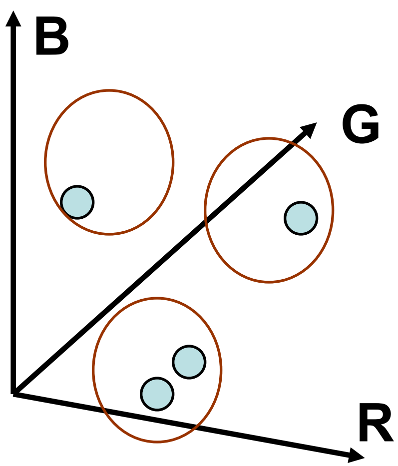
- Feature space: intensity value (1D)

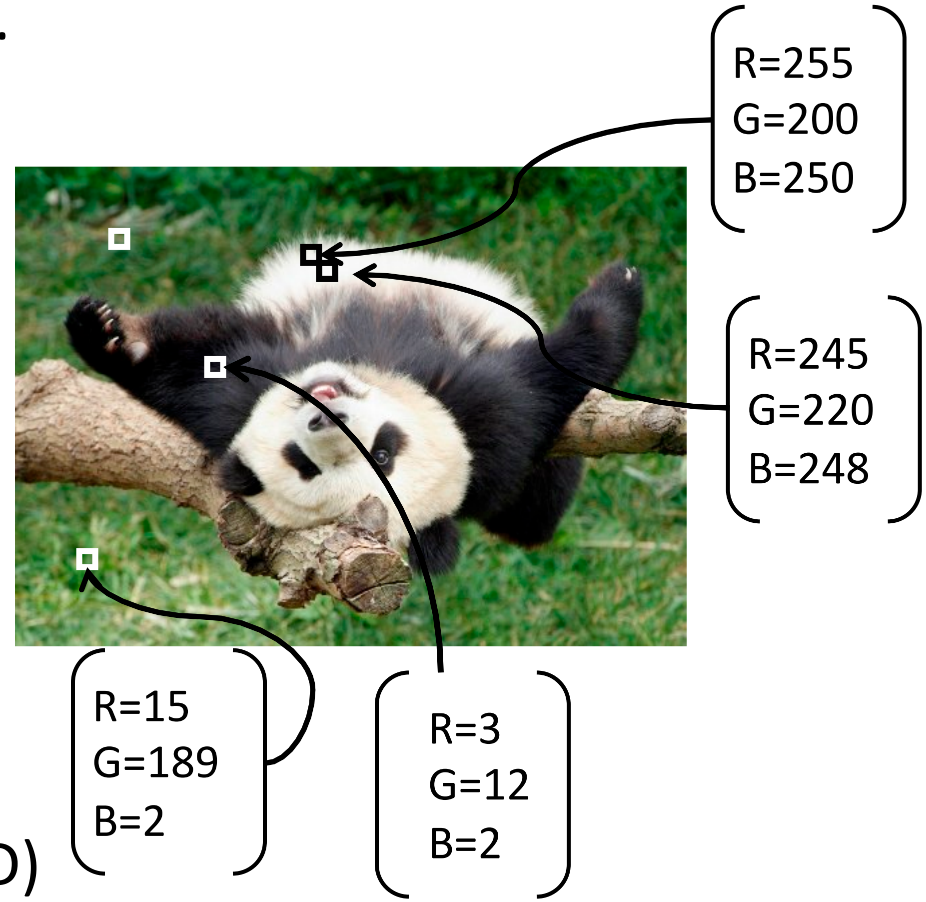
Slide credit: Kristen Grauman

Feature Space

- Depending on what we choose as the *feature space*, we can group pixels in different ways.

- Grouping pixels based on color similarity



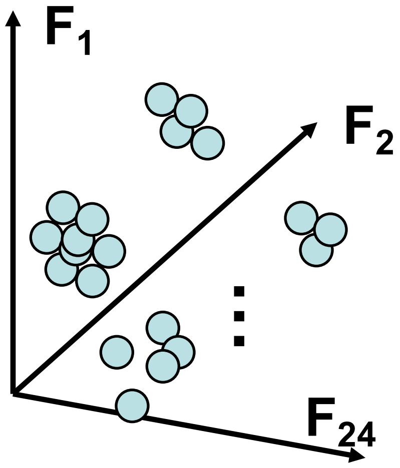


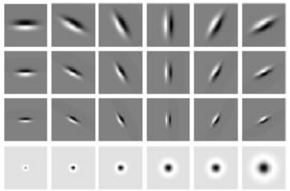
- Feature space: color value (3D)

Slide credit: Kristen Grauman

Feature Space

- Depending on what we choose as the *feature space*, we can group pixels in different ways.
- Grouping pixels based on texture similarity





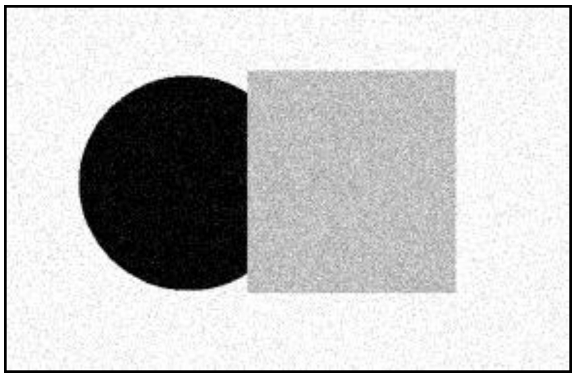
Filter bank of 24 filters

- Feature space: filter bank responses (e.g., 24D)

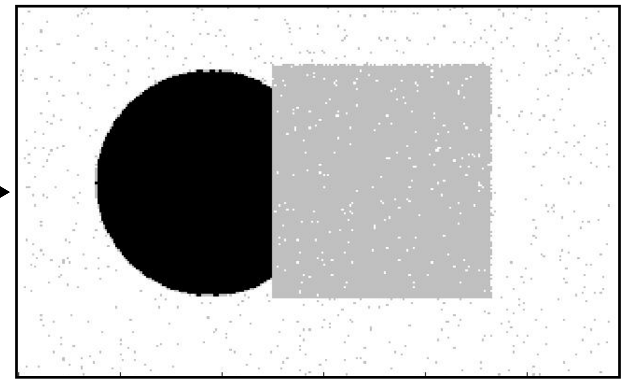
Slide credit: Kristen Grauman

Smoothing Out Cluster Assignments

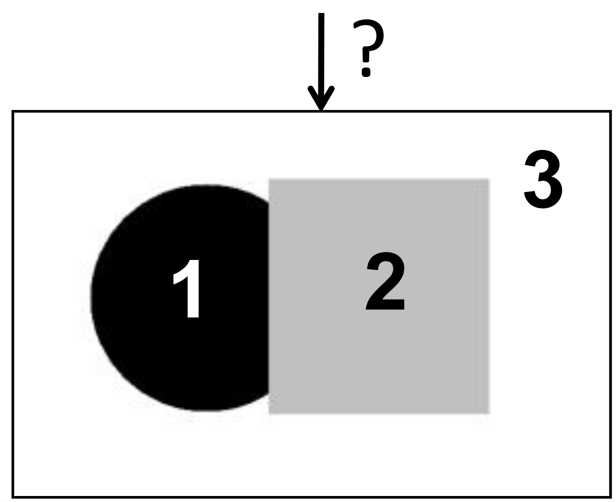
- Assigning a cluster label per pixel may yield outliers:



Original



Labeled by cluster center's intensity



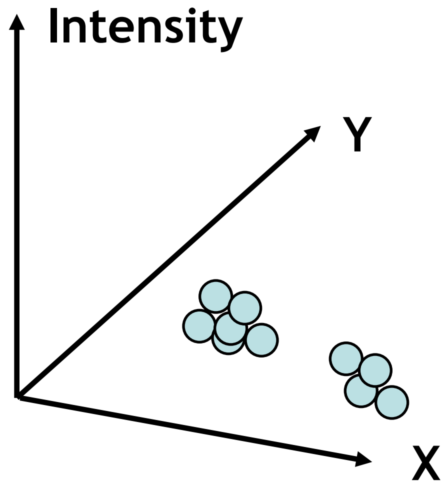
- How can we ensure they are spatially smooth?

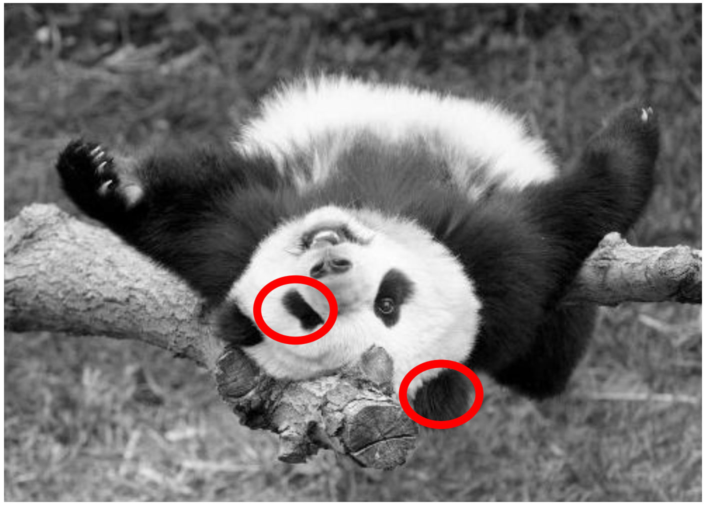
Slide credit: Kristen Grauman

Segmentation as Clustering

- Depending on what we choose as the *feature space*, we can group pixels in different ways.

- Grouping pixels based on *intensity+position* similarity





⇒ Way to encode both *similarity* and *proximity.*

Slide credit: Kristen Grauman

K-Means Clustering Results

- K-means clustering based on intensity or color is essentially vector quantization of the image attributes
  - Clusters don't have to be spatially coherent

Image

Intensity-based clusters

Color-based clusters

Image source: Forsyth & Ponce

K-Means Clustering Results

- K-means clustering based on intensity or color is essentially vector quantization of the image attributes
  - Clusters don't have to be spatially coherent
- Clustering based on (r,g,b,x,y) values enforces more spatial coherence

Image source: Forsyth & Ponce

How to evaluate clusters?

