Lecture 8: Camera Models

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What we will learn today?

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
  - Projection matrix
  - Intrinsic parameters
  - Extrinsic parameters

Reading:

[FP] Chapters 1 – 3

[HZ] Chapter 6

What we will learn today?

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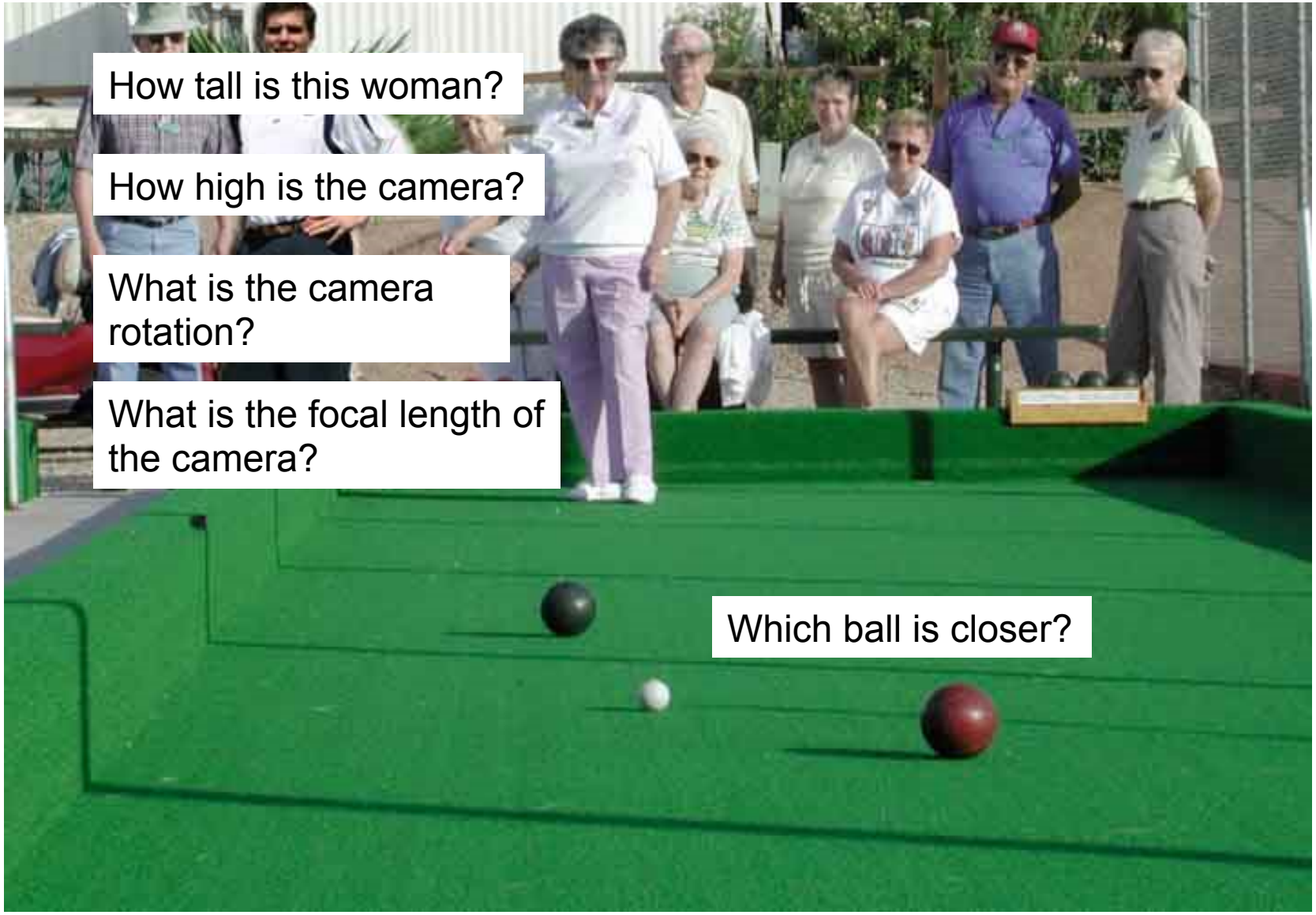
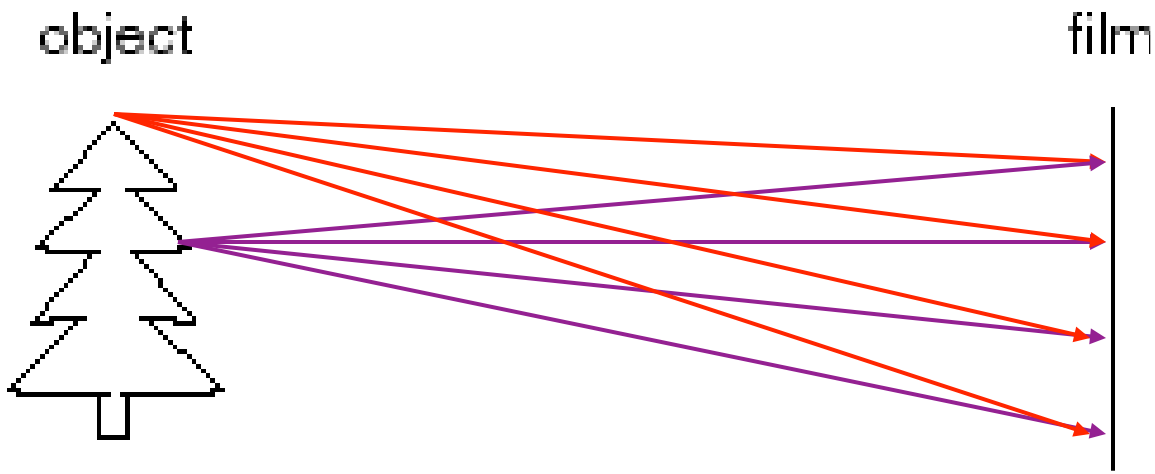
Camera and World Geometry



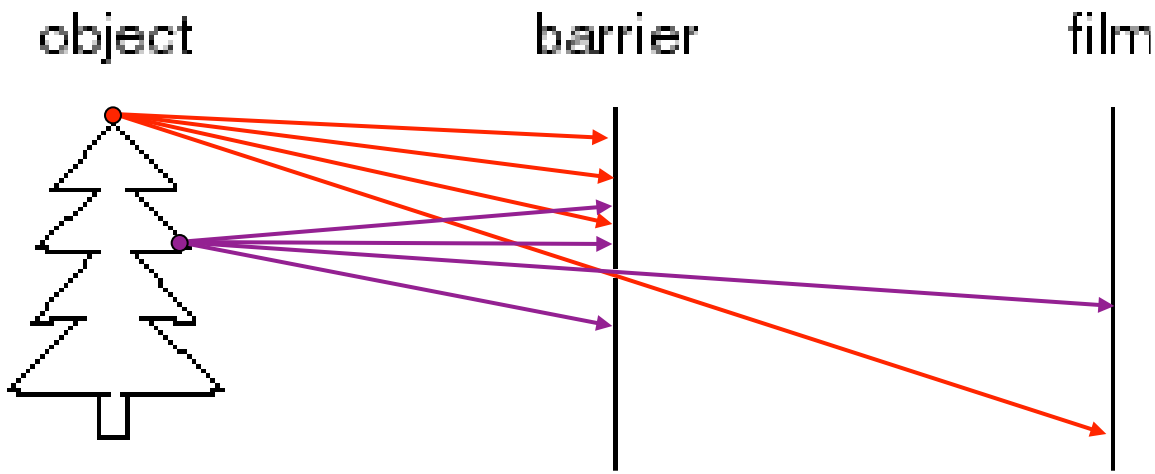
Slide credit: J. Hayes

How do we see the world?



- Let's design a camera
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

Pinhole camera



- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening known as the **aperture**

Camera obscura: the pre-camera

- Known during classical period in China and Greece (e.g. Mo-Tsi, China, 470BC to 390BC)

