Lecture 9 & 10: Stereo Vision

Professor Fei-Fei Li
Stanford Vision Lab

What we will learn today?

- Introduction to stereo vision
- Epipolar geometry: a gentle intro
- Parallel images
- Image rectification
- Solving the correspondence problem
- Active stereo vision system

Reading:
[HZ] Chapters: 4, 9, 11
[FP] Chapters: 10

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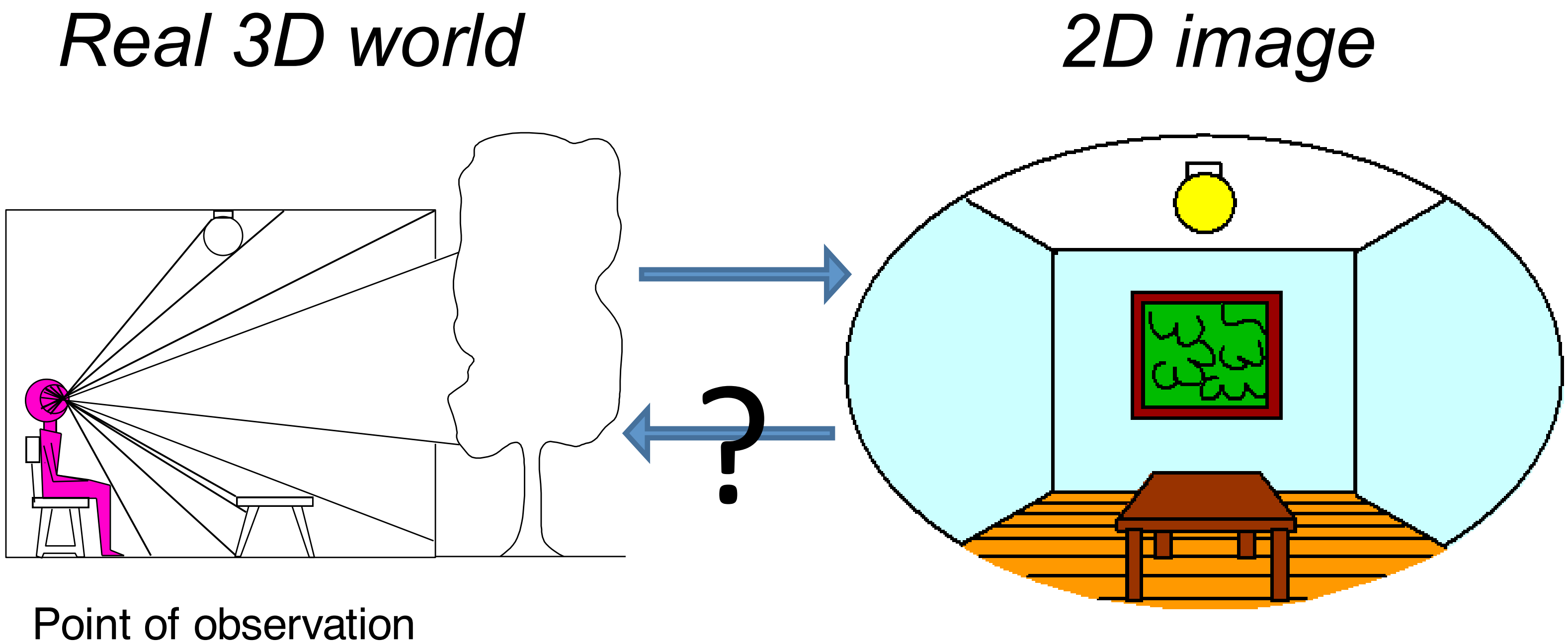
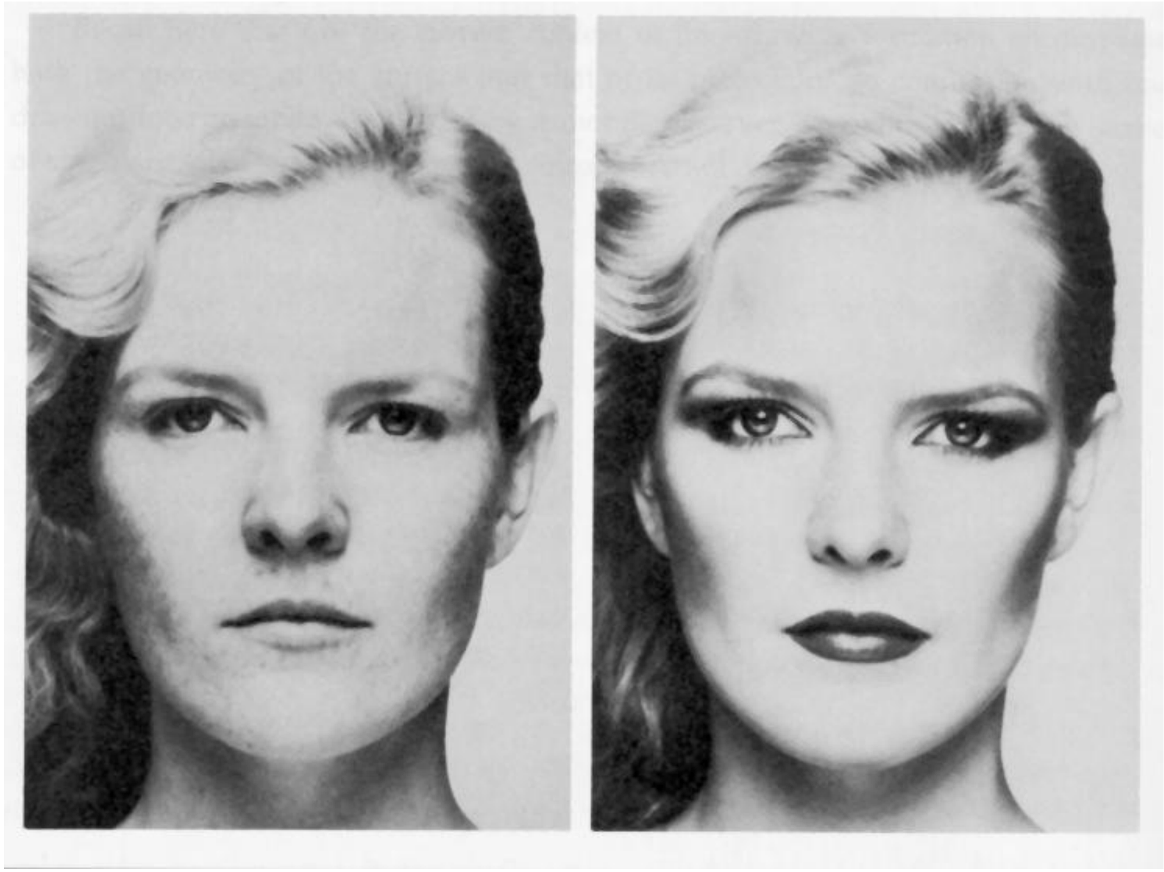
Recovering 3D from Images

- How can we automatically compute 3D geometry from images?
  – What cues in the image provide 3D information?