- Generative
  - How well are points reconstructed from the clusters?
- Discriminative
  - How well do the clusters correspond to labels?
    - Can we correctly classify which pixels belong to the panda?
  - Note: unsupervised clustering does not aim to be discriminative as we don't have the labels.

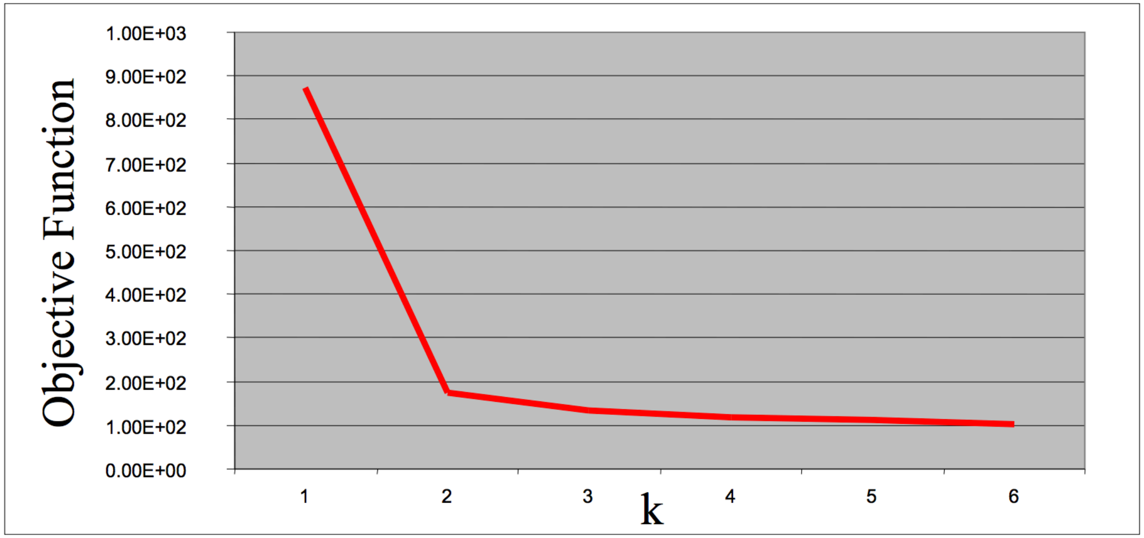
Slide: Derek Hoiem

How to choose the number of clusters?

Try different numbers of clusters in a validation set and look at performance.

We can plot the objective function values for k equals 1 to 6…

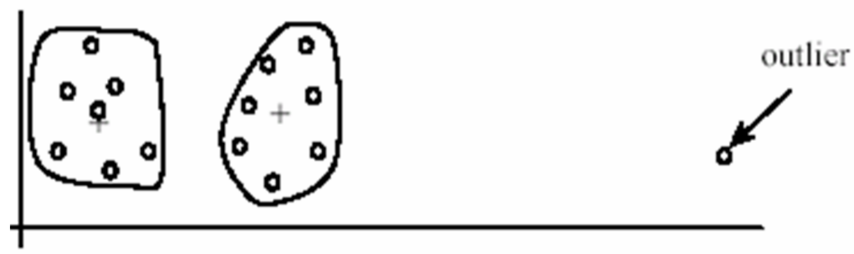
The abrupt change at k = 2, is highly suggestive of two clusters in the data. This technique for determining the number of clusters is known as "knee finding" or "elbow finding".



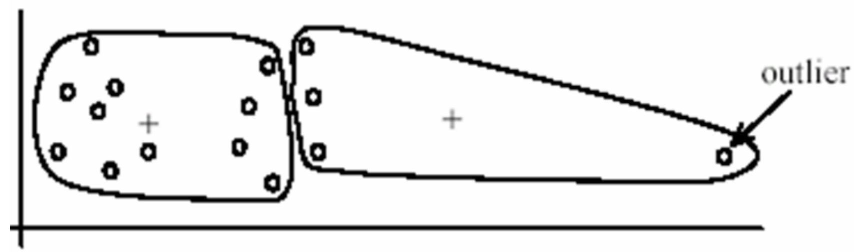
Slide: Derek Hoiem

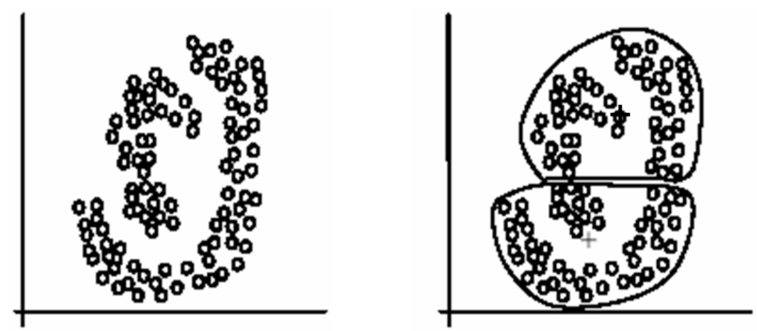
K-Means pros and cons

- Pros
  - Finds cluster centers that minimize conditional variance (good representation of data)
  - Simple and fast, Easy to implement
- Cons
  - Need to choose K
  - Sensitive to outliers
  - Prone to local minima
  - All clusters have the same parameters (e.g., distance measure is non-adaptive)
  - *Can be slow: each iteration is O(KNd) for N d-dimensional points
- Usage
  - Unsupervised clustering
  - Rarely used for pixel segmentation



(B): Ideal clusters





(A): Two natural clusters (B): *k*-means clusters

What will we learn today?

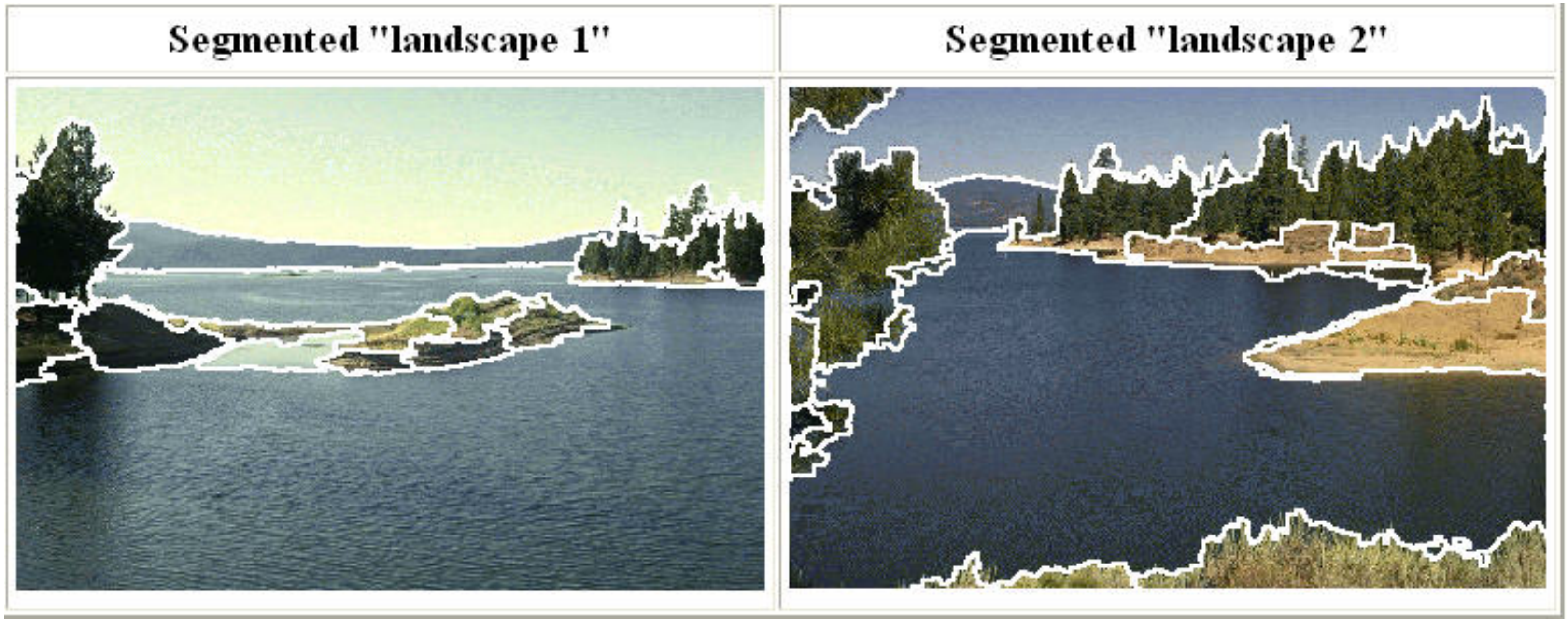
- K-means clustering
- Mean-shift clustering