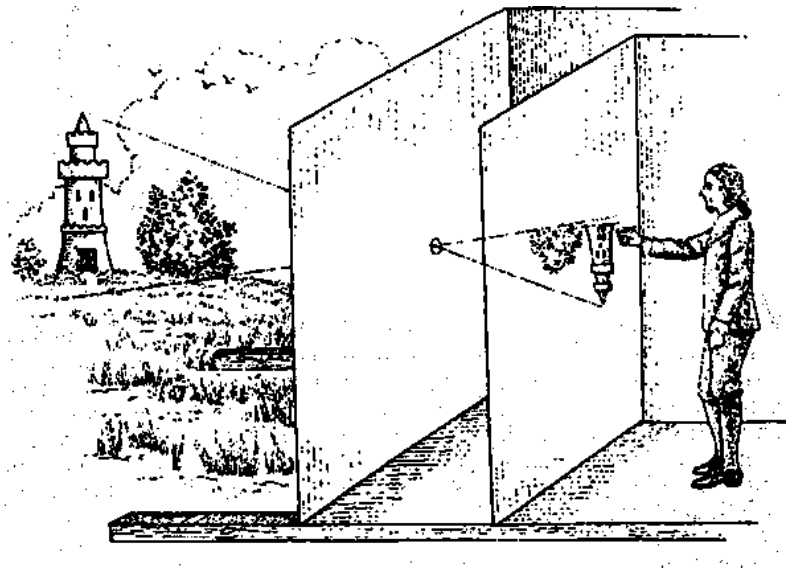


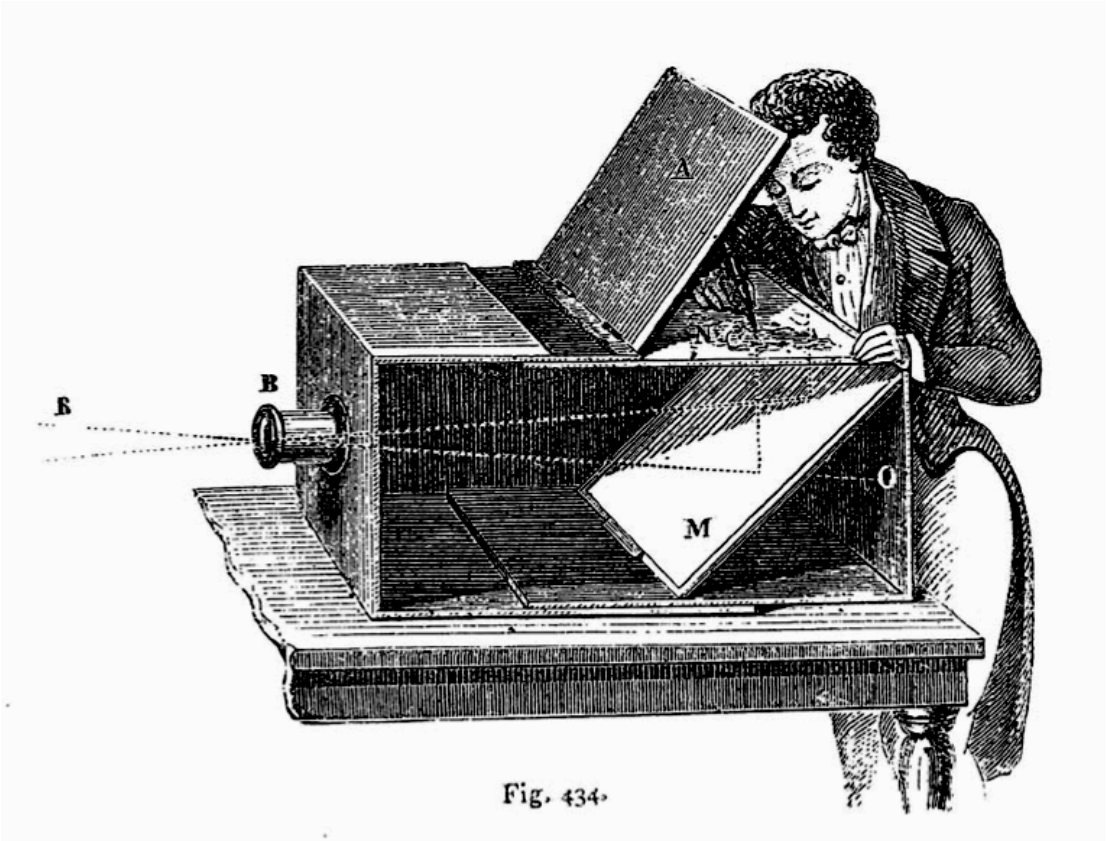
Illustration of Camera Obscura

Freestanding camera obscura at UNC Chapel Hill

Photo by Seth Ilys

Slide credit: J. Hayes

Camera Obscura used for Tracing



Lens Based Camera Obscura, 1568

Slide credit: J. Hayes

First Photograph

Oldest surviving photograph

- Took 8 hours on pewter plate

Joseph Niepce, 1826

Photograph of the first photograph

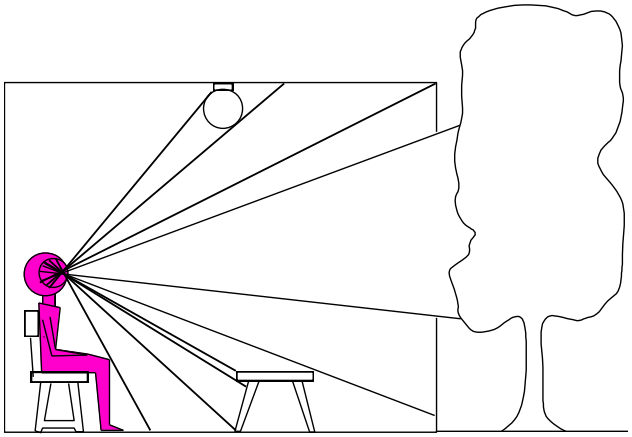
Stored at UT Austin

Niepce later teamed up with Daguerre, who eventually created Daguerrotypes

Slide credit: J. Hayes

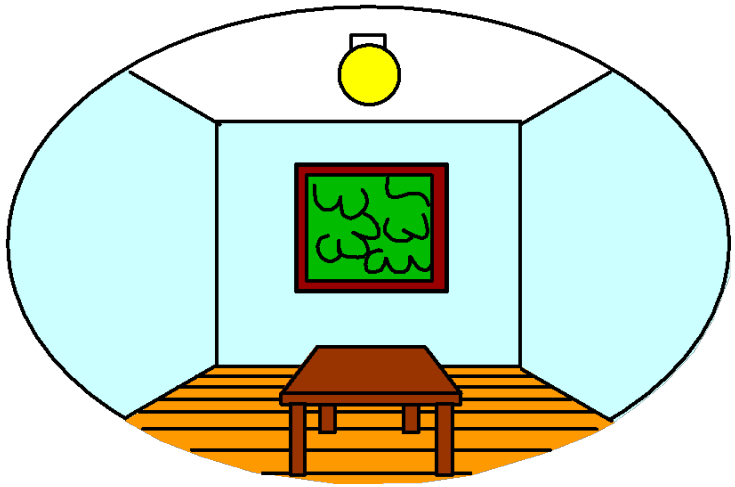
Dimensionality Reduction Machine (3D to 2D)

3D world



Point of observation

2D image



Projection can be tricky…

Slide source: Seitz

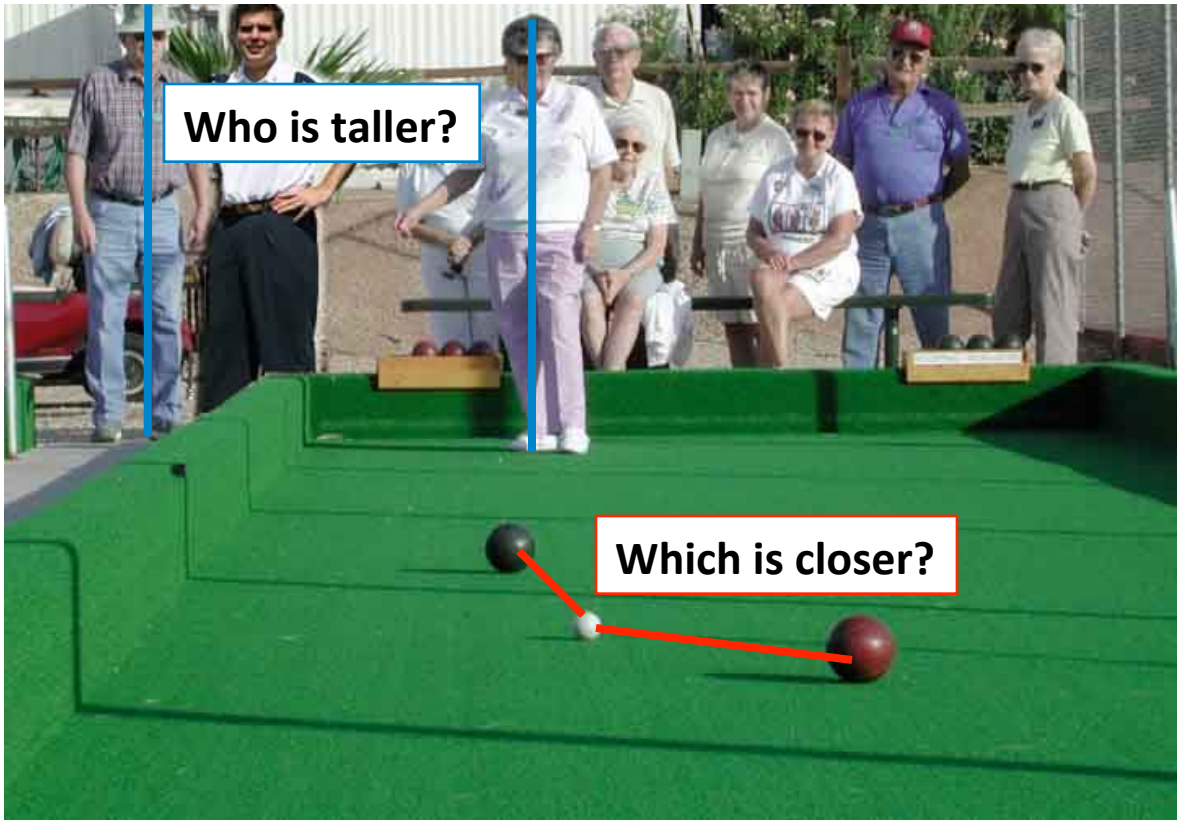
Projection can be tricky…

Slide source: Seitz

Projective Geometry

What is lost?

- Length



Slide credit: J. Hayes

Length is not preserved

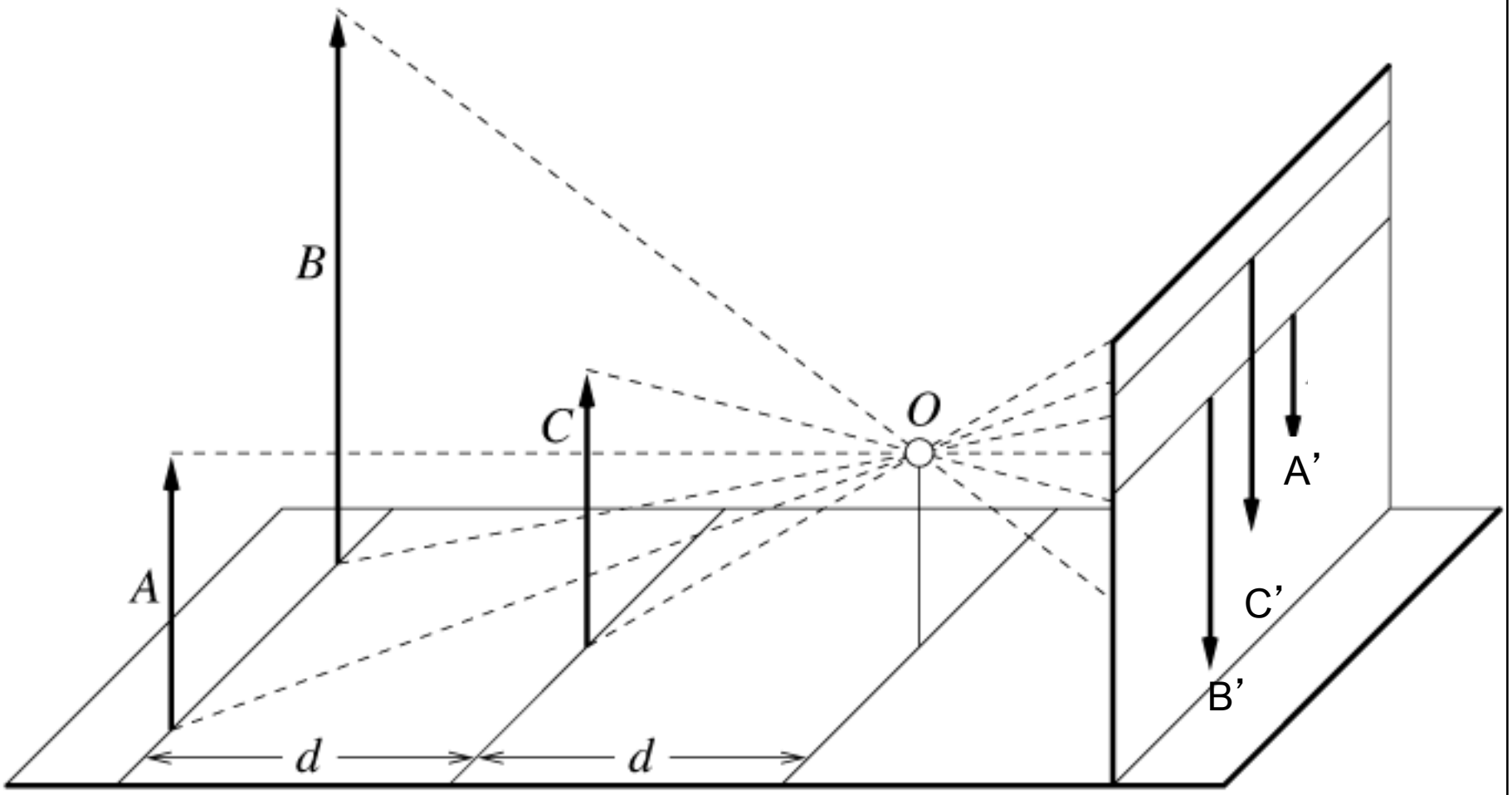
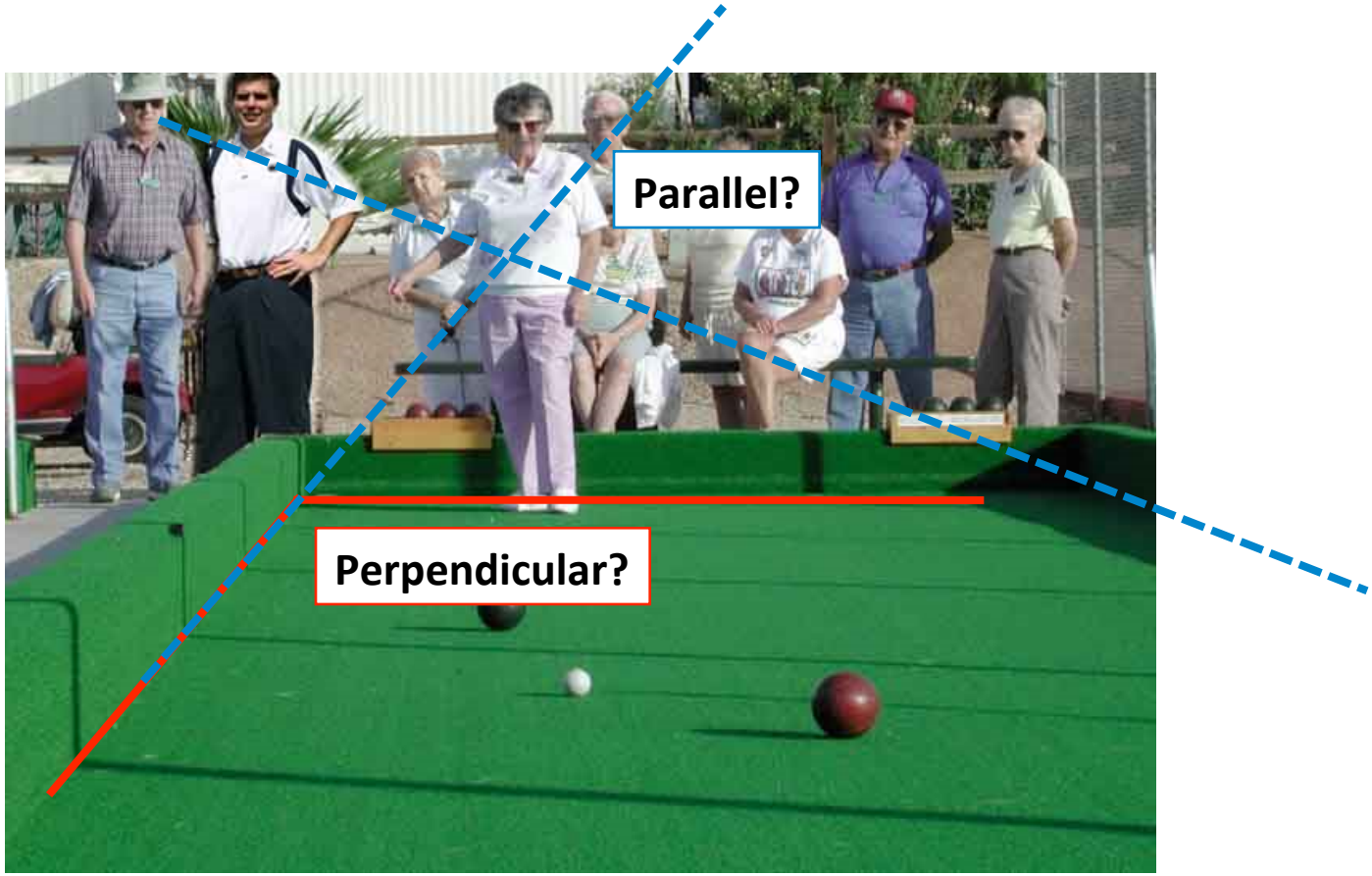


Figure by David Forsyth

Projective Geometry

What is lost?