Visual Cues for 3D

- Shading

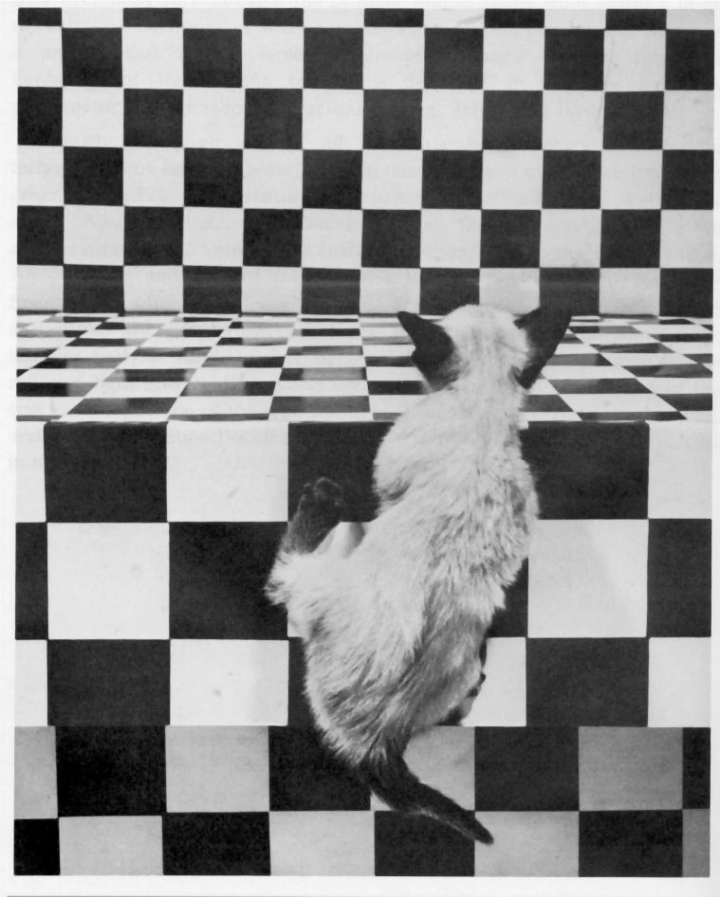


Merle Norman Cosmetics, Los Angeles

Slide credit: J. Hayes

Visual Cues for 3D

- Shading
- Texture



The Visual Cliff, by William Vandivert, 1960

Slide credit: J. Hayes

Visual Cues for 3D

- Shading
- Texture
- Focus


From *The Art of Photography*, Canon

Slide credit: J. Hayes

Visual Cues for 3D

- Shading
- Texture
- Focus
- Motion

Slide credit: J. Hayes

Visual Cues for 3D

- Shading
- Texture
- Focus
- Motion

- Others:
  - Highlights
  - Shadows
  - Silhouettes
  - Inter-reflections
  - Symmetry
  - Light Polarization
  - ...

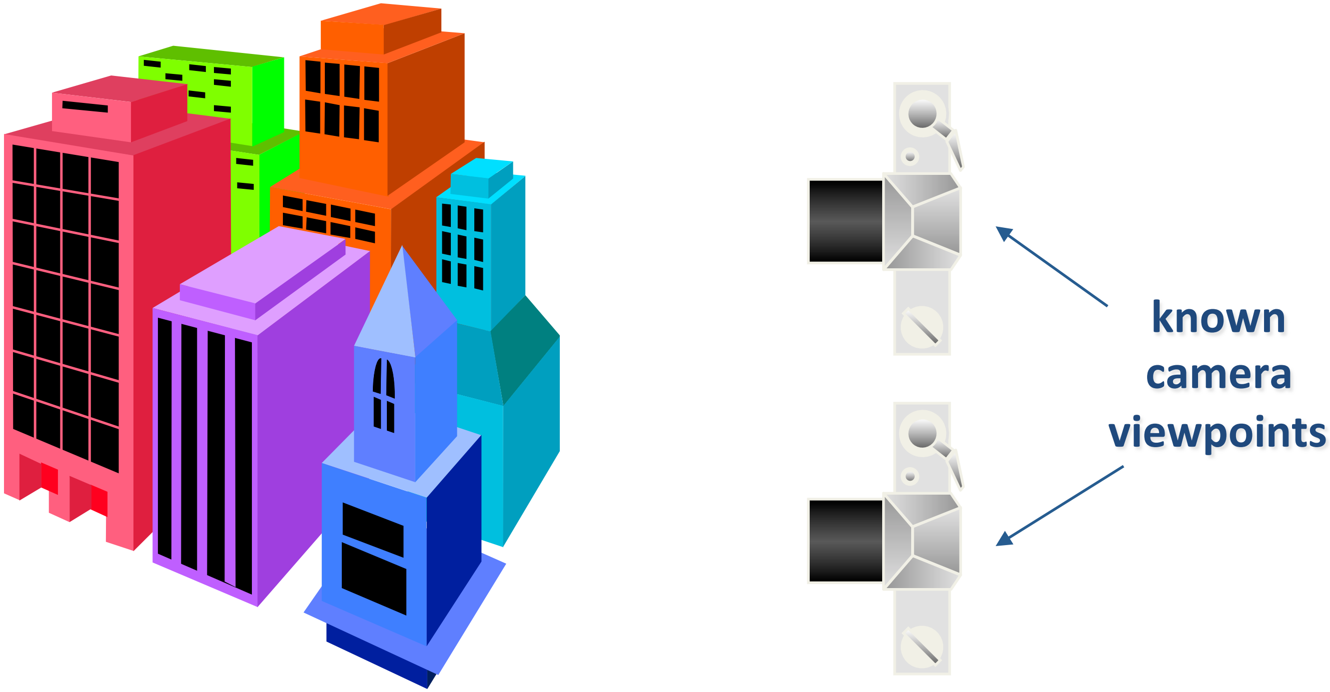
Shape From X

- X = shading, texture, focus, motion, ...
- We'll focus on the motion cue

Slide credit: J. Hayes

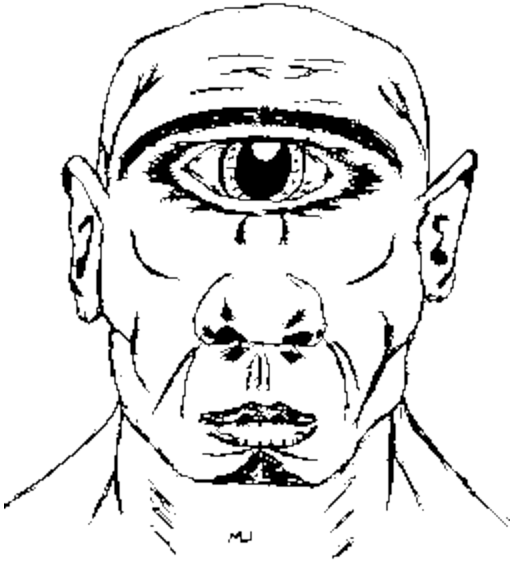
Stereo Reconstruction

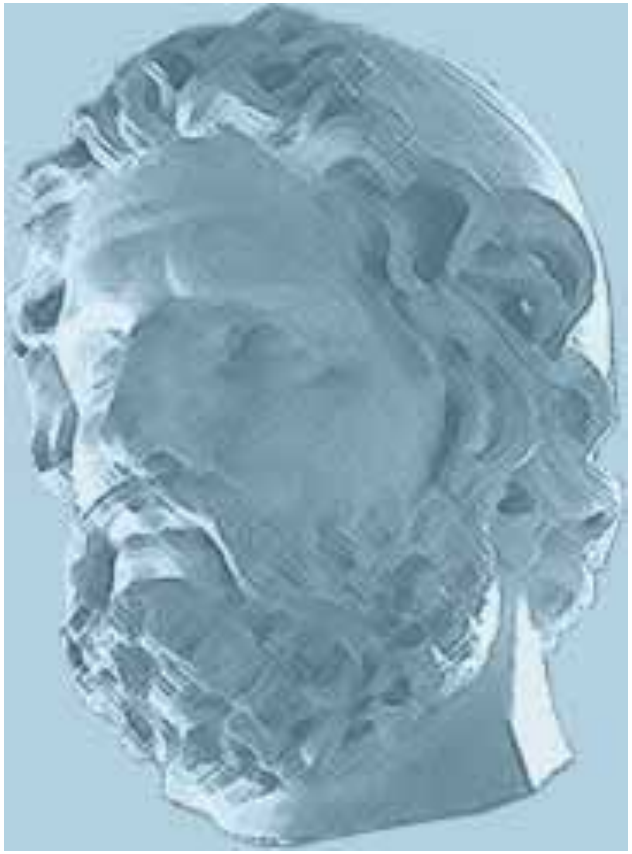
- The Stereo Problem
  - Shape from two (or more) images
  - Biological motivation



Slide credit: J. Hayes

Why do we have two eyes?