Reading: [FP] Chapters: 14.2, 14.4

D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

Mean-Shift Segmentation

- An advanced and versatile technique for clustering-based segmentation

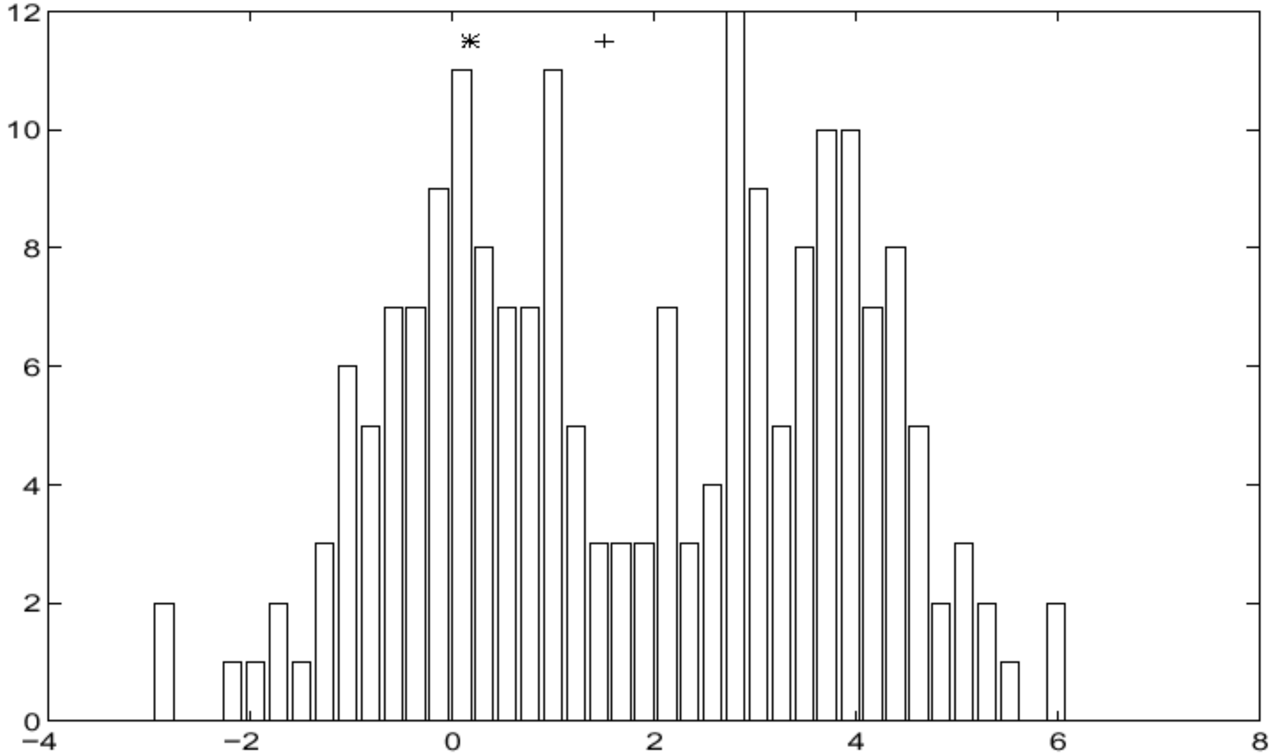


http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html

D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

Slide credit: Svetlana Lazebnik

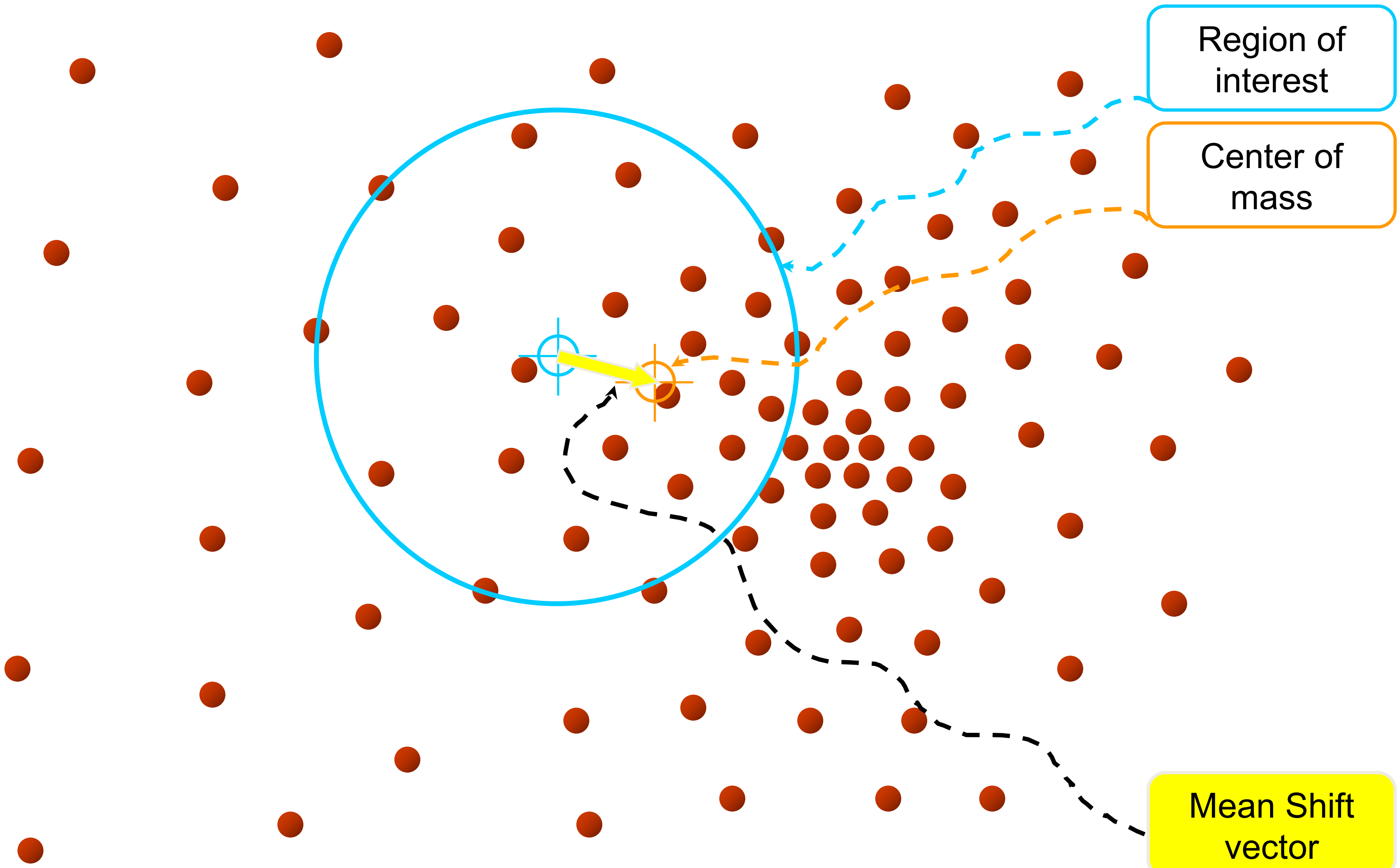
Mean-Shift Algorithm



- Iterative Mode Search
  1. Initialize random seed, and window W
  2. Calculate center of gravity (the "mean") of W: $\sum_{x \in W} xH(x)$
  3. Shift the search window to the mean
  4. Repeat Step 2 until convergence