- Length
- Angles

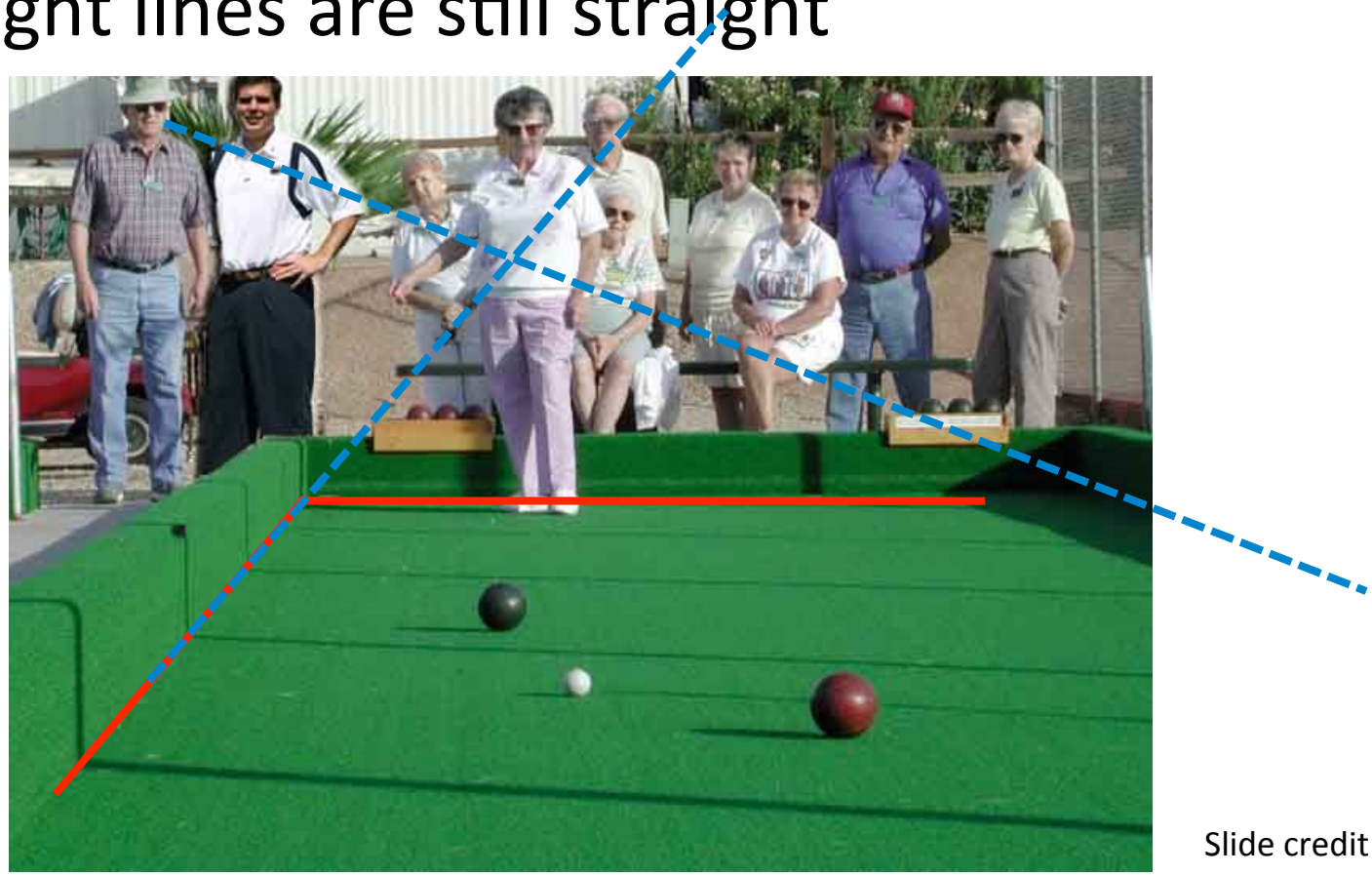


Slide credit: J. Hayes

Projective Geometry

What is preserved?

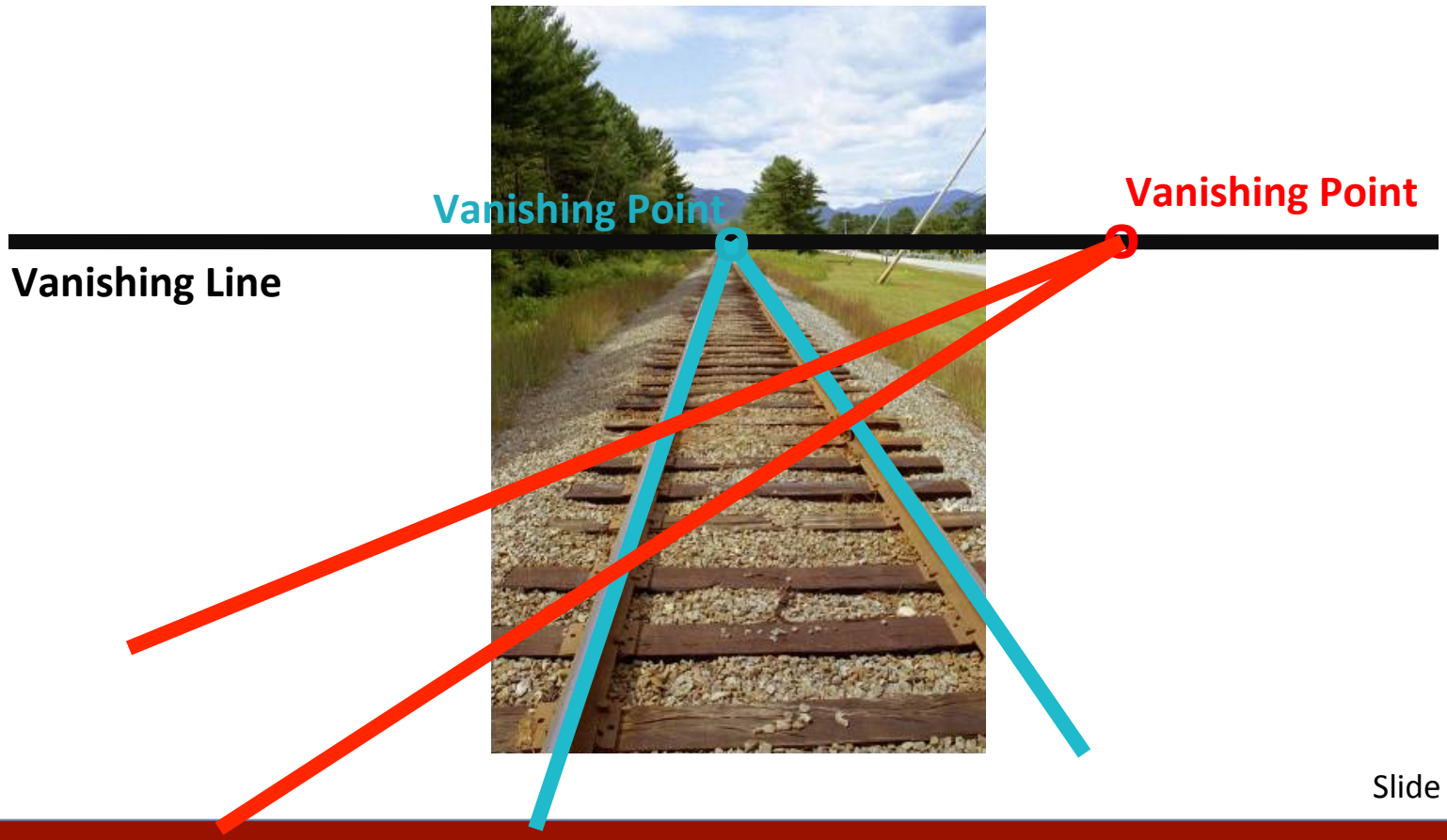
- Straight lines are still straight



Slide credit: J. Hayes

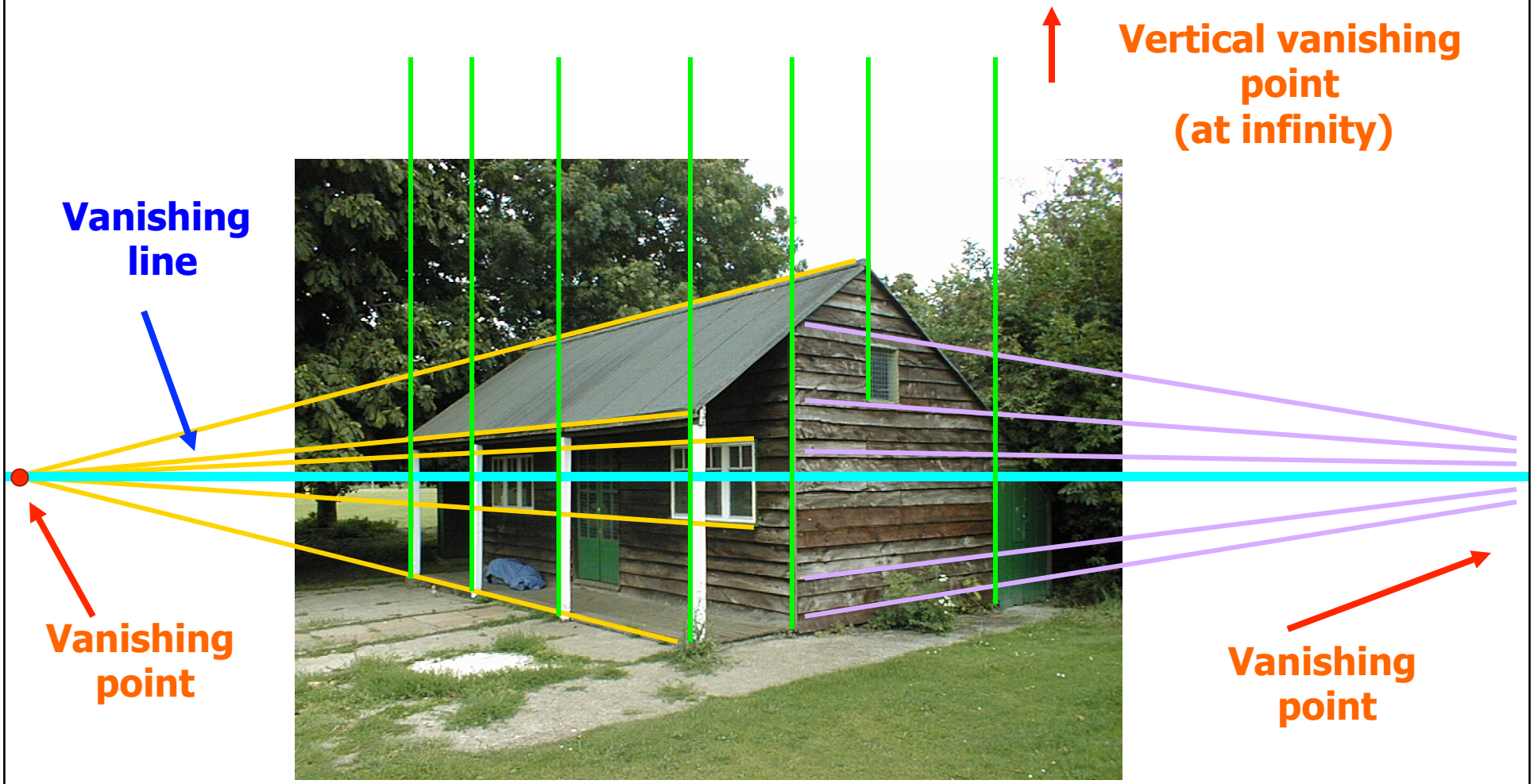
Vanishing points and lines

Parallel lines in the world intersect in the image at a "vanishing point"