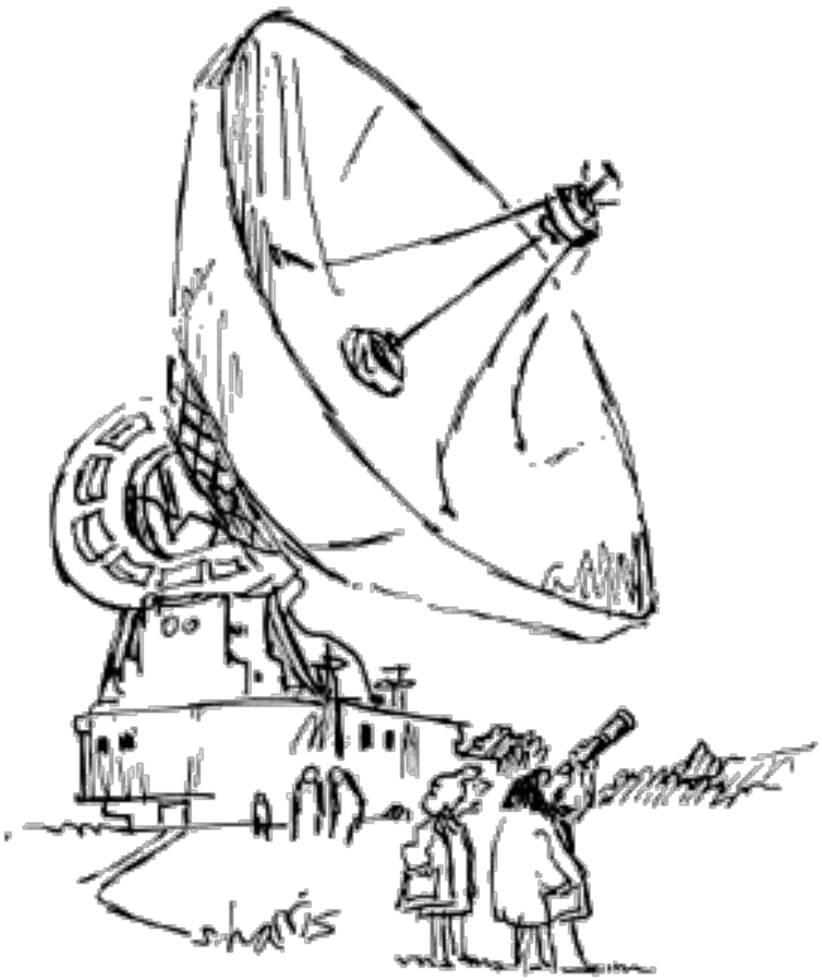




Cyclope **vs.** **Odysseus**

Slide credit: J. Hayes

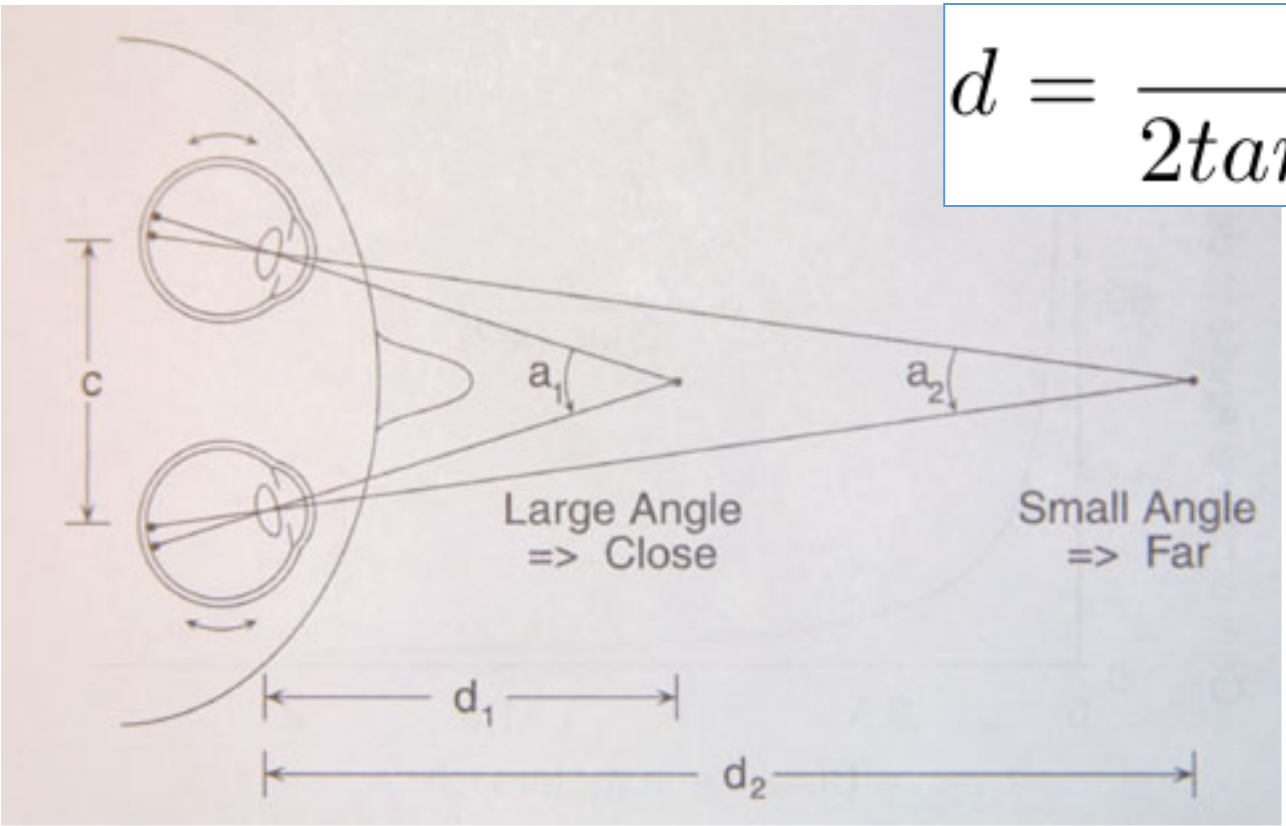
1. Two is better than one



"Just checking."