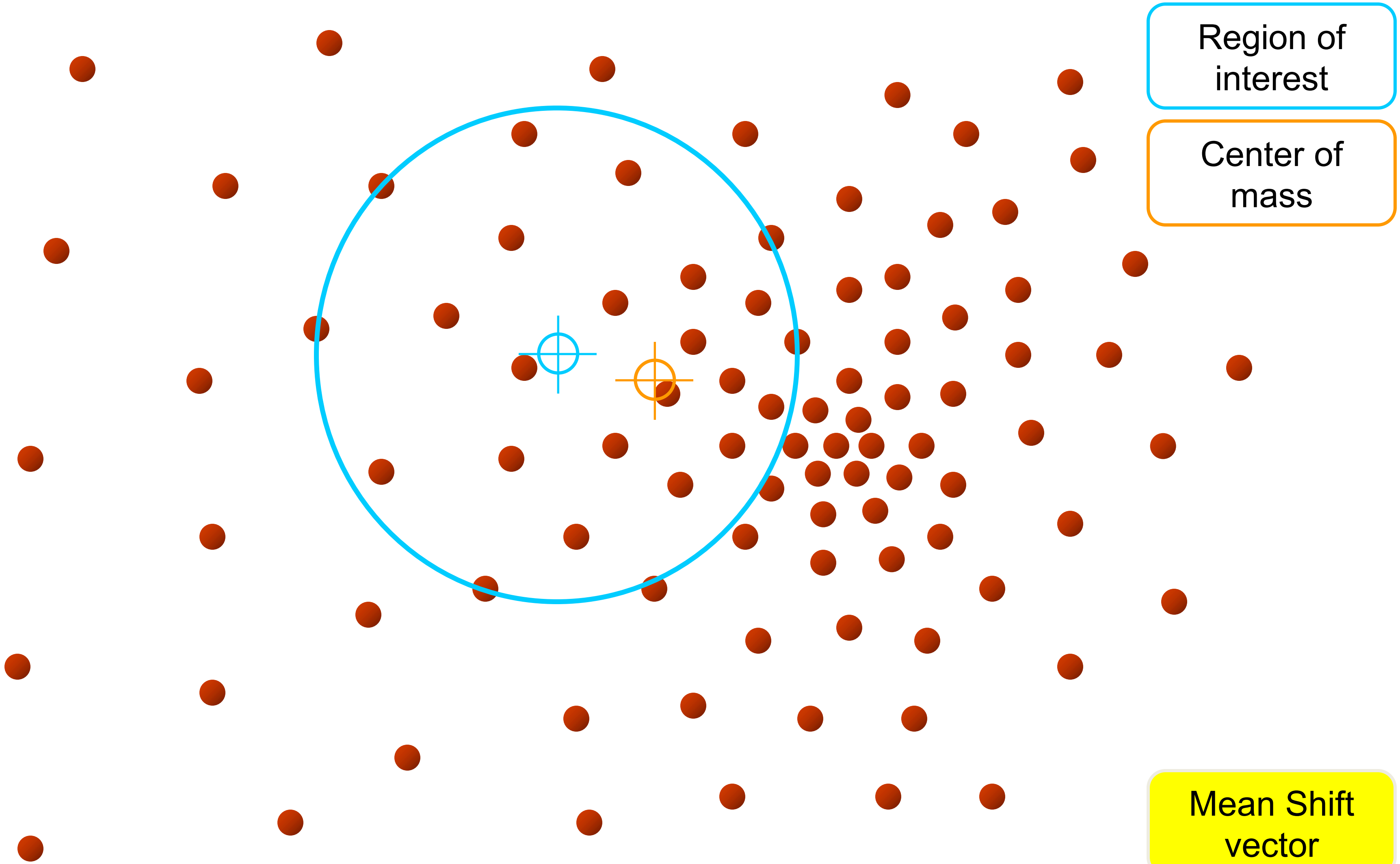
Slide credit: Steve Seitz

Mean-Shift



Slide by Y. Ukrainitz & B. Sarel

Mean-Shift



Slide by Y. Ukrainitz & B. Sarel

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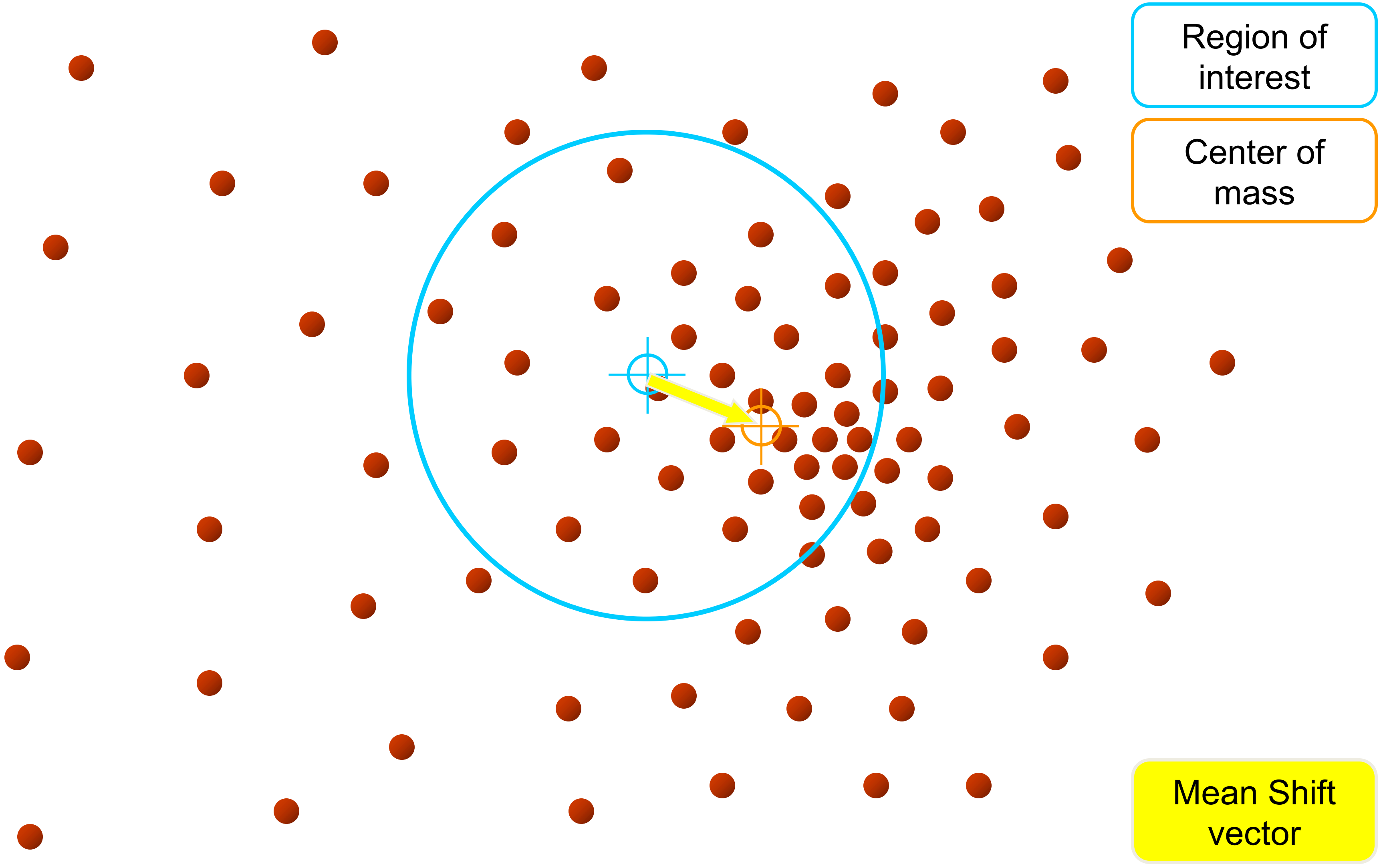

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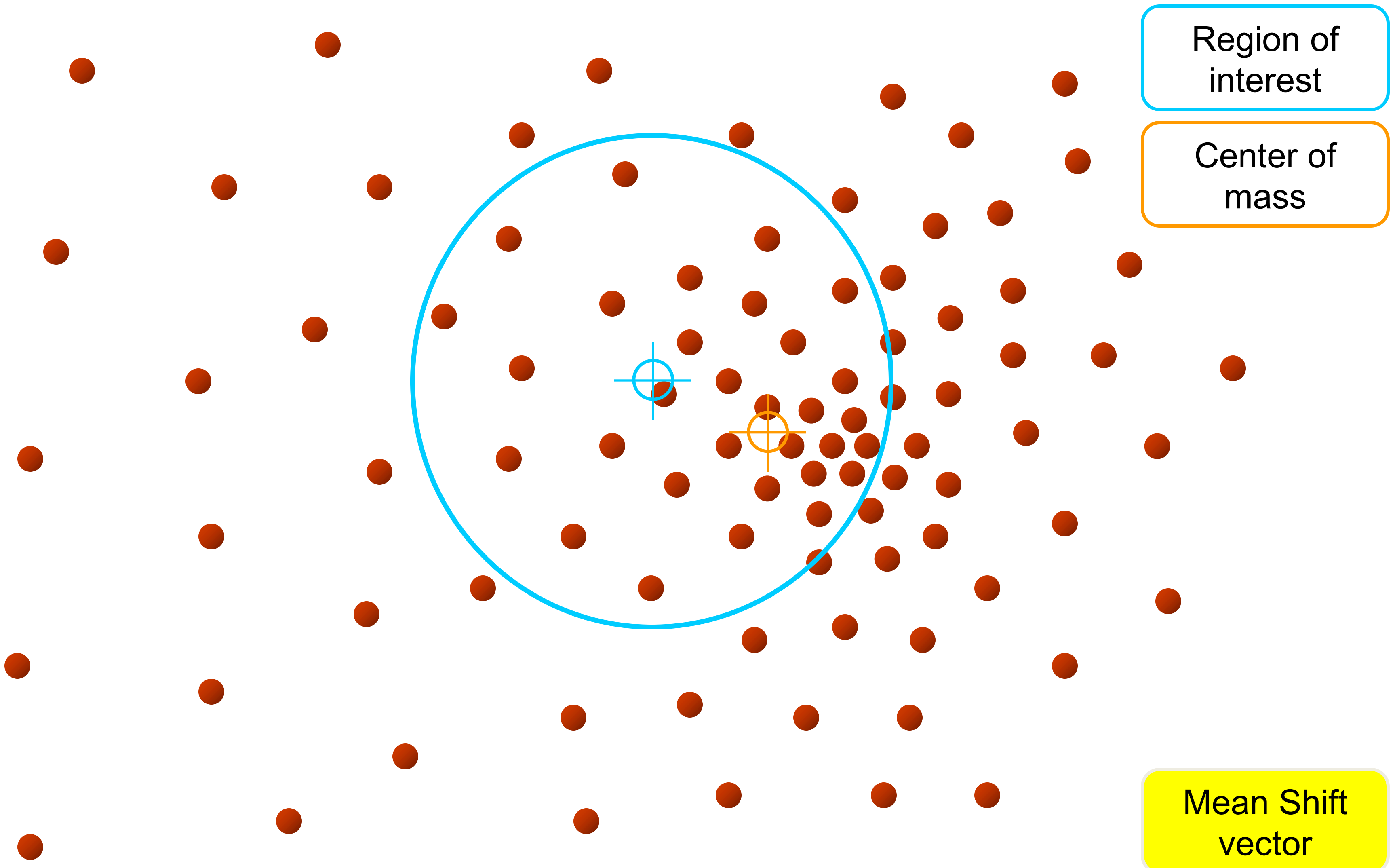
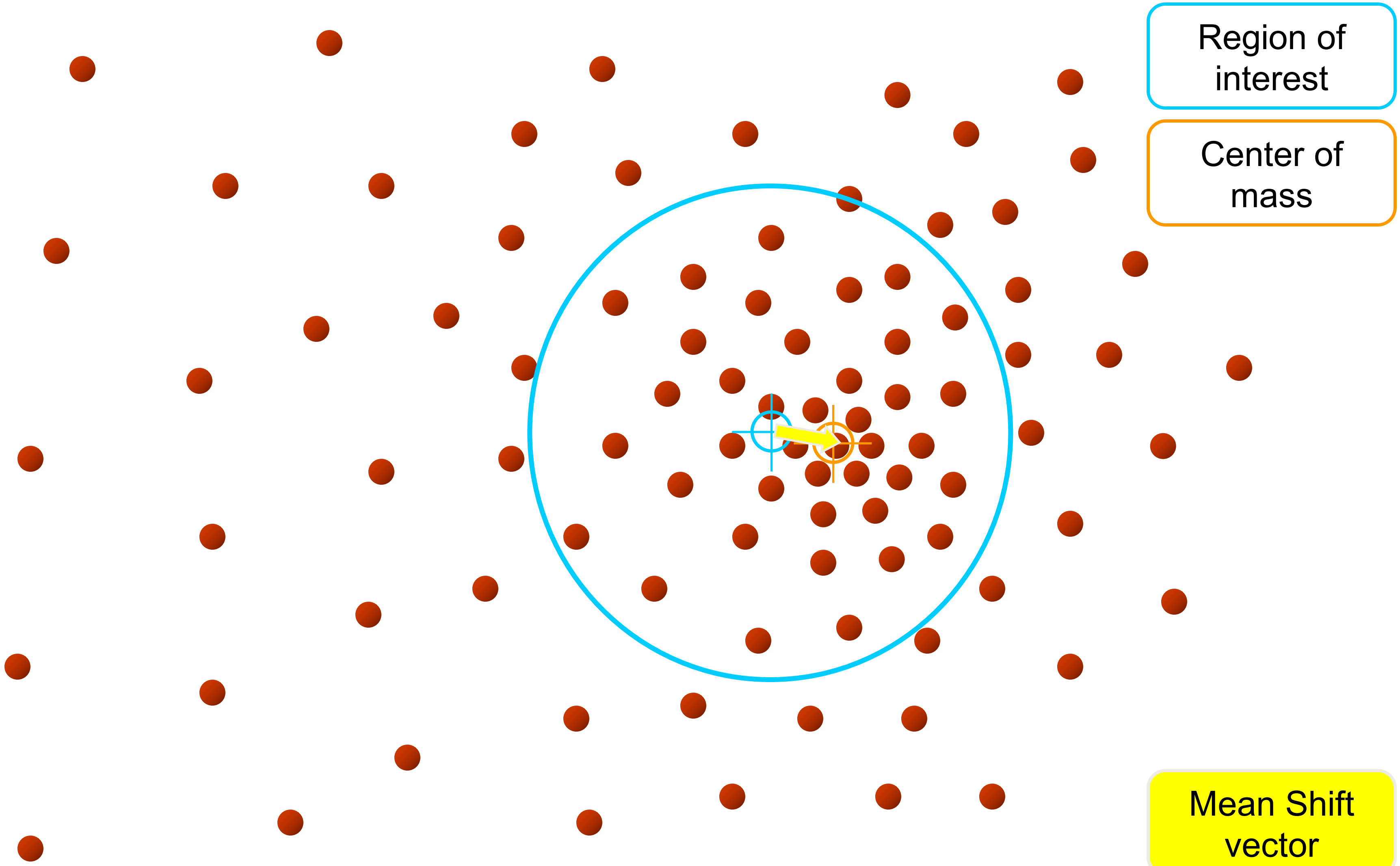