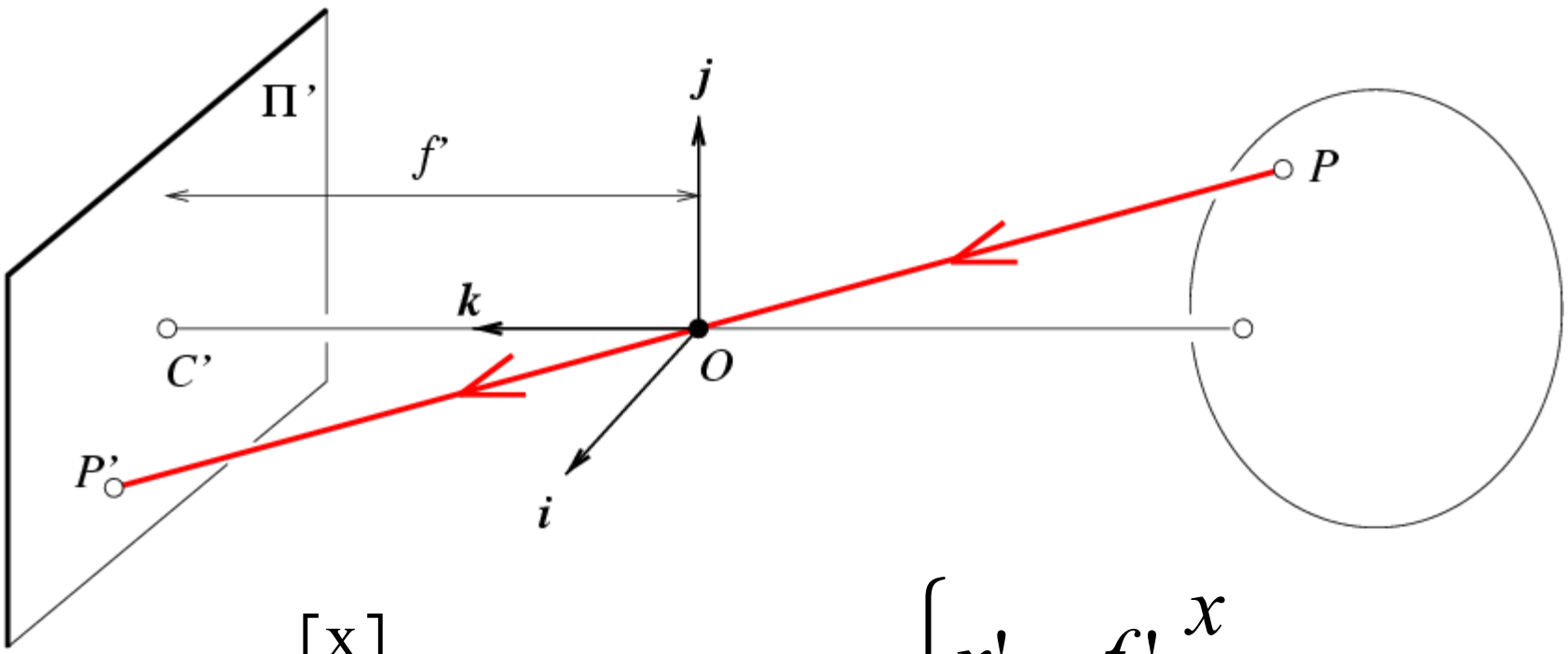
Slide credit: J. Hayes

Vanishing points and lines



Slide from Efros, Photo from Criminisi

Pinhole camera



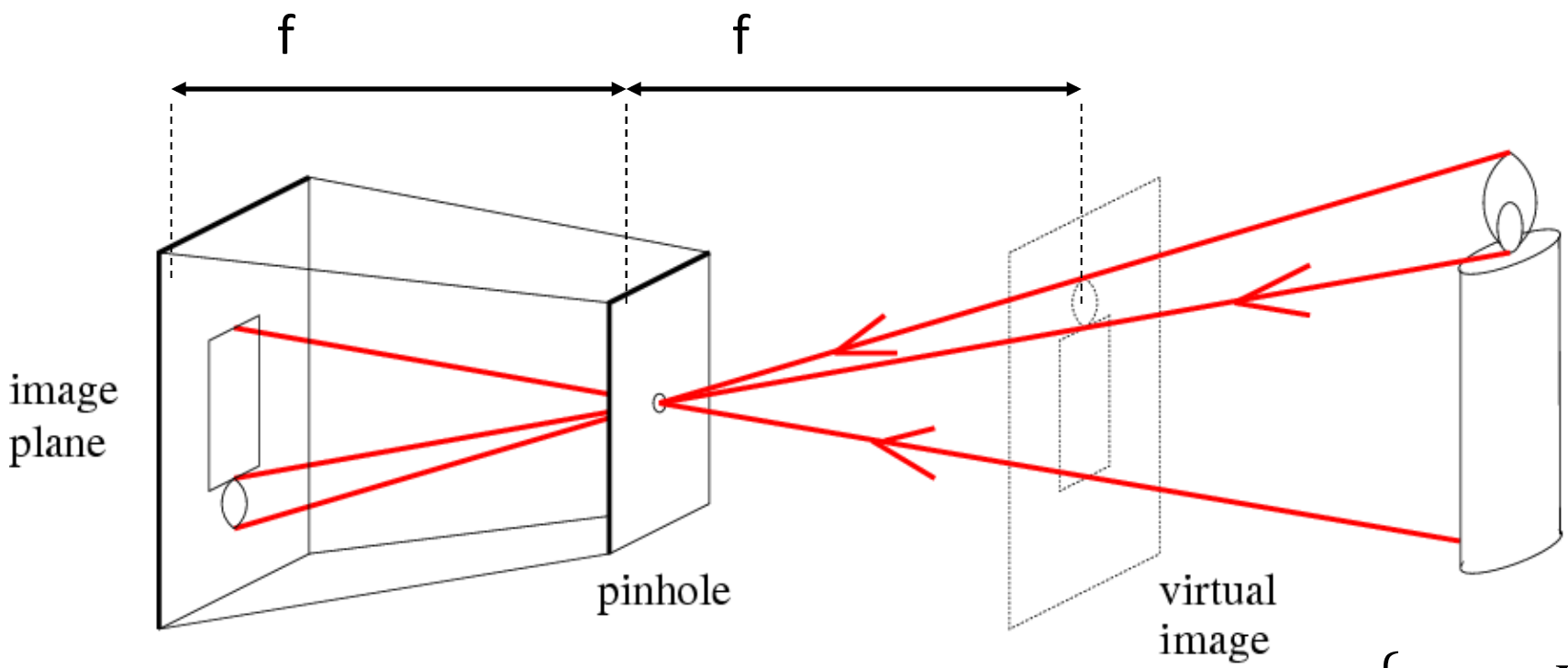
$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{cases} x' = f' \dfrac{x}{z} \\ y' = f' \dfrac{y}{z} \end{cases}$$

Note: z is always negative.

Derived using similar triangles

Pinhole camera

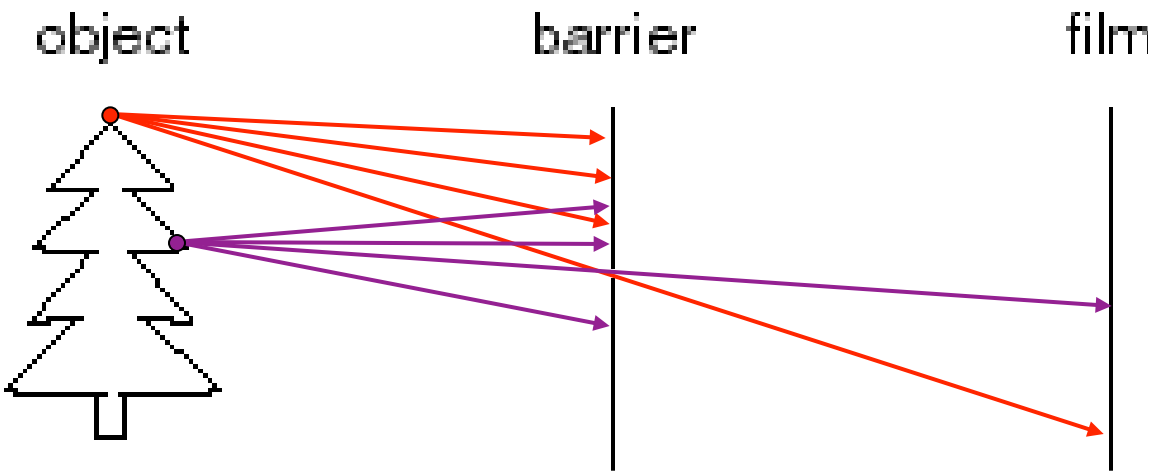


$$\begin{cases} x' = f\,\dfrac{x}{z} \\ y' = f\,\dfrac{y}{z} \end{cases}$$

- Common to draw image plane *in front* of the focal point
- Moving the image plane merely scales the image.

Pinhole camera

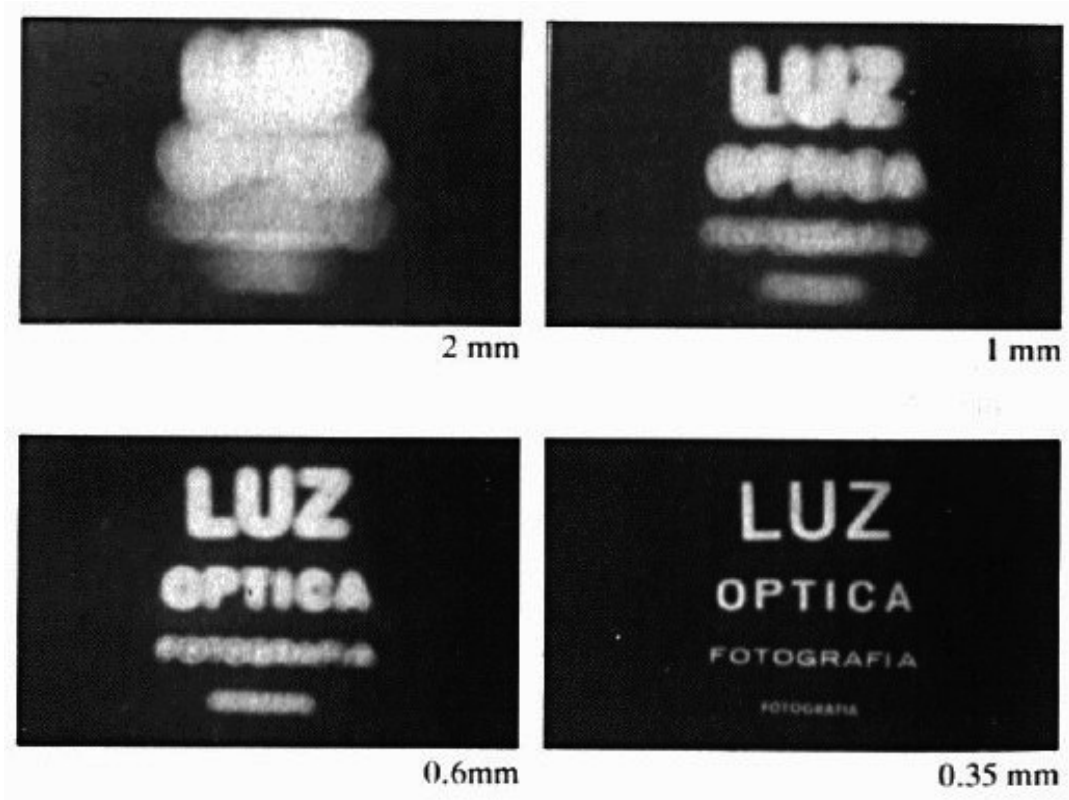
Is the size of the aperture important?



Cameras & Lenses

Shrinking aperture size

- Rays are mixed up



-Why the aperture cannot be too small?

-Less light passes through

-Diffraction effect

Adding lenses!