Slide credit: J. Hayes

2. Depth from Convergence

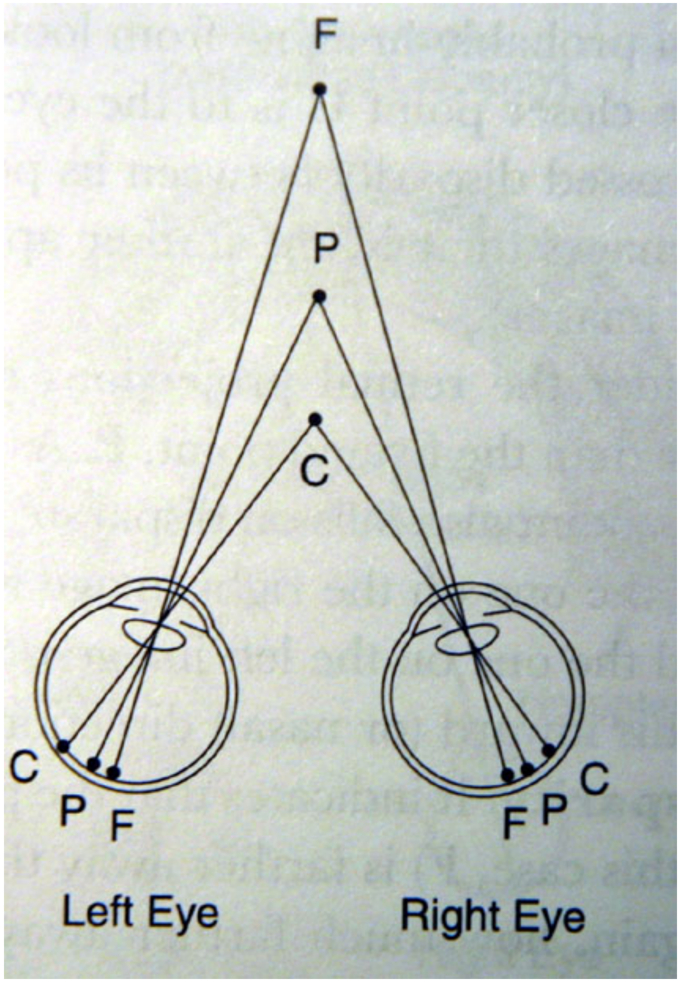


$$d = \frac{c}{2tan(a/2)}$$

Human performance: up to 6-8 feet

Slide credit: J. Hayes

3. Depth from binocular disparity



Sign and magnitude of disparity

P: converging point

C: object nearer projects to the outside of the P, disparity = +

F: object farther projects to the inside of the P, disparity = -

Slide credit: J. Hayes

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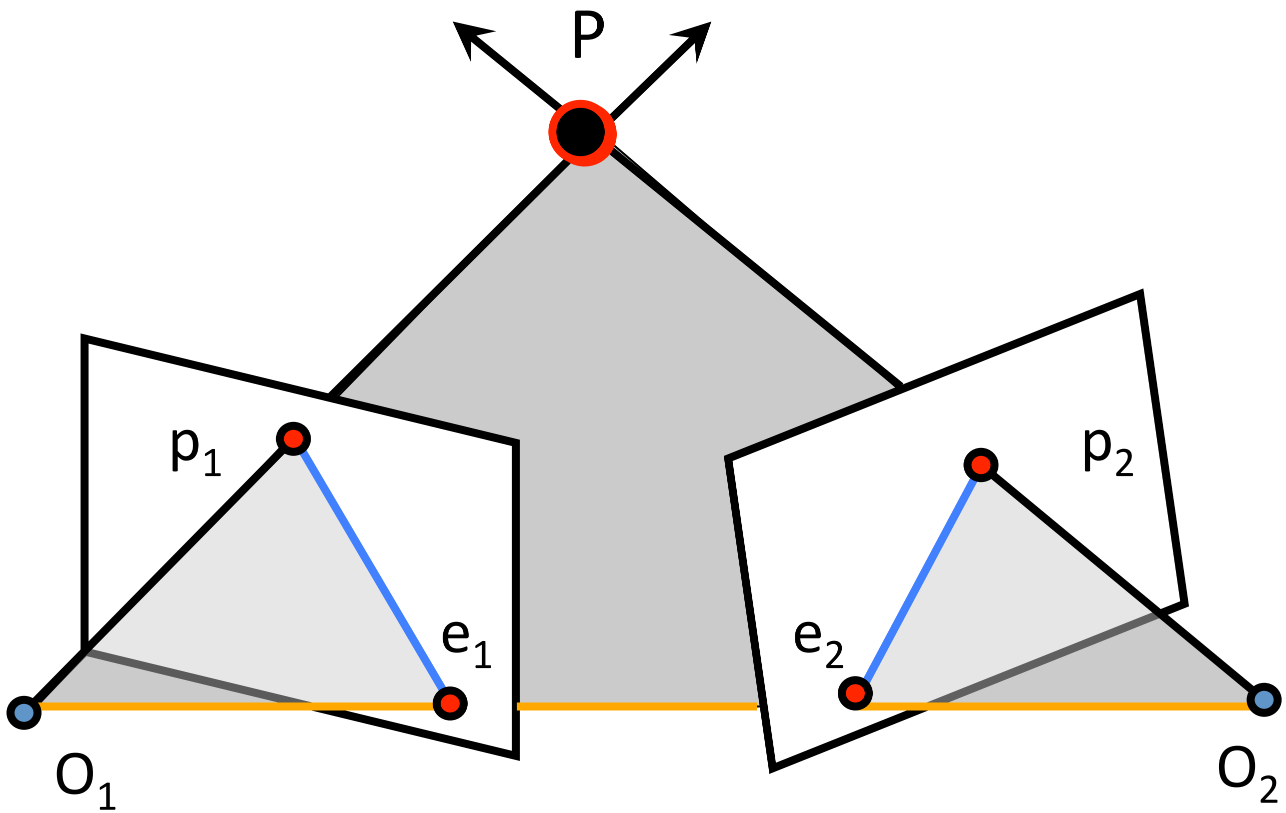
Epipolar geometry