Slide by Y. Ukrainitz & B. Sarel

Mean-Shift



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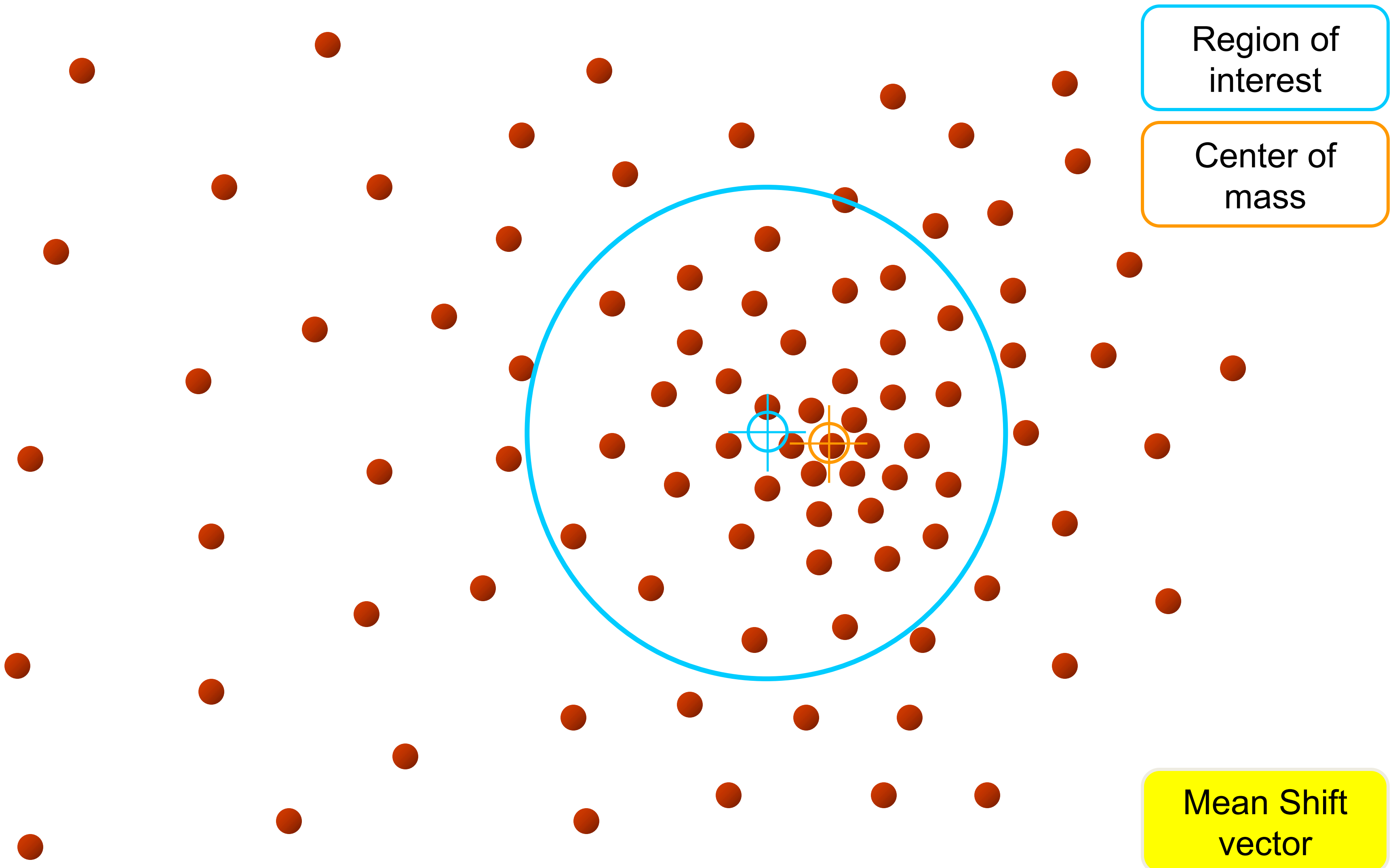
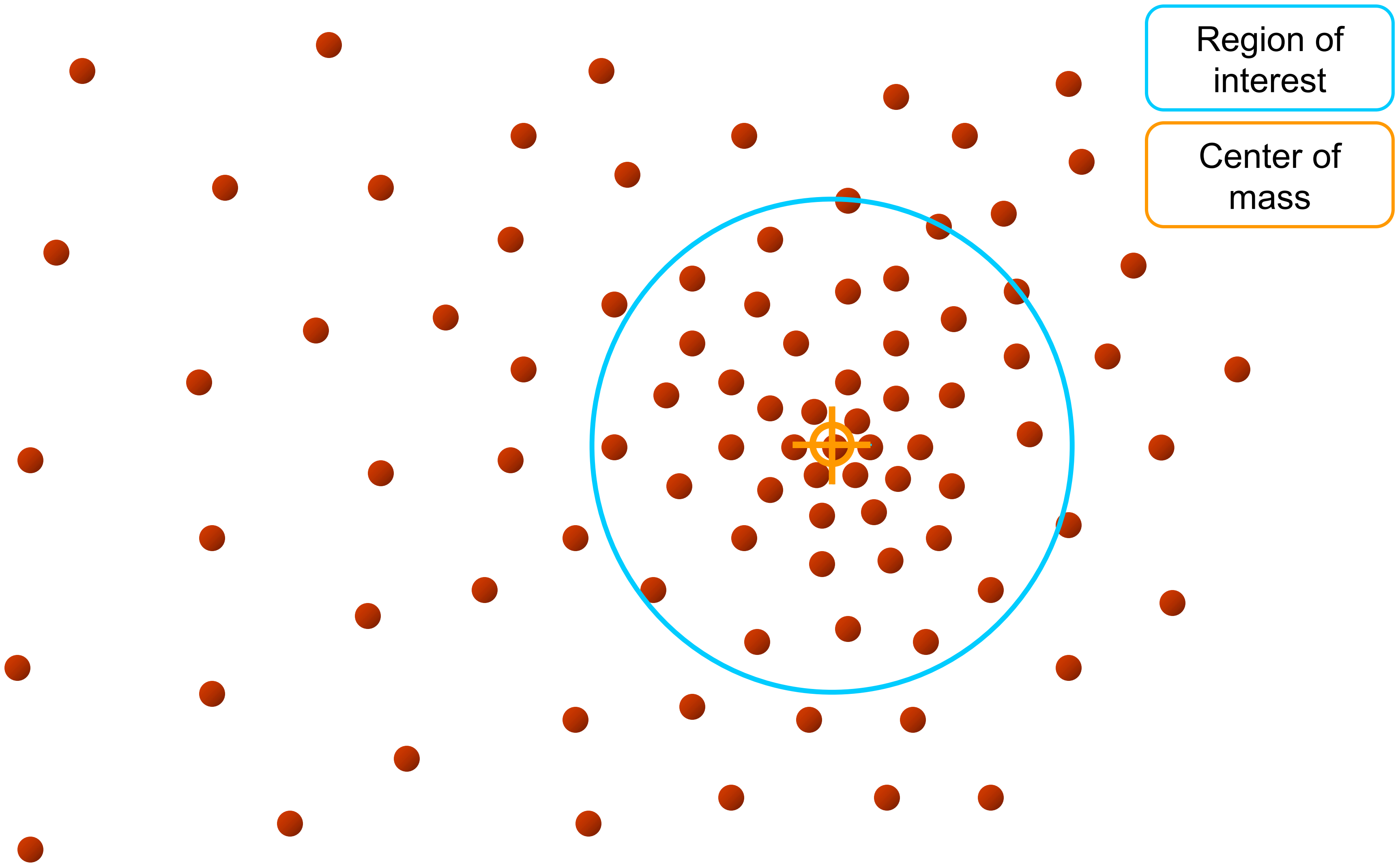
Mean-Shift