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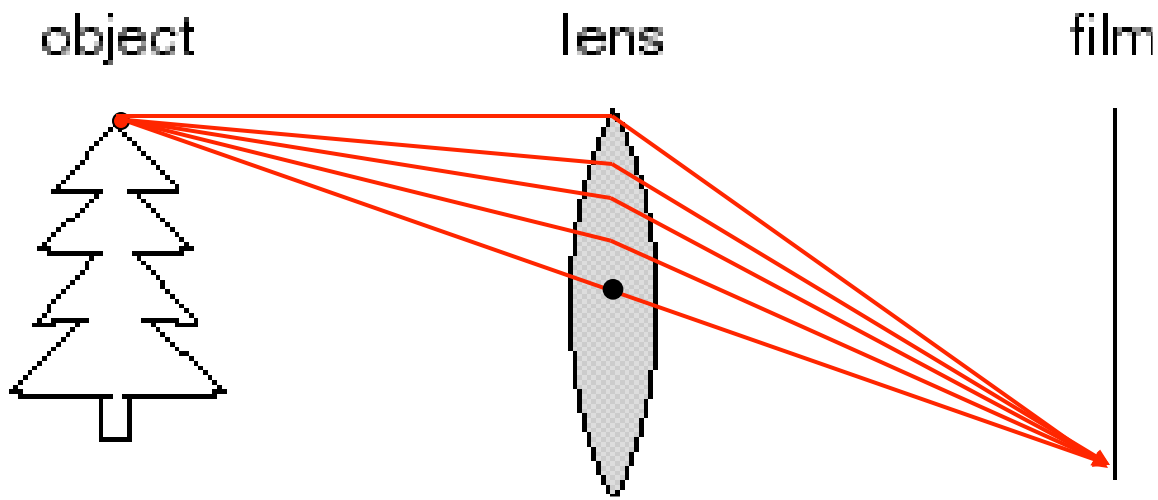
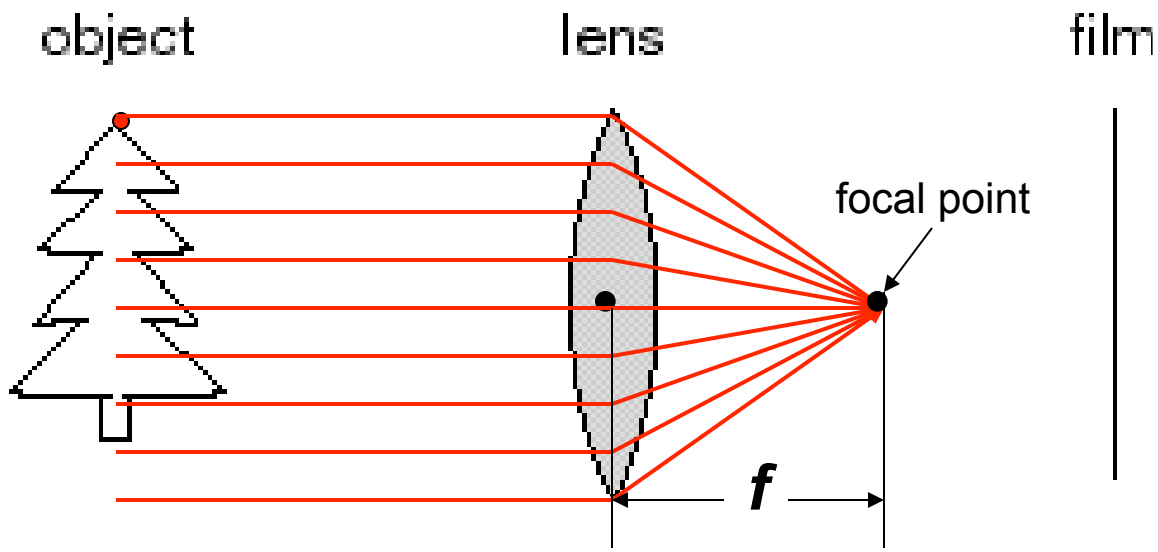
Cameras & Lenses



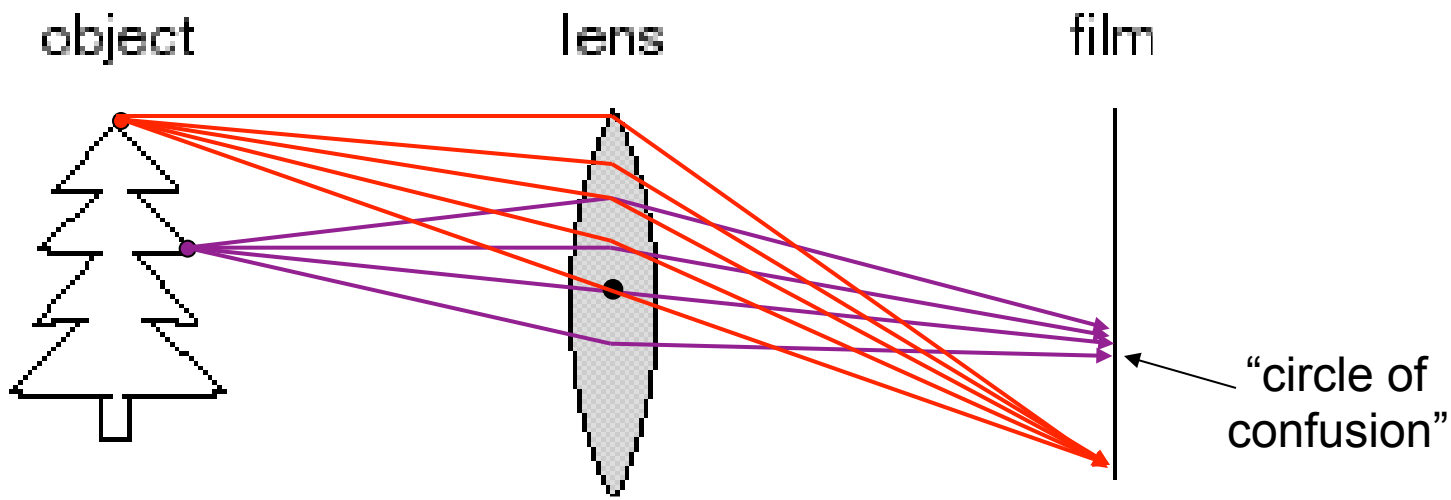
- A lens focuses light onto the film

Cameras & Lenses



- A lens focuses light onto the film
  - Rays passing through the center are not deviated
  - All parallel rays converge to one point on a plane located at the *focal length f*

Cameras & Lenses



- A lens focuses light onto the film
  - There is a specific distance at which objects are "in focus"
    [other points project to a "circle of confusion" in the image]

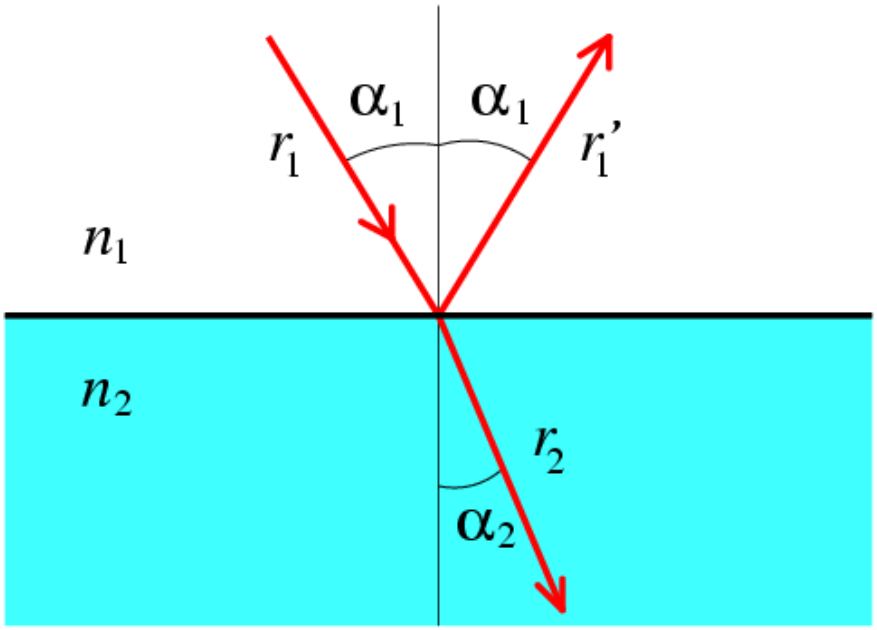
Cameras & Lenses

- Laws of geometric optics
  - Light travels in straight lines in homogeneous medium
  - Reflection upon a surface: incoming ray, surface normal, and reflection are co-planar
  - Refraction: when a ray passes from one medium to another

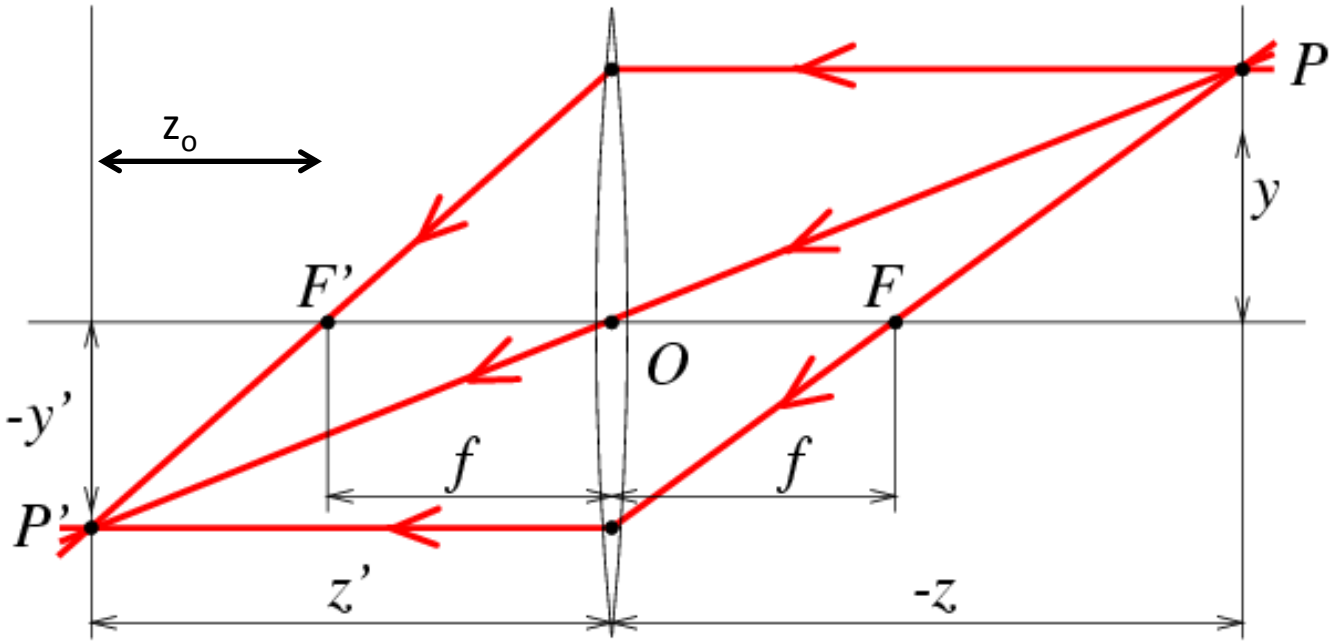
Snell's law

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

α_1 = incident angle
α_2 = refraction angle
n_i = index of refraction



Thin Lenses



$$z' = f + z_o$$

$$f = \frac{R}{2(n-1)}$$

Snell's law:

$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$

Small angles:

$n_1 \alpha_1 \approx n_2 \alpha_2$

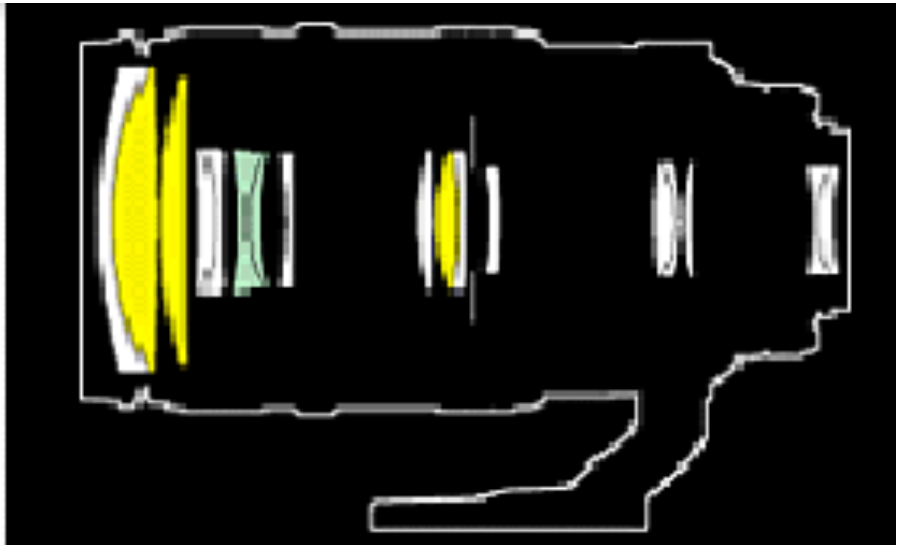
$n_2 = n$ (lens)