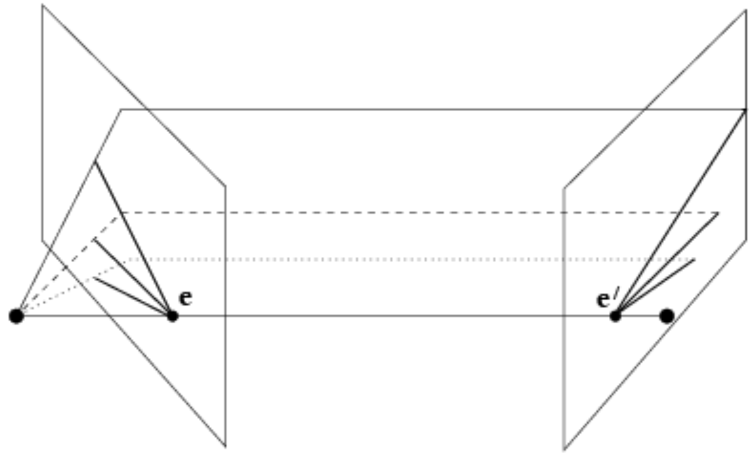


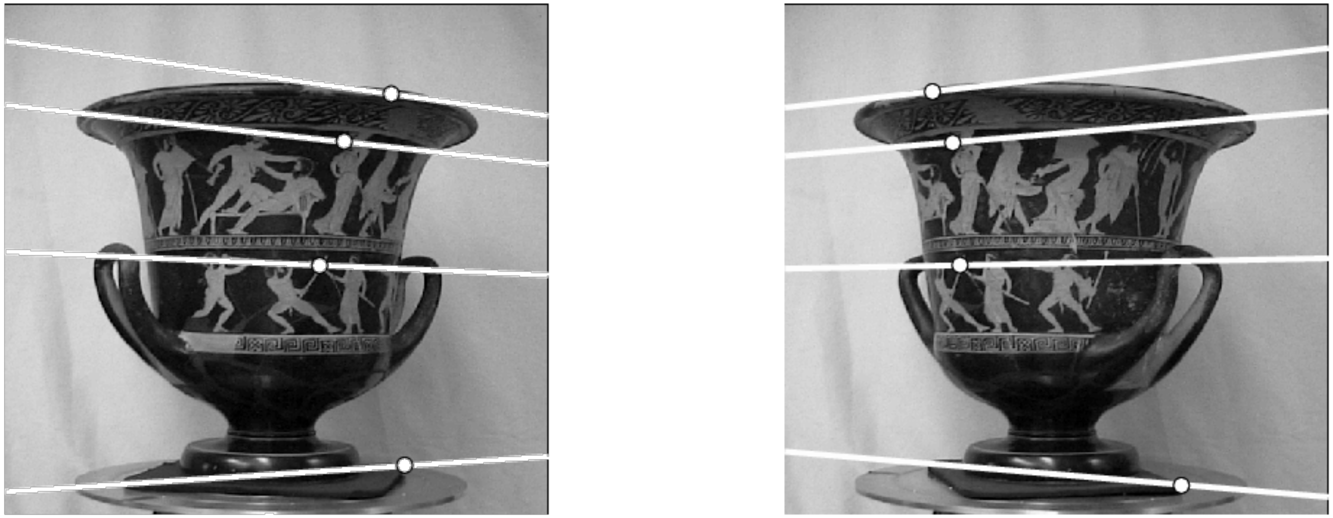
- Epipolar Plane
- Baseline
- Epipolar Lines

- Epipoles e_1, e_2

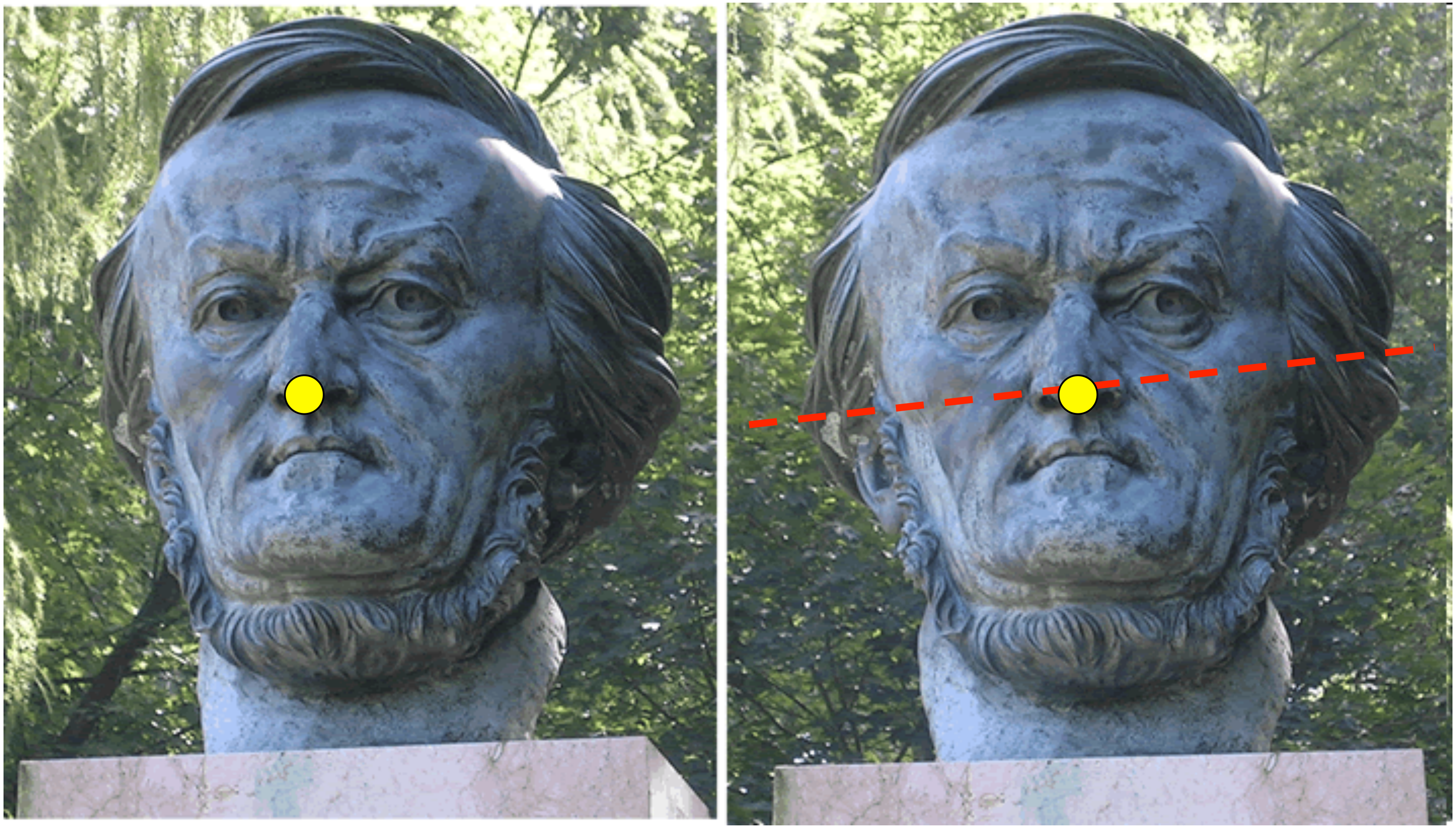
  = intersections of baseline with image planes
  = projections of the other camera center
  = vanishing points of camera motion direction