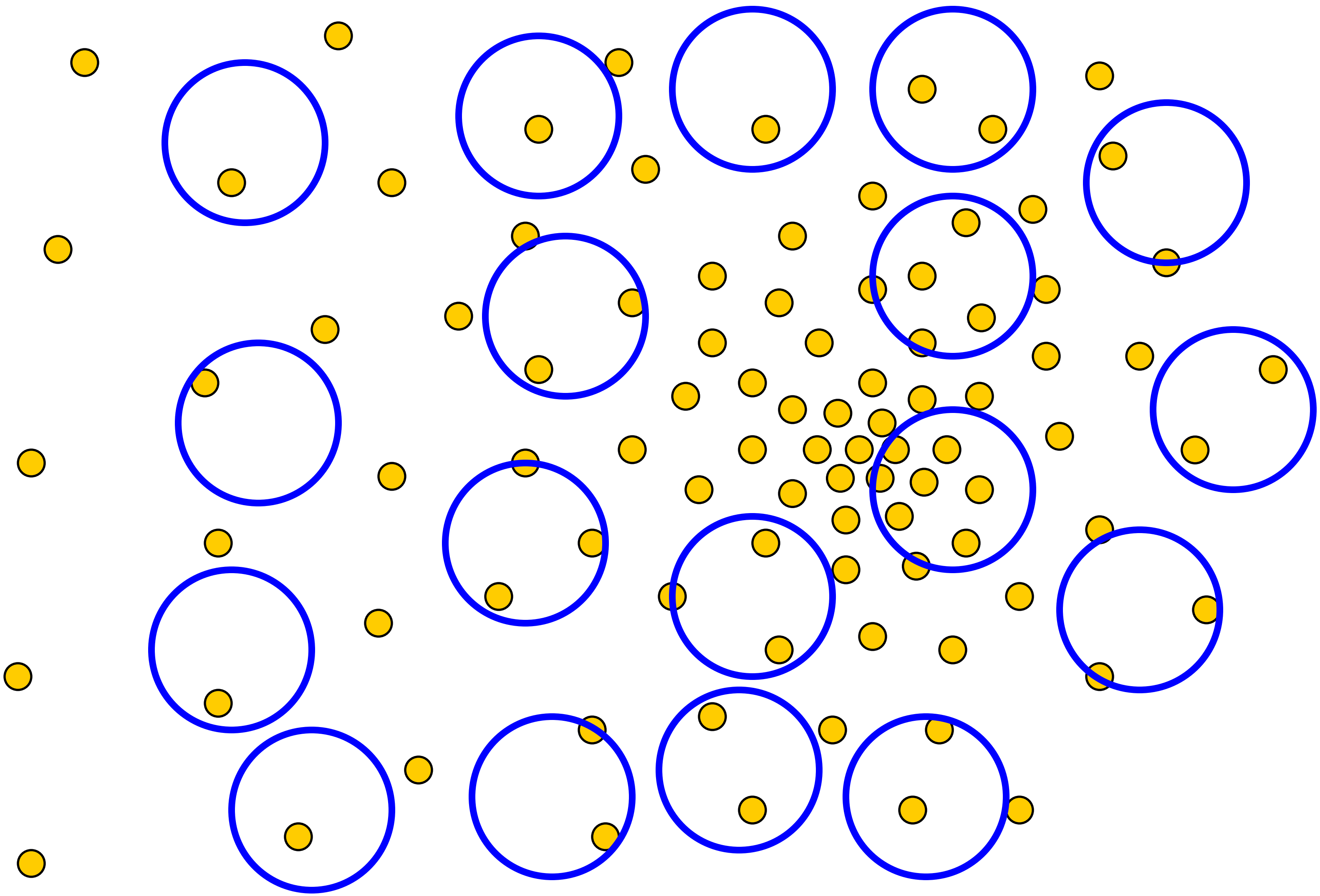
Slide by Y. Ukrainitz & B. Sarel

Mean-Shift



Slide by Y. Ukrainitz & B. Sarel

Real Modality Analysis

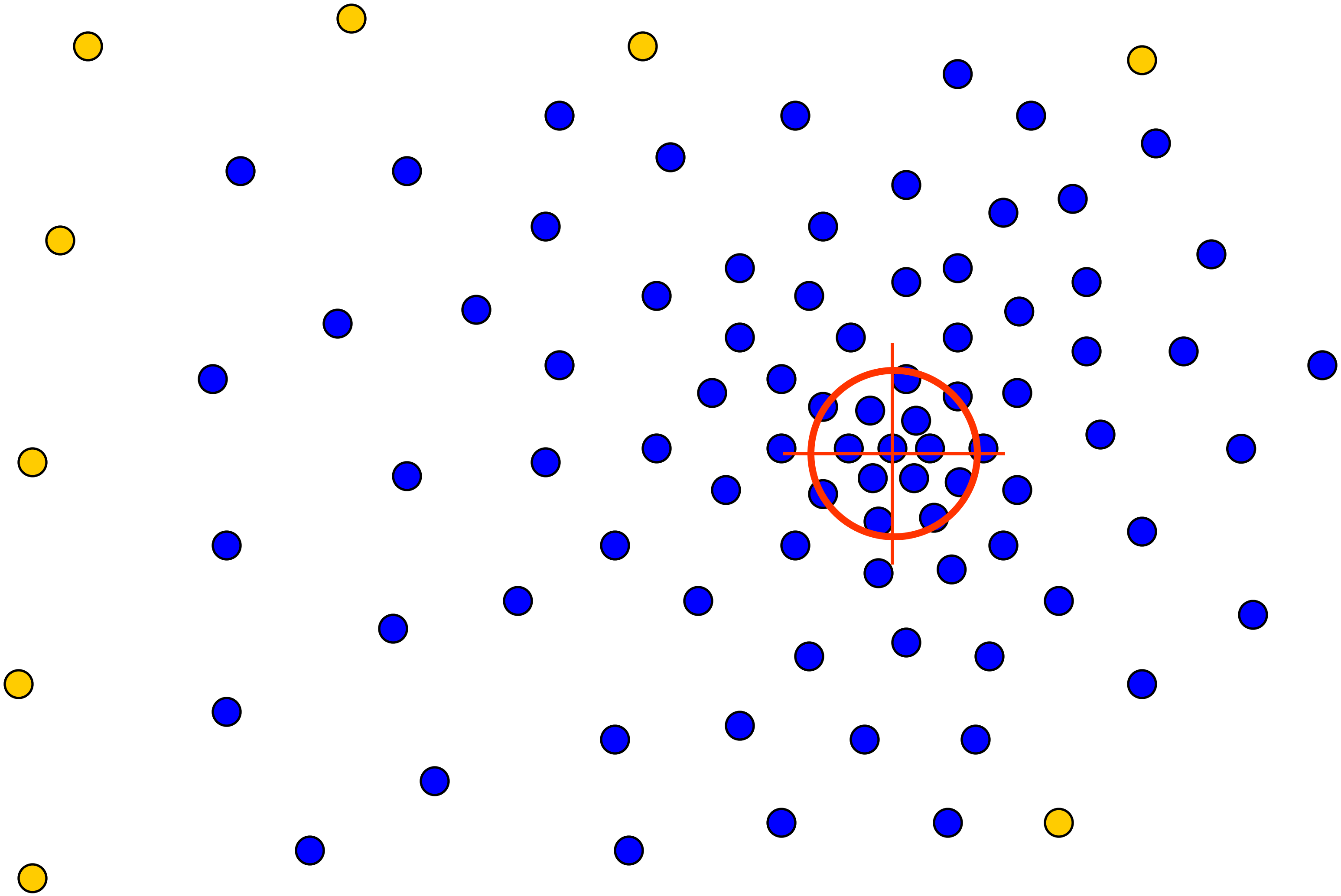


Tessellate the space with windows

Run the procedure in parallel

Slide by Y. Ukrainitz & B. Sarel

Real Modality Analysis

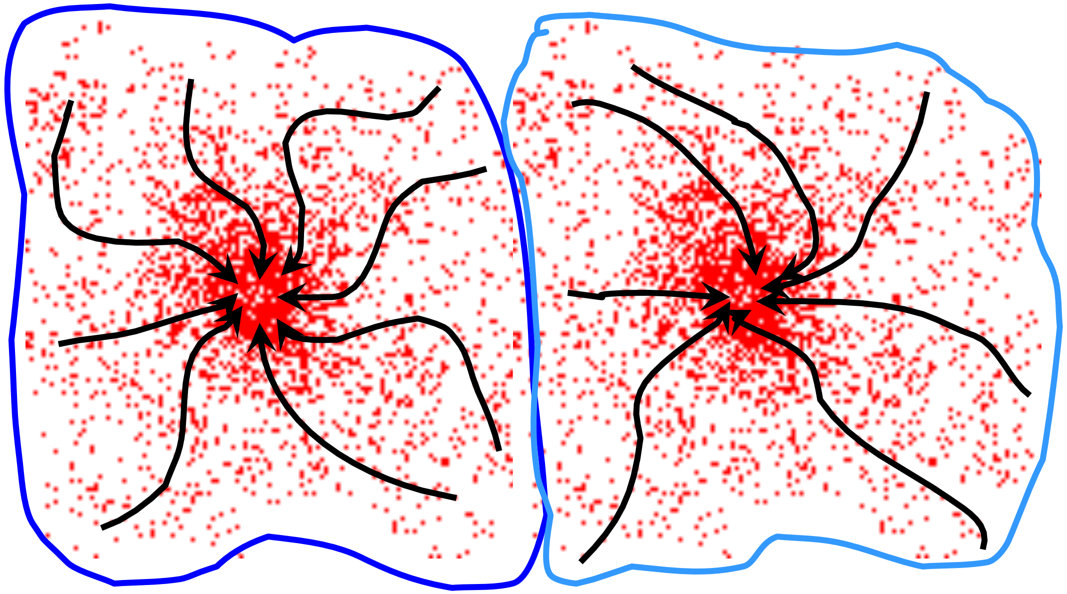


The **blue** data points were traversed by the windows towards the mode.