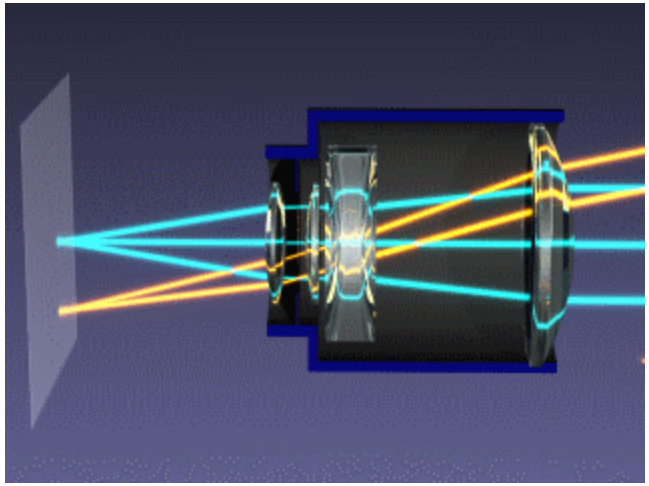
$n_1 = 1$ (air)

$$x' = z' \frac{x}{z}$$

$$y' = z' \frac{y}{z}$$

Cameras & Lenses



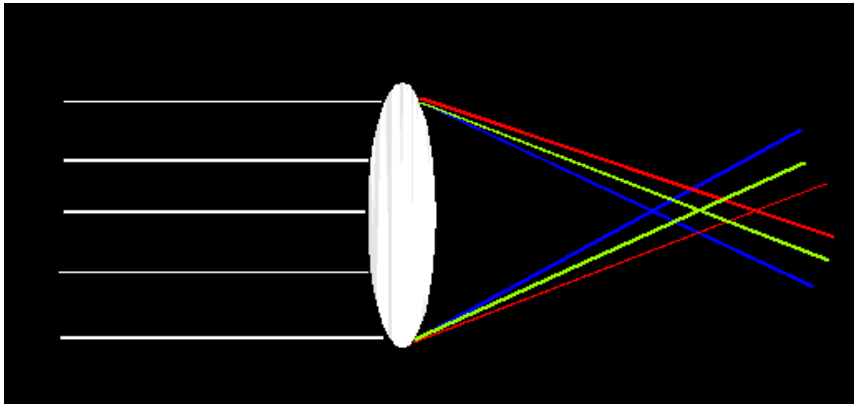


Source wikipedia

Issues with lenses: Chromatic Aberration

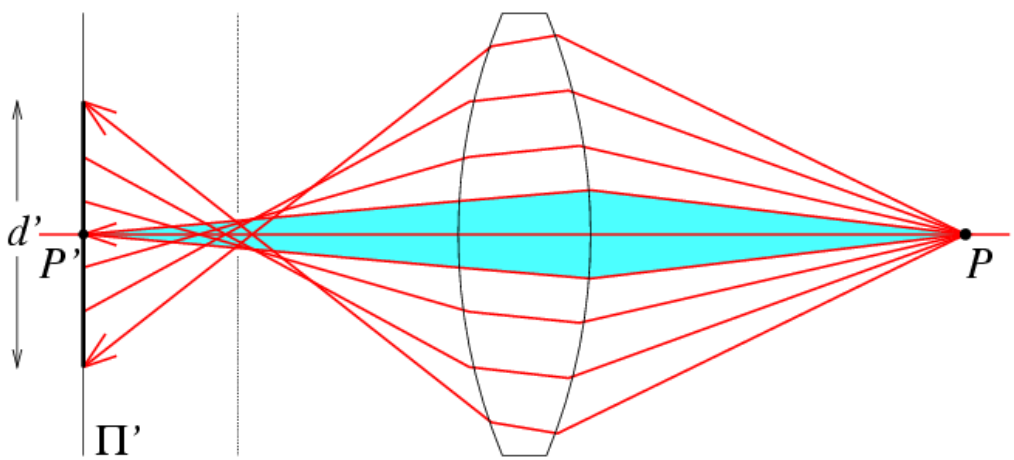
- Lens has different refractive indices for different wavelengths: causes color fringing

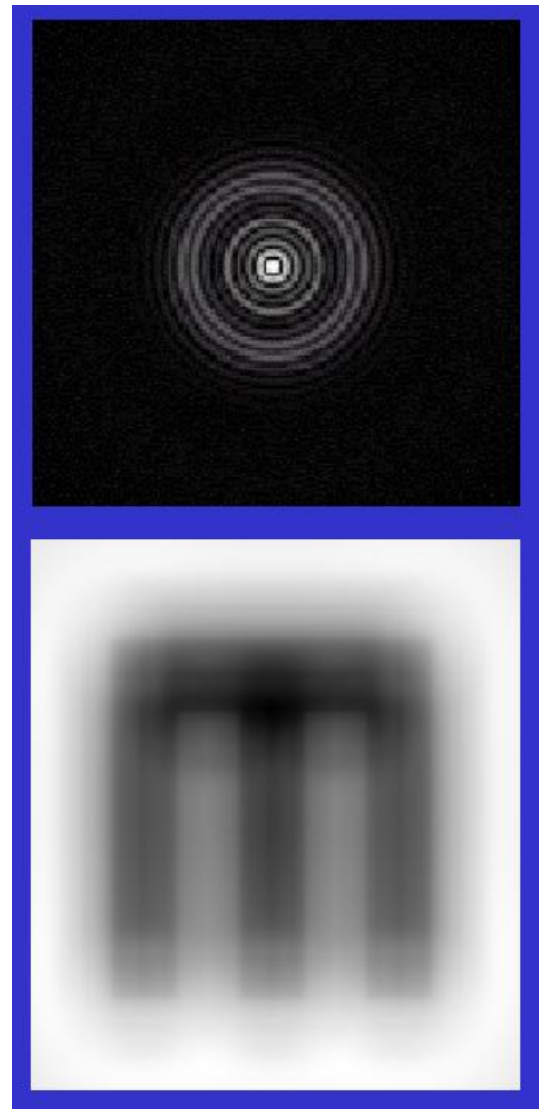
$$f = \frac{R}{2(n-1)}$$



Issues with lenses: Spherical aberration

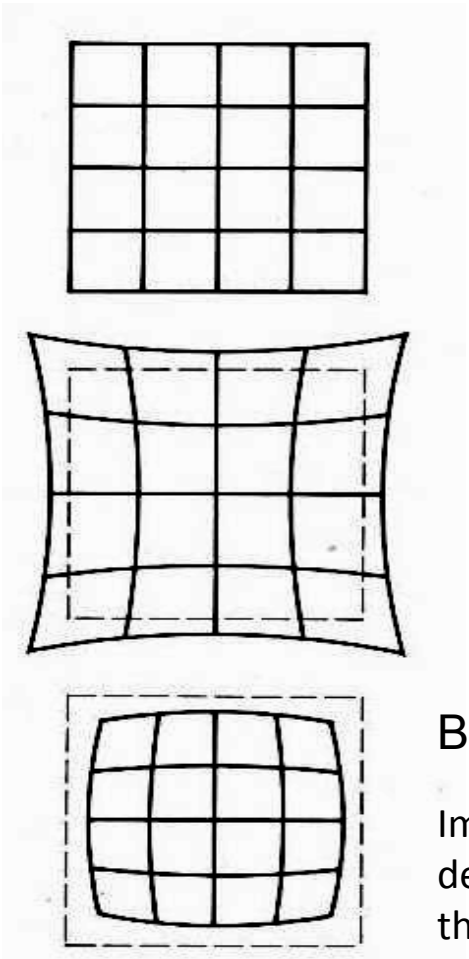
- Rays farther from the optical axis focus closer





Issues with lenses: Radial Distortion

– Deviations are most noticeable for rays that pass through the edge of the lens



No distortion

Pin cushion

Barrel (fisheye lens)

Image magnification decreases with distance from the optical axis

What we will learn today?

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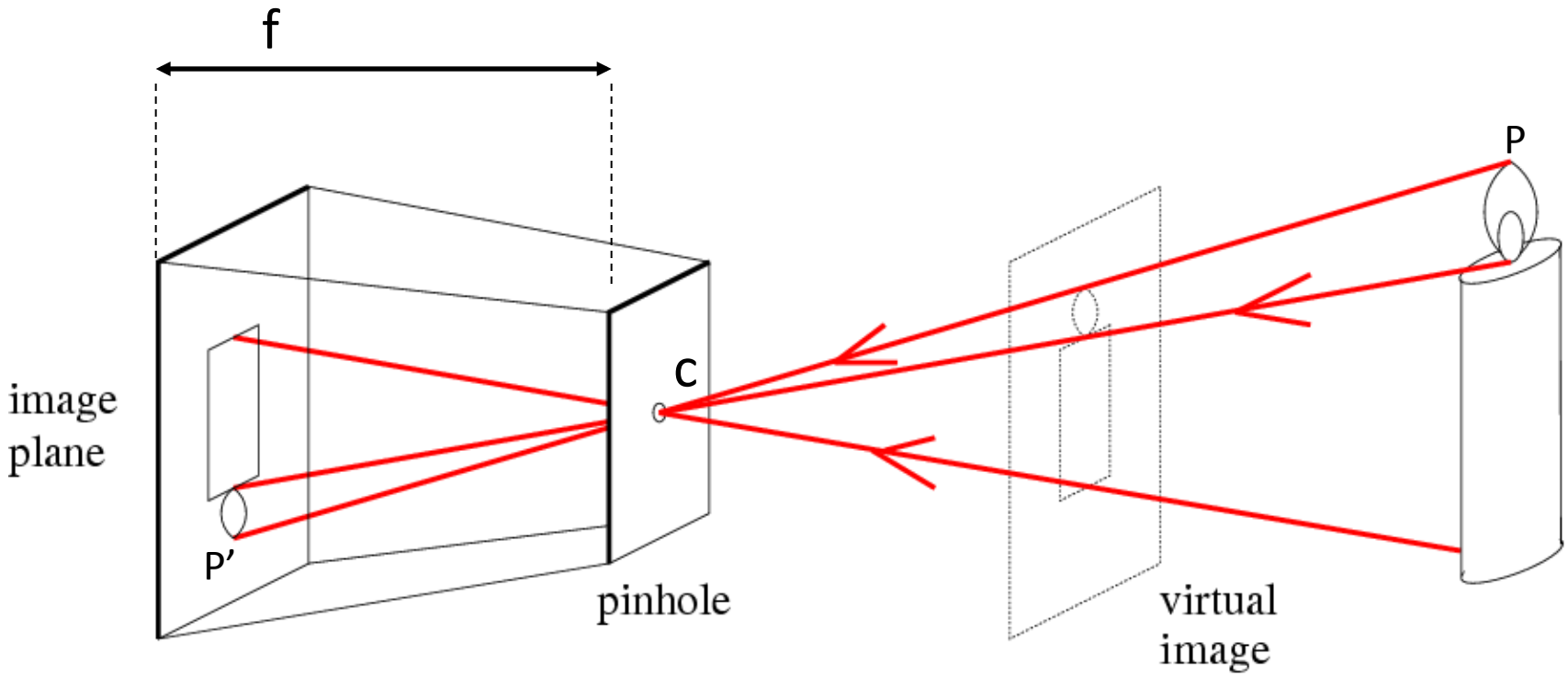
Relating real-world point to a point on a camera



$$P = (x, y, z) \rightarrow P' = (f\frac{x}{z}, f\frac{y}{z})$$

f = focal length

c = center of the camera

$$\Re^3 \xrightarrow{E} \Re^2$$

Relating real-world point to a point on a camera

Is this a linear transformation?

$$P = (x, y, z) \rightarrow P' = (f\frac{x}{z}, f\frac{y}{z})$$

No — division by z is nonlinear!

How to make it linear?