Example: Converging image planes



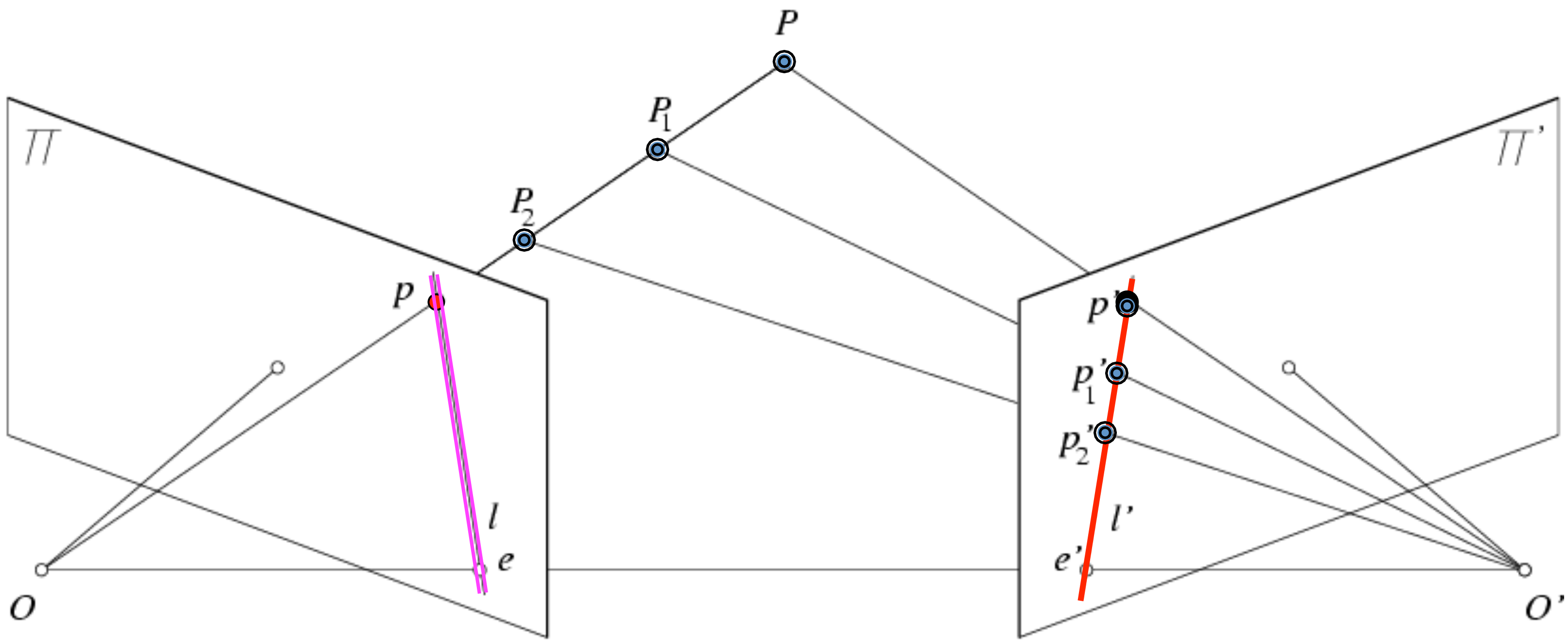


Epipolar Constraint



- Two views of the same object
- Suppose I know the camera positions and camera matrices
- Given a point on left image, how can I find the corresponding point on right image?

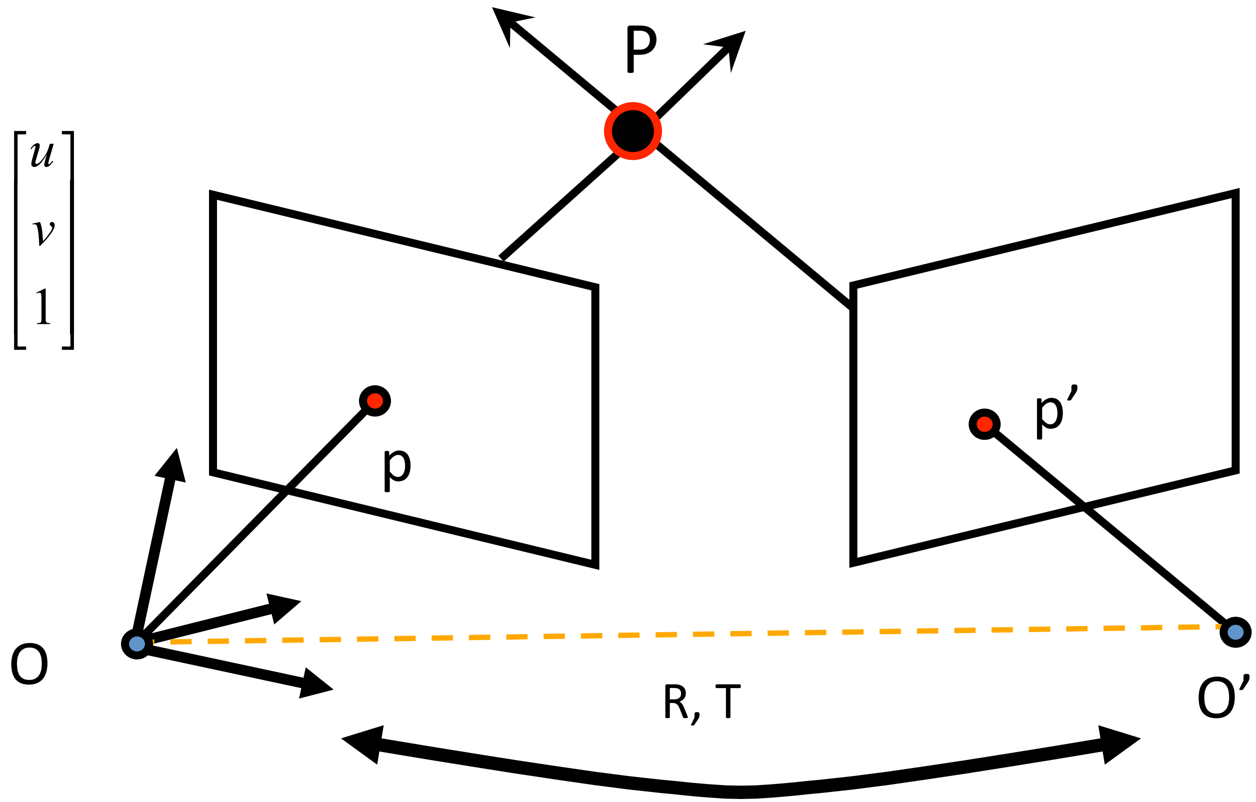
Epipolar Constraint



- Potential matches for *p* have to lie on the corresponding epipolar line *l'*.
- Potential matches for *p'* have to lie on the corresponding epipolar line *l*.