Slide by Y. Ukrainitz & B. Sarel

Mean-Shift Clustering

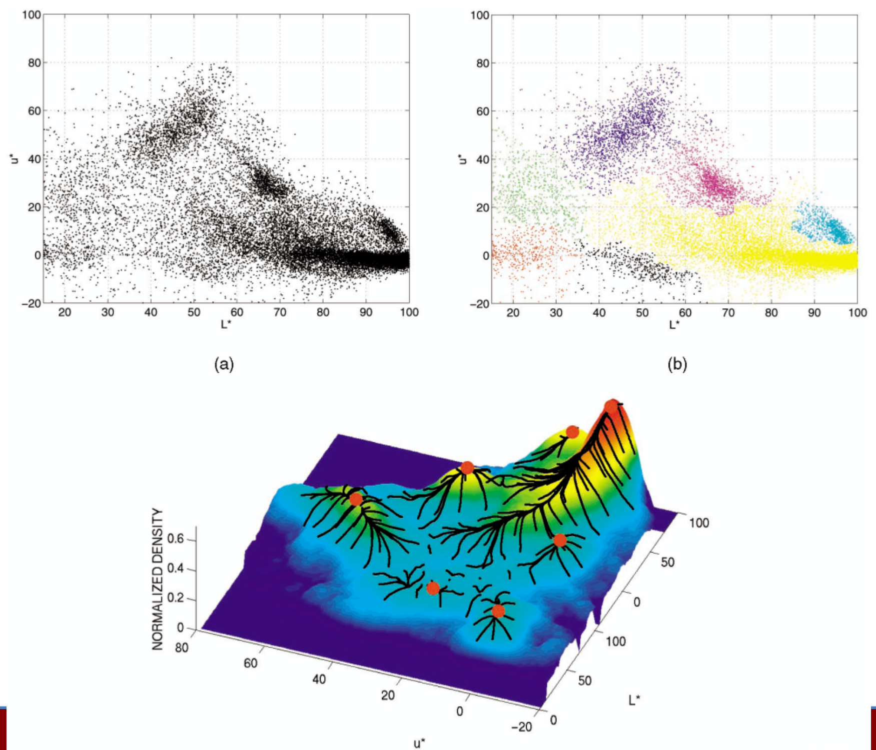
- Cluster: all data points in the attraction basin of a mode
- Attraction basin: the region for which all trajectories lead to the same mode



Slide by Y. Ukrainitz & B. Sarel

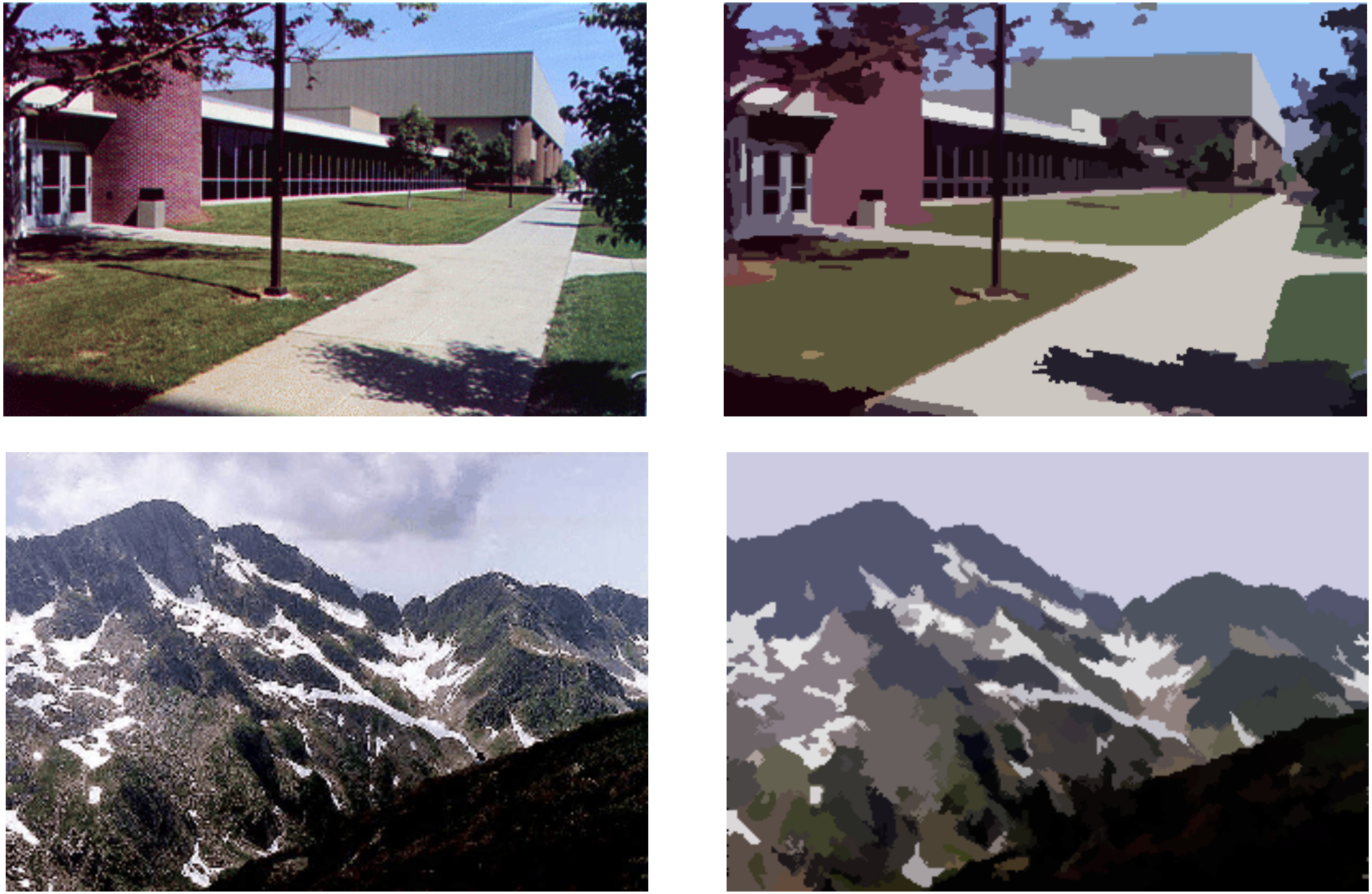
Mean-Shift Clustering/Segmentation

- Find features (color, gradients, texture, etc)
- Initialize windows at individual pixel locations
- Perform mean shift for each window until convergence
- Merge windows that end up near the same "peak" or mode