Homogeneous coordinates – a reminder

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

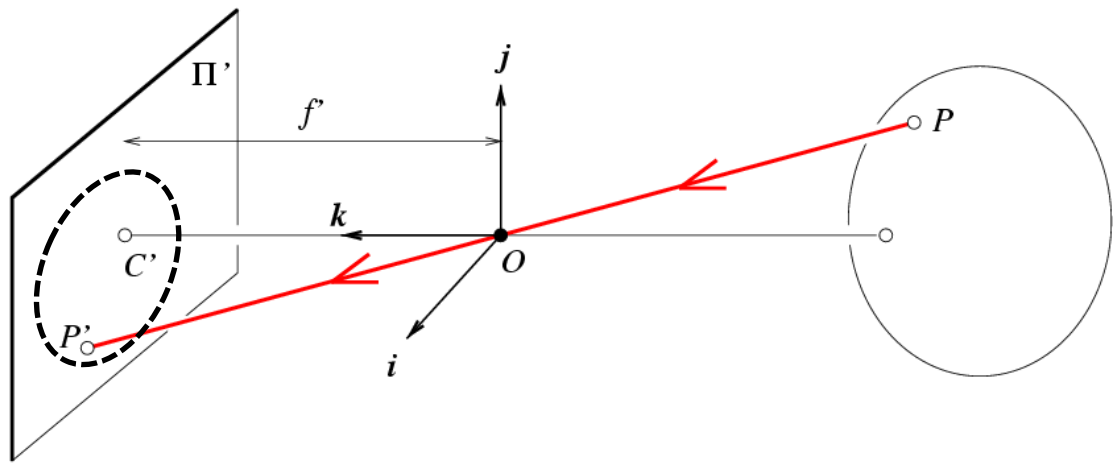
homogeneous scene coordinates

- Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Relating a real-world point to a point on the camera



In Cartesian coordinates:

$$P = (x, y, z) \rightarrow P' = (f\frac{x}{z}, f\frac{y}{z})$$

In homogeneous coordinates:

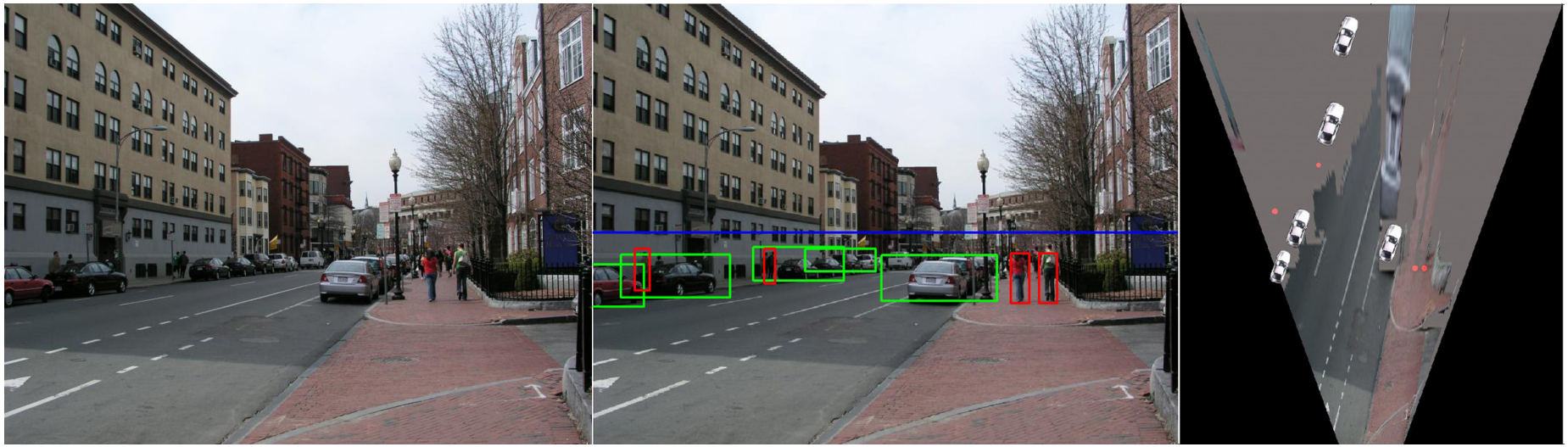
$$P' = \begin{bmatrix} f\ x \\ f\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{M}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

"Projection matrix"

$$P' = M\ P \qquad \Re^4 \xrightarrow{\text{H}} \Re^3$$

Interlude: why does this matter?

Object Recognition (CVPR 2006)



Slide credit: J. Hayes

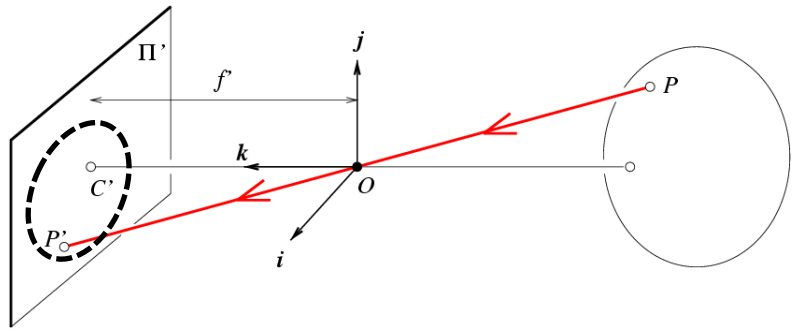
Inserting photographed objects into images (SIGGRAPH 2007)

Original

Created

Slide credit: J. Hayes

Relating a real-world point to a point on the camera



In homogeneous coordinates:

$$P' = \begin{bmatrix} f\,x \\ f\,y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{M} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

ideal world

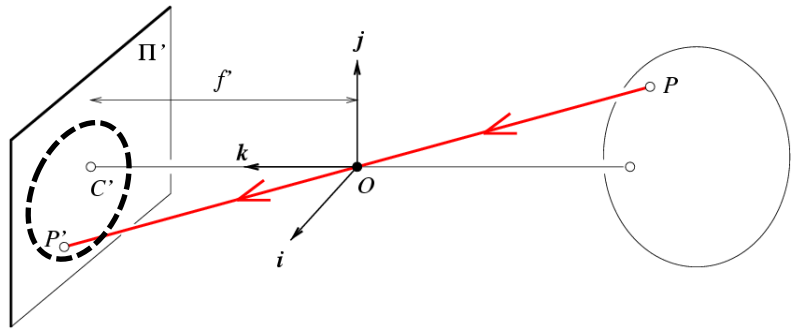
Intrinsic Assumptions

- Unit aspect ratio
- Optical center at (0,0)
- No skew

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

Relating a real-world point to a point on the camera



In homogeneous coordinates:

$$P' = \begin{bmatrix} f\,x \\ f\,y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = K\begin{bmatrix} I & 0 \end{bmatrix} P$$

Intrinsic Assumptions

- Unit aspect ratio
- Optical center at (0,0)
- No skew

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

Remove assumption: known optical center

Intrinsic Assumptions
- ~~Optical center at (0,0)~~
- Optical center at (u_0, v_0)
- Square pixels
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$P' = K\begin{bmatrix} I & 0 \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & u_0 & 0 \\ 0 & f & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide inspiration: S. Savarese

Remove assumption: square pixels

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- ~~Square pixels~~
- Rectangular pixels
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$P' = K\begin{bmatrix} I & 0 \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

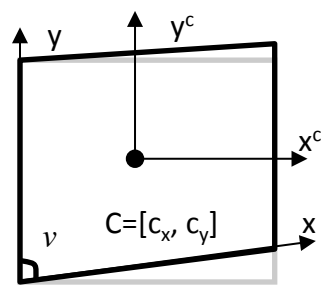
Slide inspiration: S. Savarese

Remove assumption: non-skewed pixels

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- ~~No skew~~
- Small skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$P' = K\begin{bmatrix} I & 0 \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Slide inspiration: S. Savarese

Remove assumption: non-skewed pixels

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
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Extrinsic Assumptions
- No rotation
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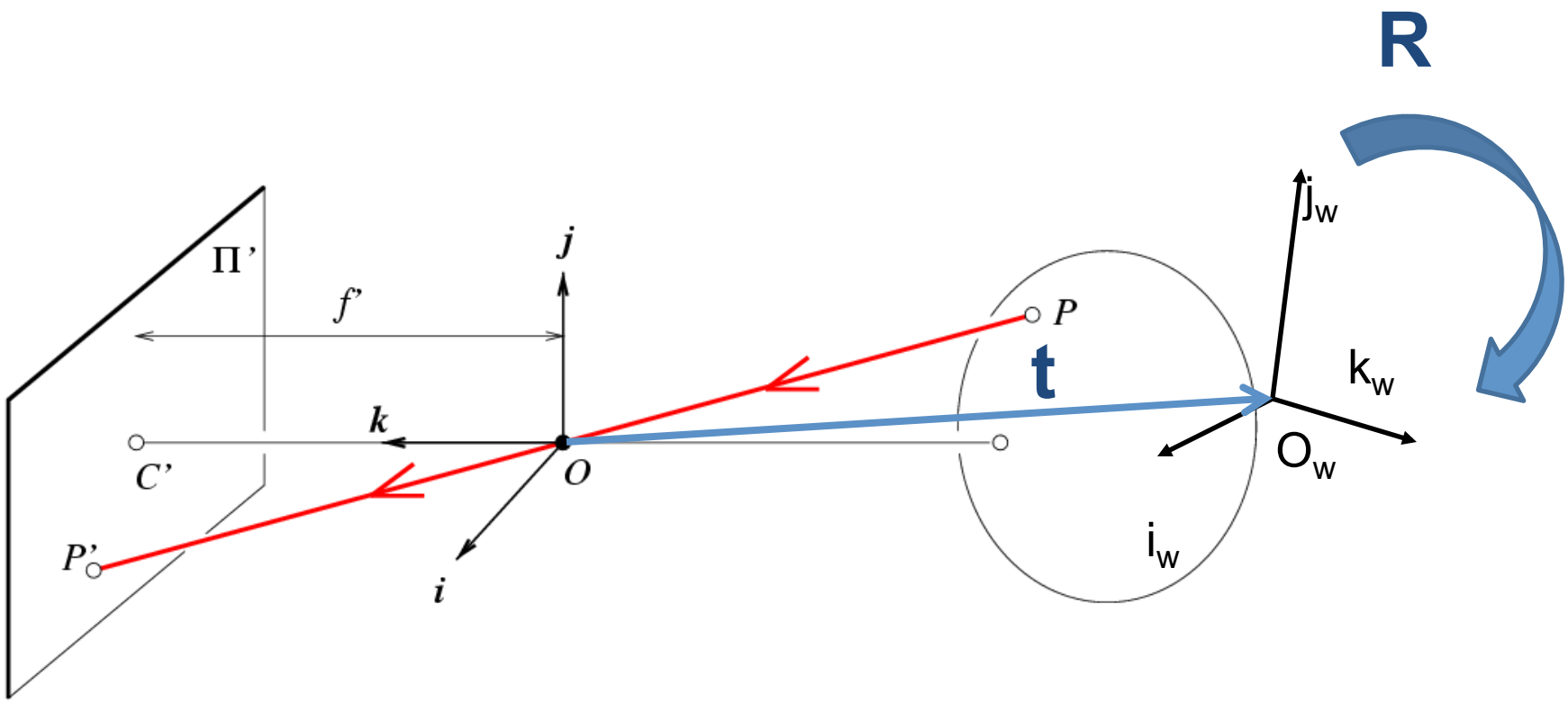
Intrinsic parameters

$$P' = K\begin{bmatrix} I & 0 \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide inspiration: S. Savarese

Real world camera: Translate + Rotate



Slide inspiration: S. Savarese, J. Hayes

Remove assumption: allow translation

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- No rotation
- Camera at ~~(0,0,0)~~ -> (t_x,t_y,t_z)

$$P' = K\begin{bmatrix} I & \bar{t} \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide inspiration: S. Savarese

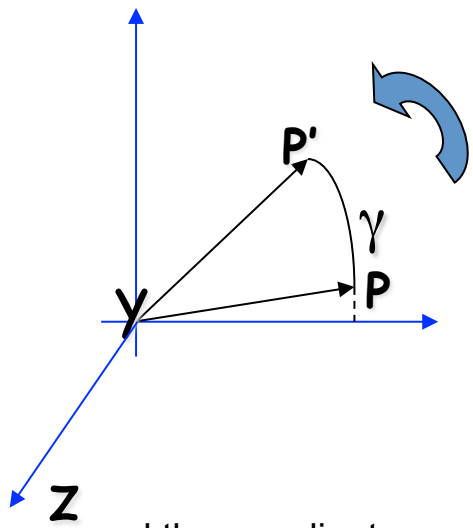
Remove assumption: allow rotation

Intrinsic Assumptions

- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions

- ~~No~~ rotation
- Camera at (t_x,t_y,t_z)



Rotation around the coordinate axes, counter-clockwise

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Slide inspiration: S. Savarese

Remove assumption: allow rotation

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- ~~No~~ rotation
- Camera at (t_x,t_y,t_z)

$$P' = K\begin{bmatrix} R & \bar{t} \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide inspiration: S. Savarese

A generic projection matrix

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- Allow rotation
- Camera at (t_x,t_y,t_z)

$$P' = K\begin{bmatrix} R & \bar{t} \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide inspiration: S. Savarese

A generic projection matrix

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- Allow rotation
- Camera at (t_x,t_y,t_z)

$$P' = K\begin{bmatrix} R & \bar{t} \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Degrees of freedom??