Epipolar Constraint

$$p \rightarrow M\ P = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

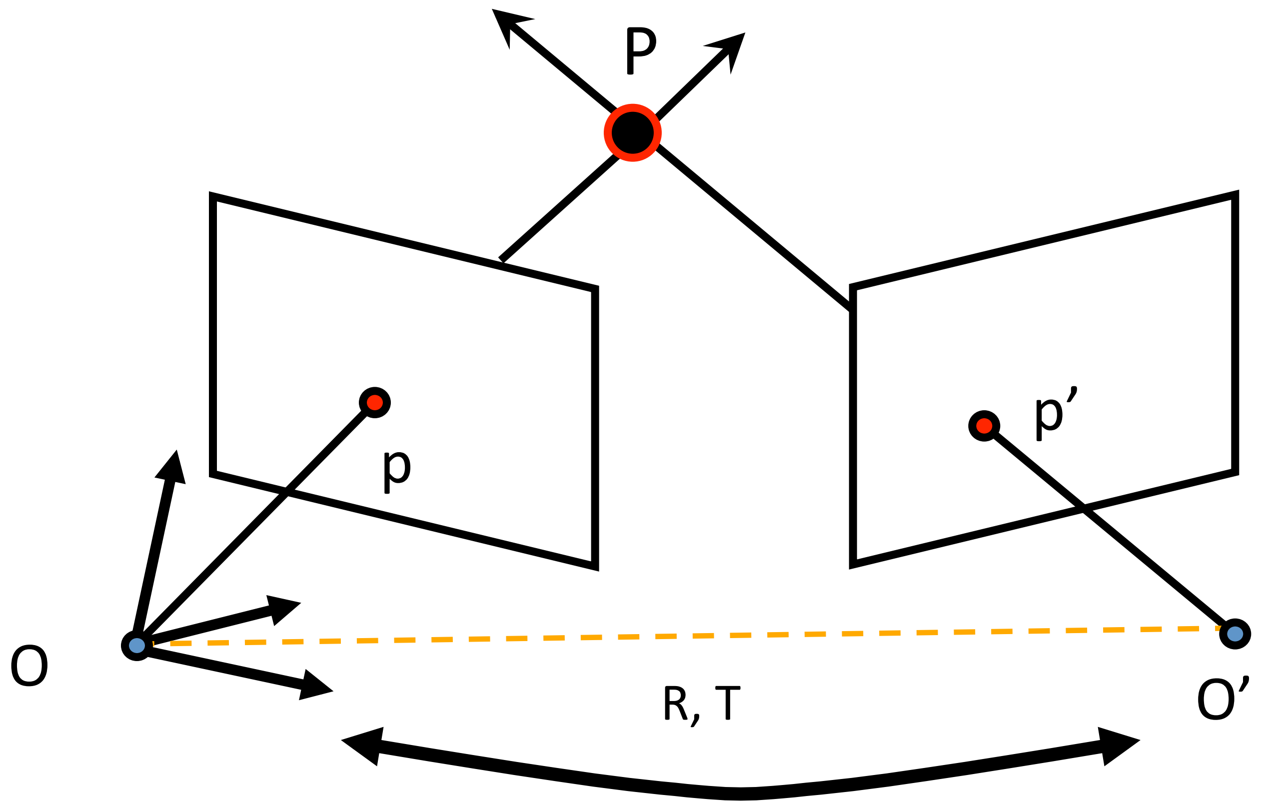


$$p \rightarrow M'\ P = \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix}$$

$$M = K_1\begin{bmatrix} I & 0 \end{bmatrix}$$

$$M' = K_2\begin{bmatrix} R & T \end{bmatrix}$$

Epipolar Constraint



$$M = K_1 \begin{bmatrix} I & 0 \end{bmatrix}$$

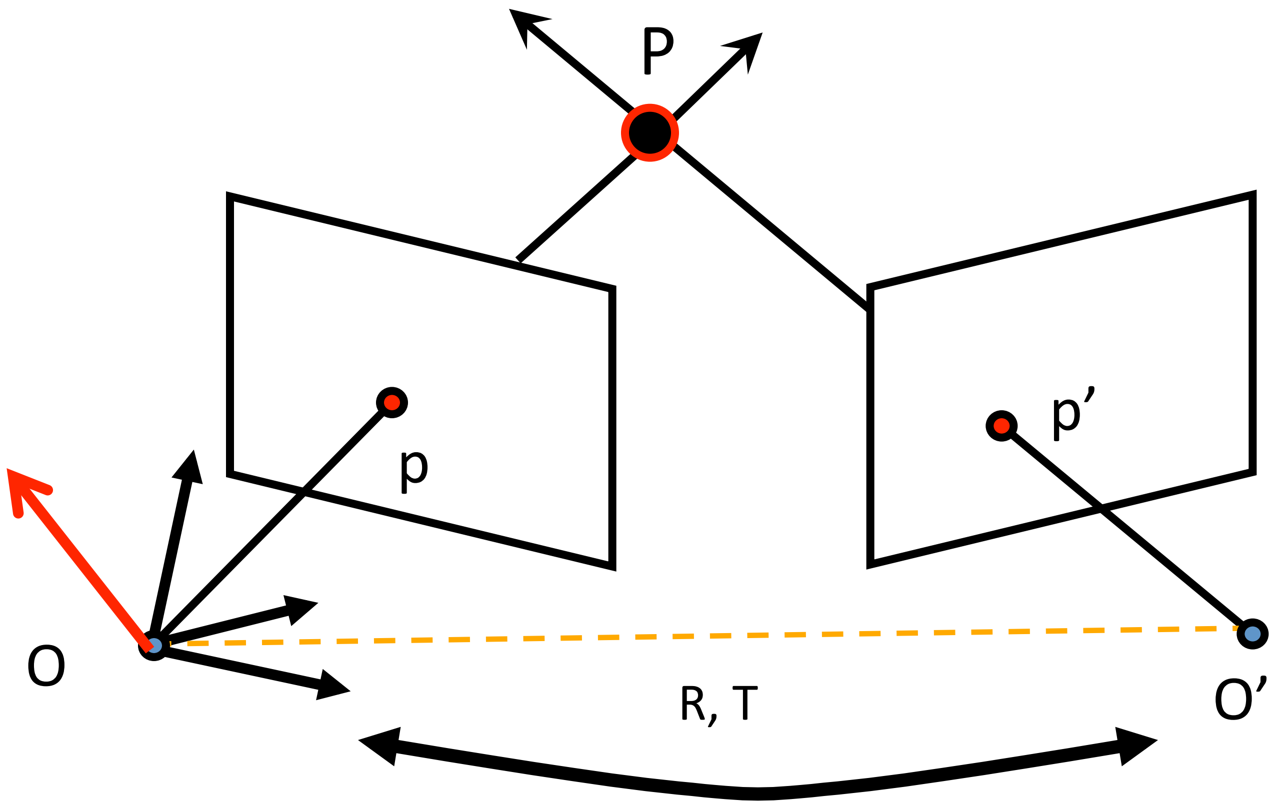
K₁ and K₂ are known (calibrated cameras)

$$M' = K_2 \begin{bmatrix} R & T \end{bmatrix}$$

$$M = \begin{bmatrix} I & 0 \end{bmatrix}$$

$$M' = \begin{bmatrix} R & T \end{bmatrix}$$

Epipolar Constraint



$$T \times (R\, p')$$

Perpendicular to epipolar plane

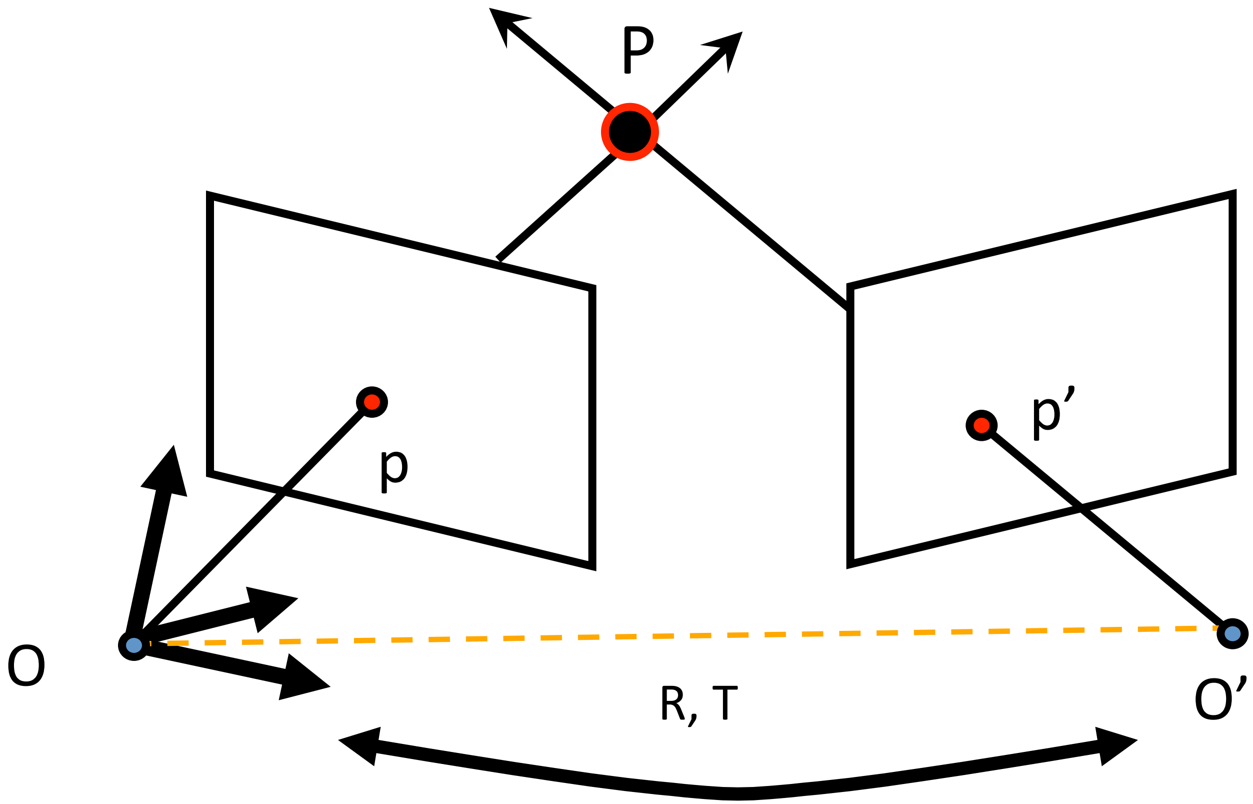
$$p^T \cdot \left[T \times (R\, p') \right] = 0$$