Slide credit: Svetlana Lazebnik

Mean-Shift Segmentation Results



http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html

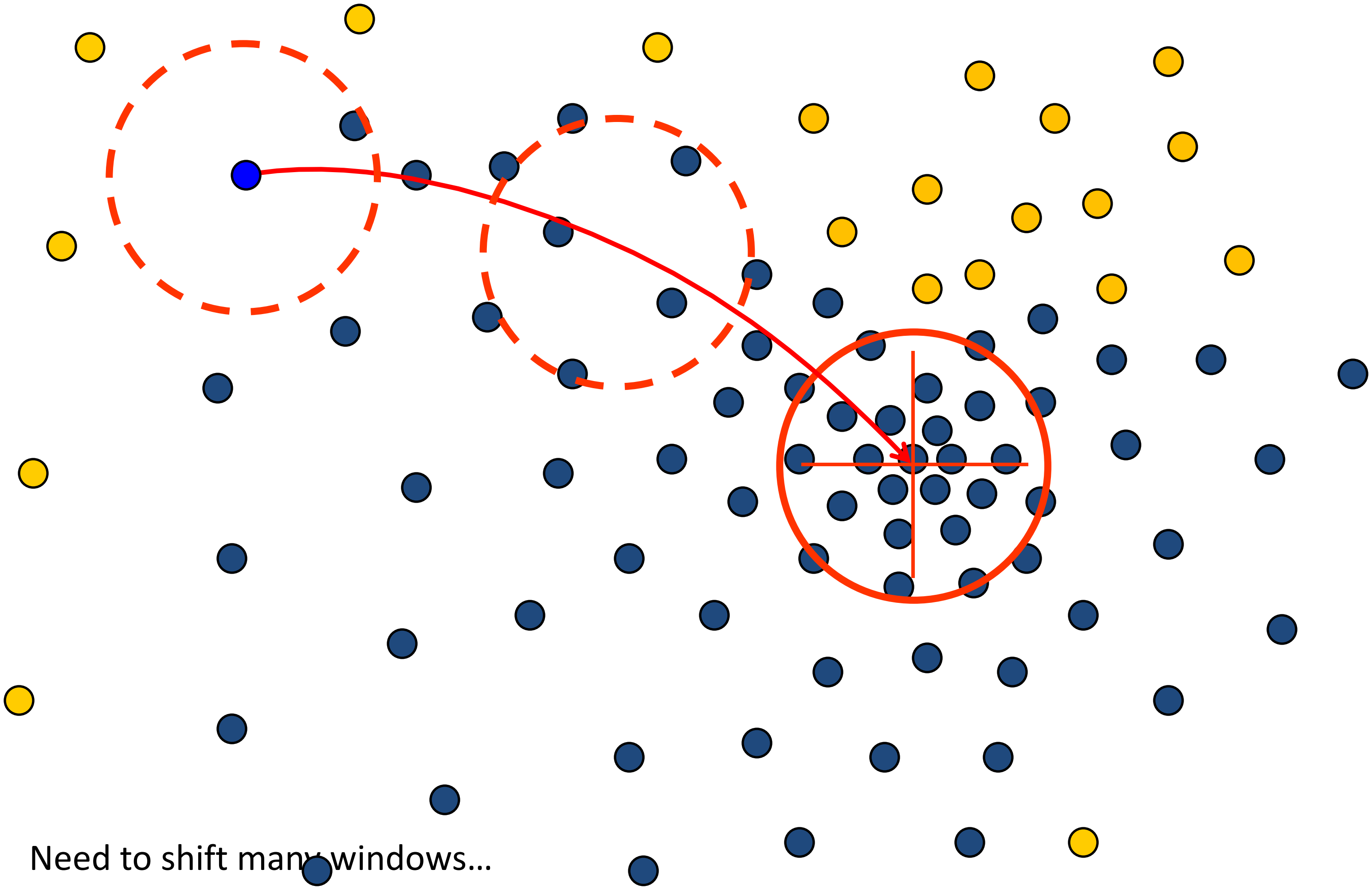
Slide credit: Svetlana Lazebnik

More Results

Slide credit: Svetlana Lazebnik

More Results

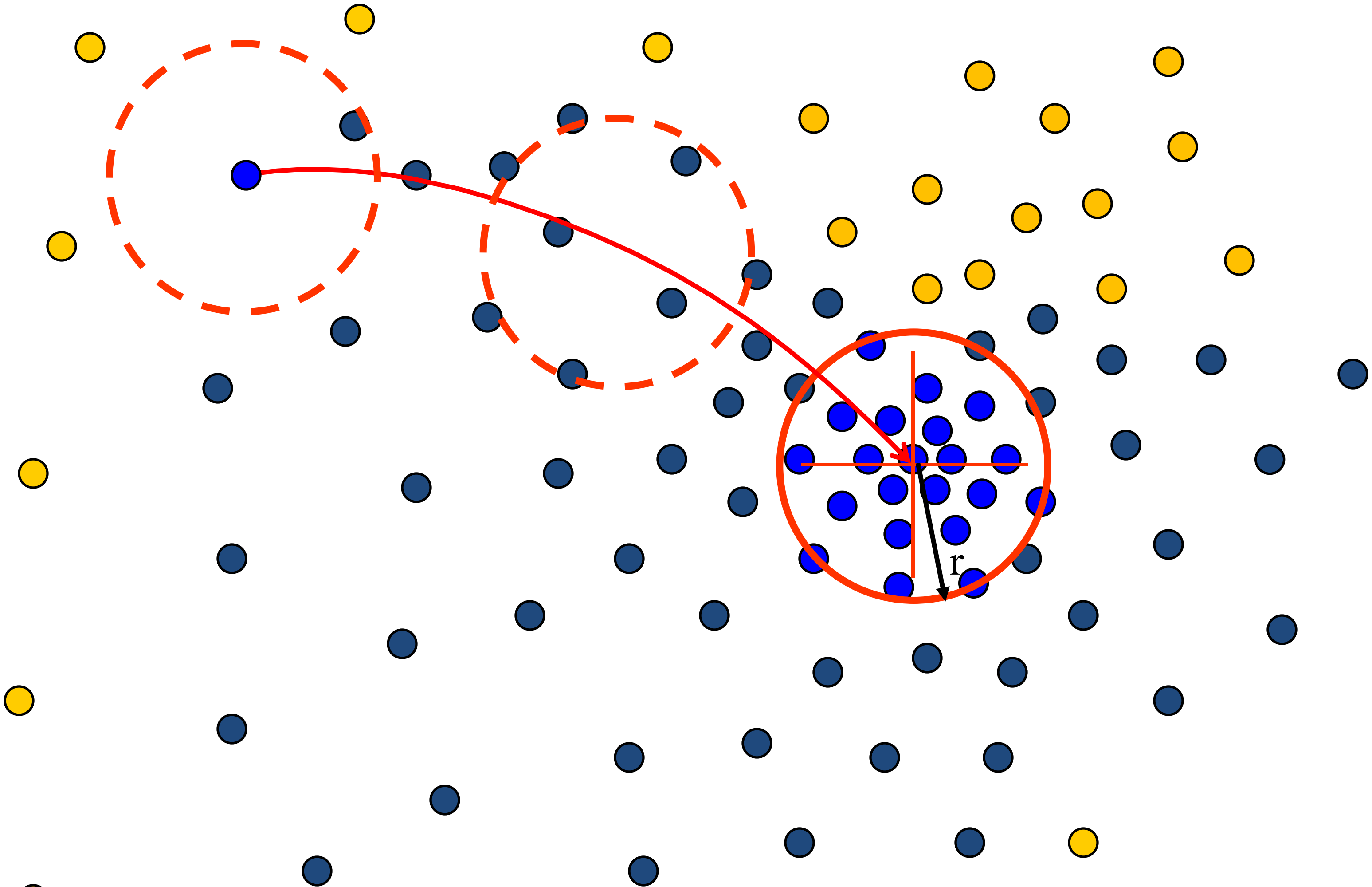
Problem: Computational Complexity



- Need to shift many windows…
- Many computations will be redundant.

Slide credit: Bastian Leibe

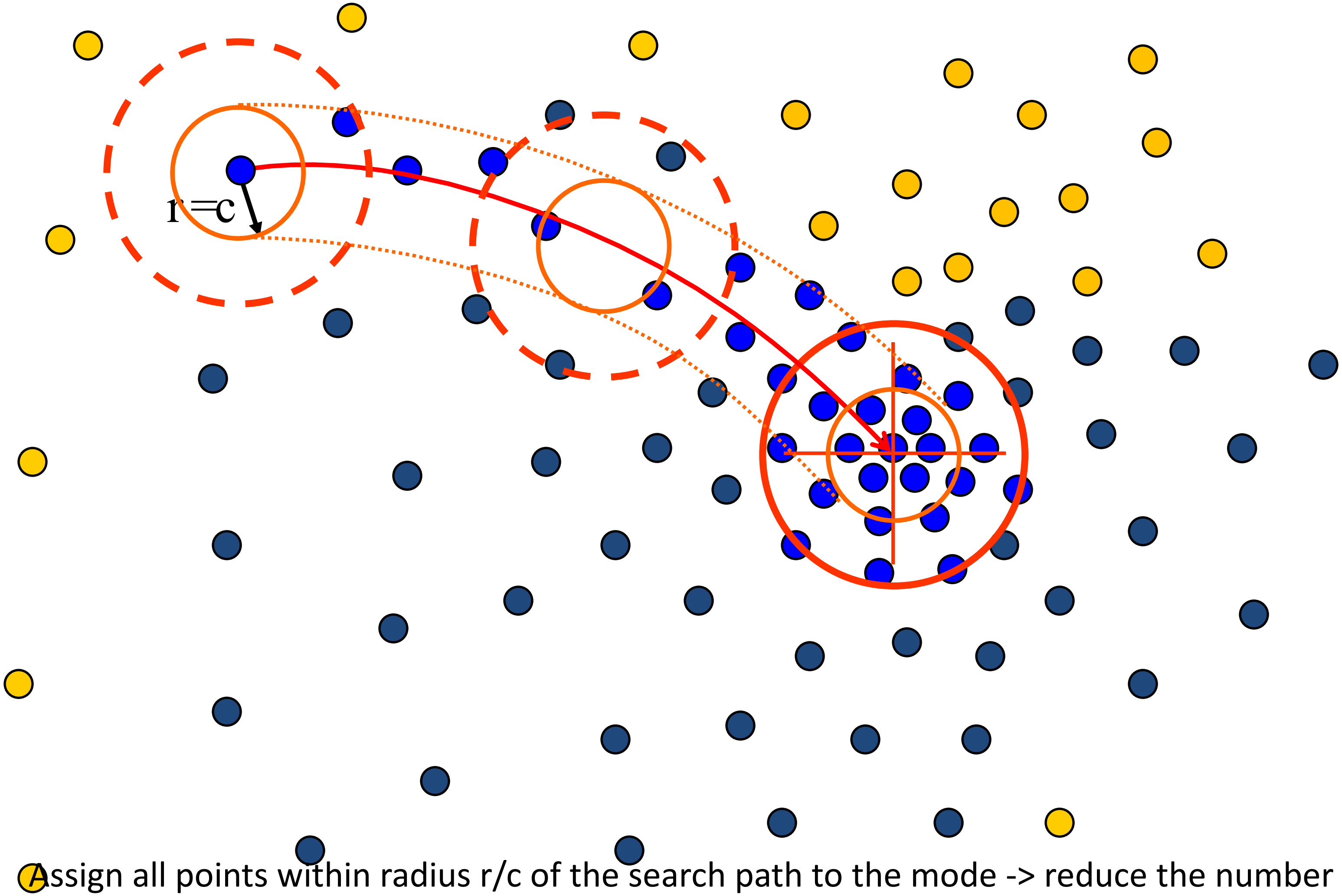
Speedups: Basin of Attraction



1. Assign all points within radius r of end point to the mode.

Slide credit: Bastian Leibe

Speedups



2. Assign all points within radius r/c of the search path to the mode -> reduce the number of data points to search.

Slide credit: Bastian Leibe

Technical Details

Given n data points $\mathbf{x}_i \in \mathbb{R}^d$, the multivariate kernel density estimate using a radially symmetric kernel[1] (e.g., Epanechnikov and Gaussian kernels), $K(\mathbf{x})$, is given by,

$$\hat{f}_K = \frac{1}{nh^d} \sum_{i=1}^{n} K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right), \tag{1}$$

where h (termed the *bandwidth* parameter) defines the radius of kernel. The radially symmetric kernel is defined as,

$$K(\mathbf{x}) = c_k k(\|\mathbf{x}\|^2), \tag{2}$$

where c_k represents a normalization constant.

Comaniciu & Meer, 2002

Technical Details

$$\nabla \hat{f}(\mathbf{x}) = \underbrace{\frac{2c_{k,d}}{nh^{d+2}} \left[\sum_{i=1}^{n} g\left(\left\| \frac{\mathbf{x} - \mathbf{x}_i}{h} \right\|^2 \right) \right]}_{\text{term 1}} \underbrace{\left[\frac{\sum_{i=1}^{n} \mathbf{x}_i g\left(\left\| \frac{\mathbf{x} - \mathbf{x}_i}{h} \right\|^2 \right)}{\sum_{i=1}^{n} g\left(\left\| \frac{\mathbf{x} - \mathbf{x}_i}{h} \right\|^2 \right)} - \mathbf{x} \right]}_{\text{term 2}}, \qquad (3)$$

where $g(x) = -k'(x)$ denotes the derivative of the selected kernel profile.

- Term1: this is proportional to the density estimate at x (similar to equation 1 from the previous slide).
- Term2: this is the mean-shift vector that points towards the direction of maximum density.

Comaniciu & Meer, 2002

Technical Details

Finally, the mean shift procedure from a given point x_t is:

1. Computer the mean shirt vector m:

$$\left[\frac{\sum_{i=1}^{n} \mathbf{x}_i g\left(\left\| \frac{\mathbf{x}-\mathbf{x}_i}{h} \right\|^2 \right)}{\sum_{i=1}^{n} g\left(\left\| \frac{\mathbf{x}-\mathbf{x}_i}{h} \right\|^2 \right)} - \mathbf{x} \right]$$

2. Translate the density window:

$$\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \mathbf{m}(\mathbf{x}_i^t).$$

3. Iterate steps 1 and 2 until convergence.

$$\nabla f(\mathbf{x}_i) = 0.$$

Comaniciu & Meer, 2002

Summary Mean-Shift

- Pros
  - General, application-independent tool
  - Model-free, does not assume any prior shape (spherical, elliptical, etc.) on data clusters
  - Just a single parameter (window size h)
    - h has a physical meaning (unlike k-means)
  - Finds variable number of modes
  - Robust to outliers
- Cons
  - Output depends on window size
  - Window size (bandwidth) selection is not trivial
  - Computationally (relatively) expensive (~2s/image)
  - Does not scale well with dimension of feature space

Slide credit: Svetlana Lazebnik

What will we have learned today

- K-means clustering
- Mean-shift clustering

Reading: [FP] Chapters: 14.2, 14.4

D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.