Slide inspiration: S. Savarese

A generic projection matrix

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- Allow rotation
- Camera at (t_x,t_y,t_z)

$$P' = K\begin{bmatrix} R & \bar{t} \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

5 6

Degrees of freedom??

Slide inspiration: S. Savarese

CS231a: Camera Calibration
estimate all intrinsic and extrinsic parameters

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- Allow rotation
- Camera at (t_x,t_y,t_z)

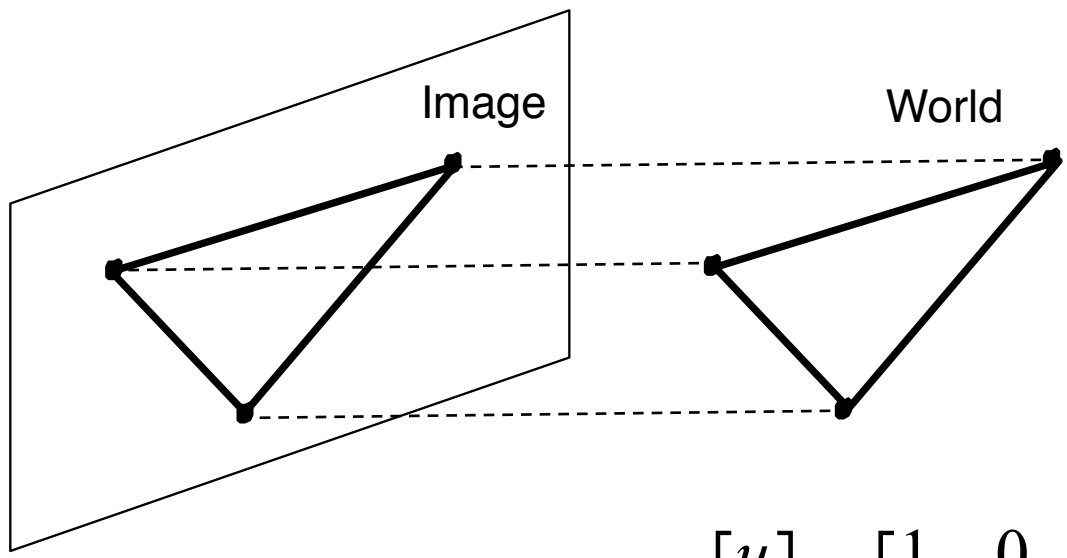
$$P' = K\begin{bmatrix} R & \bar{t} \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

5 6

Slide inspiration: S. Savarese

Orthographic Projection

- Special case of perspective projection
  - Distance from the COP to the image plane is infinite



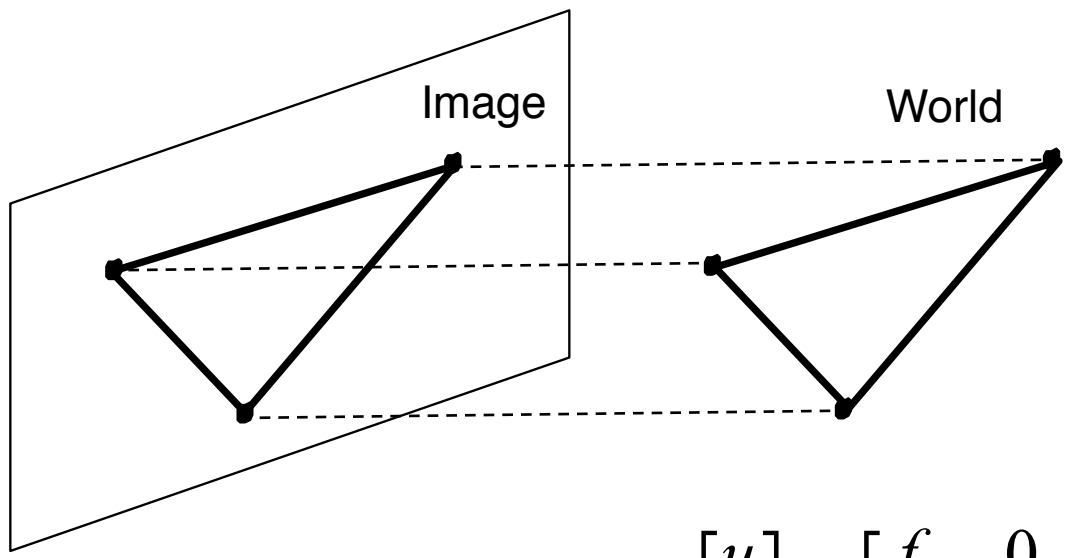
  - Also called "parallel projection"
  - What's the projection matrix?

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide credit: Steve Seitz

Scaled Orthographic Projection

- Special case of perspective projection
  - Object dimensions are small compared to distance to camera

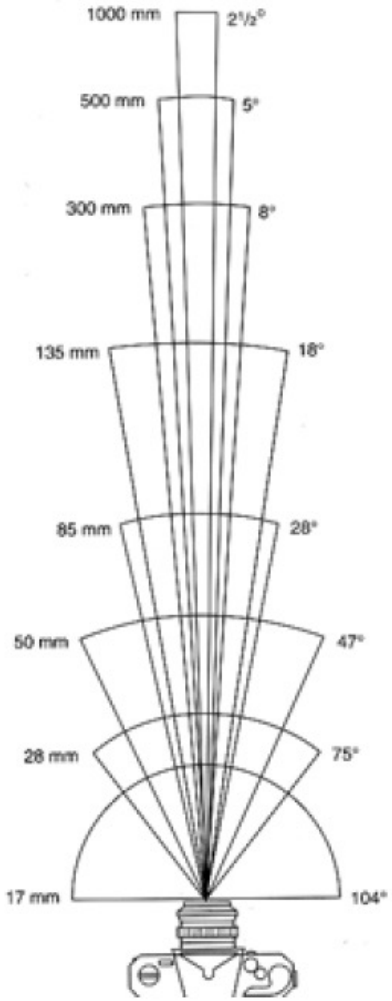


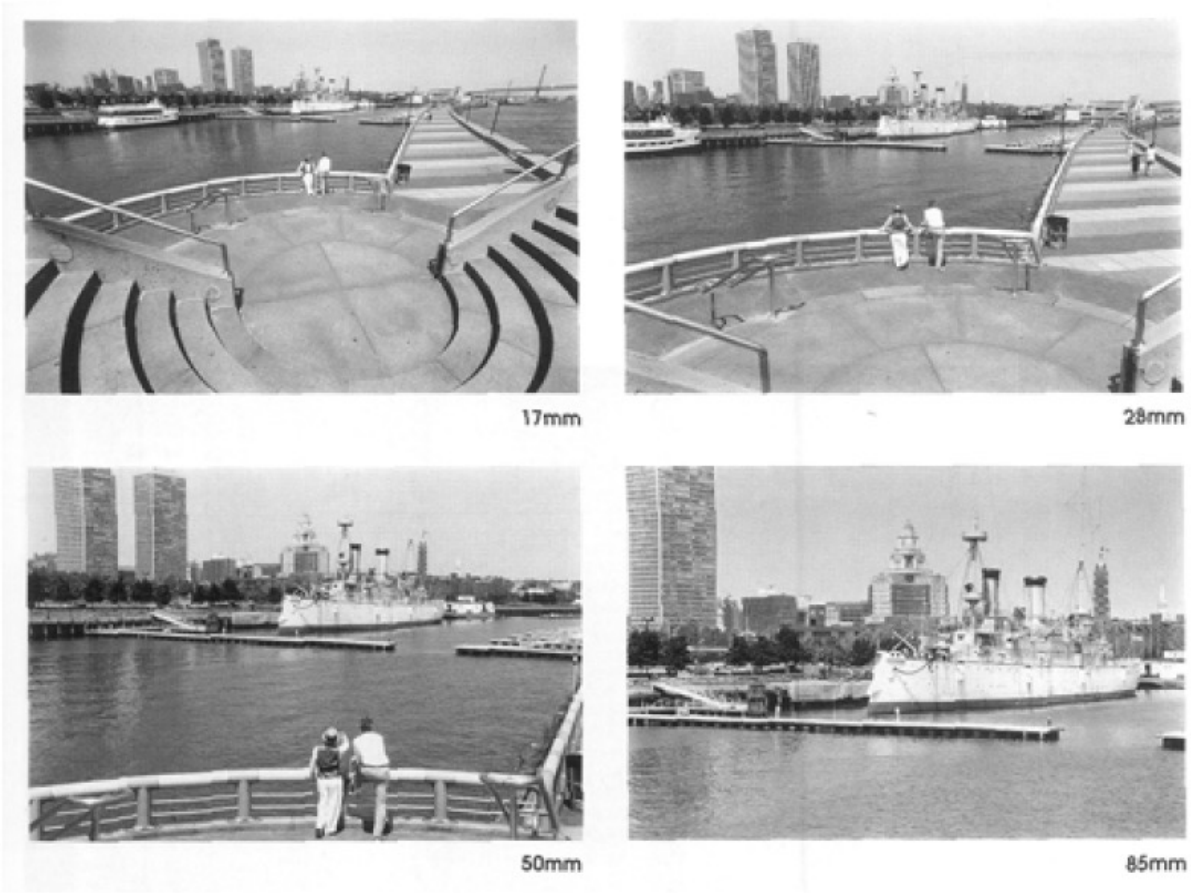
  - Also called "weak perspective"
  - What's the projection matrix?

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 0 & s \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide credit: Steve Seitz

Field of View (Zoom)

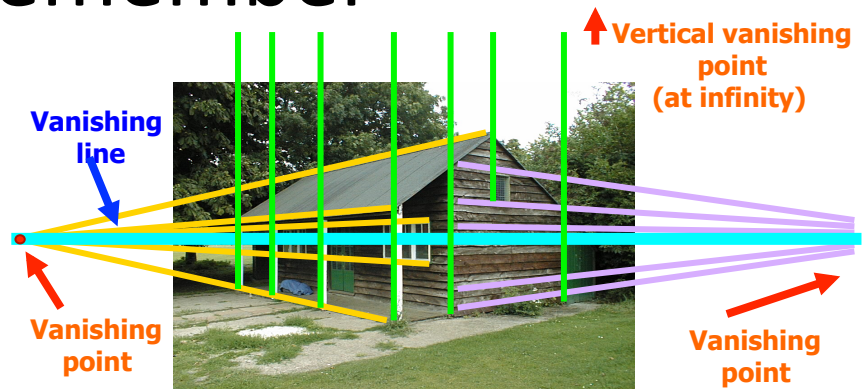




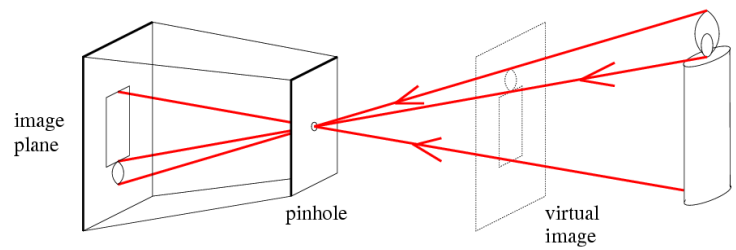
From London and Upton

Things to remember

- Vanishing points and vanishing lines



- Pinhole camera model and camera projection matrix M
  - Intrinsic parameters
  - Extrinsic parameters



$$P' = K\begin{bmatrix} R & \bar{t} \end{bmatrix} P$$

- Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Slide inspiration: J. Hayes

What we have learned today?

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
  - Projection matrix
  - Intrinsic parameters
  - Extrinsic parameters

Reading:

[FP] Chapters 1 – 3

[HZ] Chapter 6