Cross product as matrix multiplication

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [\mathbf{a}_\times]\mathbf{b}$$

"skew symmetric matrix"

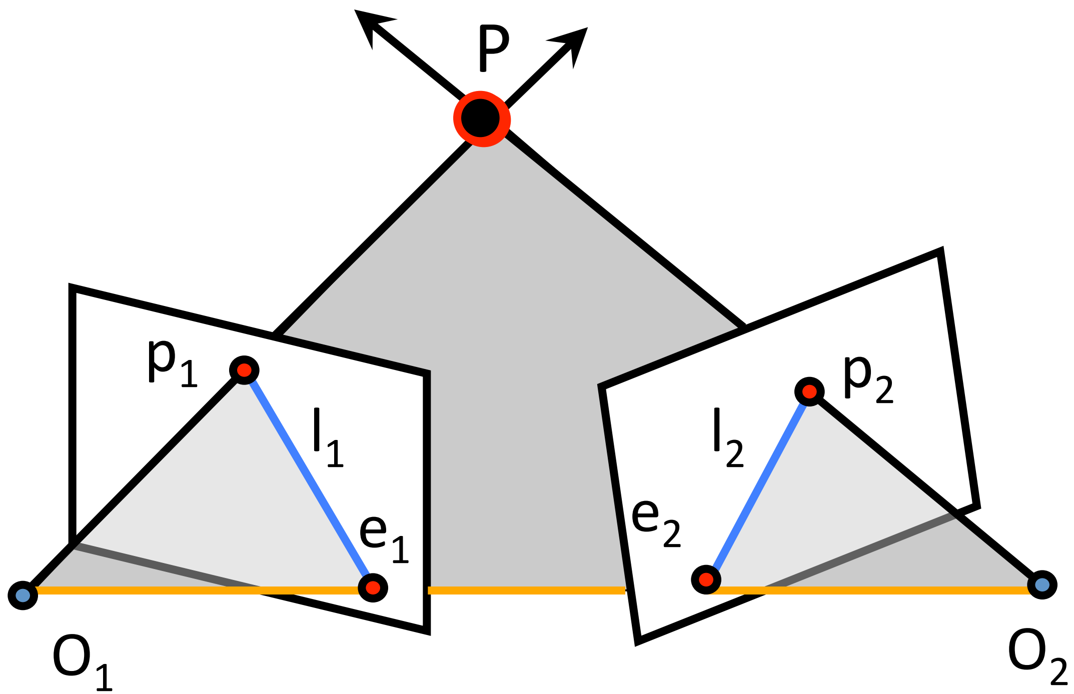
Epipolar Constraint



$$p^T \cdot \left[T \times (R\, p') \right] = 0 \rightarrow p^T \cdot \left[T_\times \right] \cdot R\, p' = 0$$

(Longuet-Higgins, 1981) E = essential matrix

Epipolar Constraint



- $E\,p_2$ is the epipolar line associated with p_2 ($l_1 = E\,p_2$)
- $E^T p_1$ is the epipolar line associated with p_1 ($l_2 = E^T p_1$)
- E is singular (rank two)
- $E\,e_2 = 0$ and $E^T e_1 = 0$
- E is 3x3 matrix; 5 DOF

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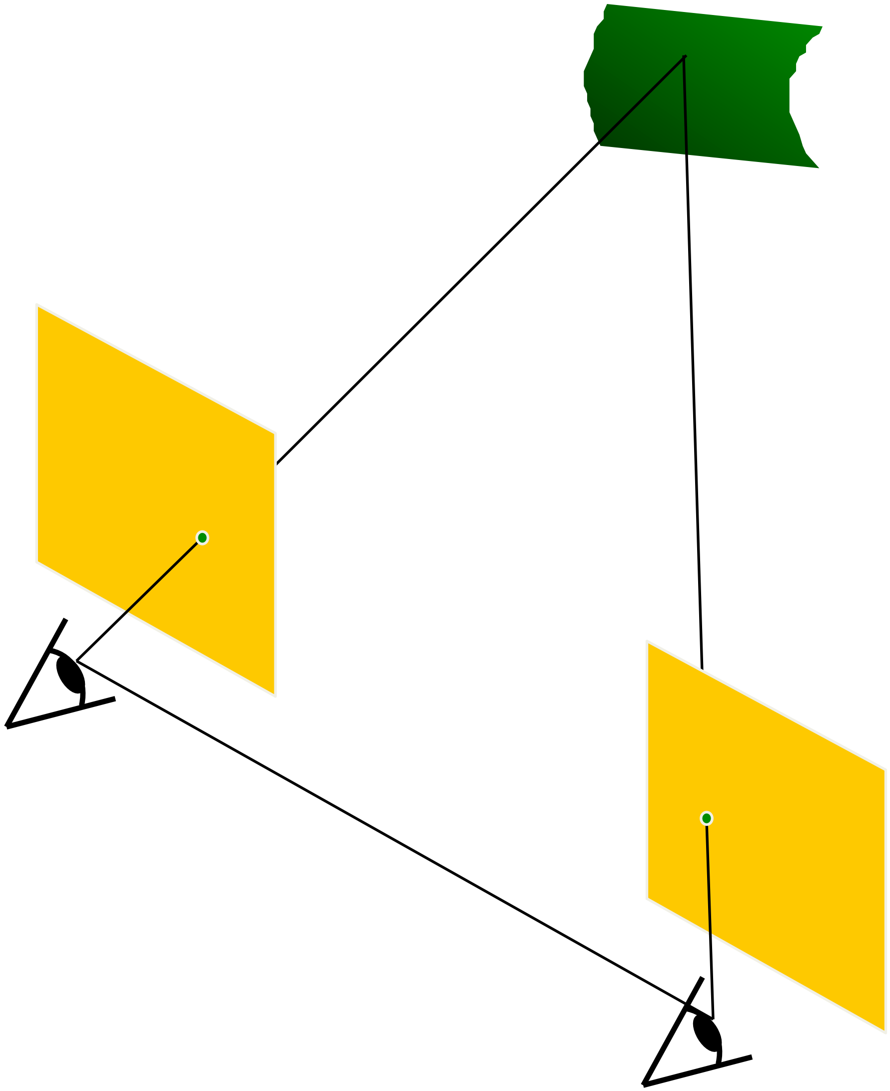
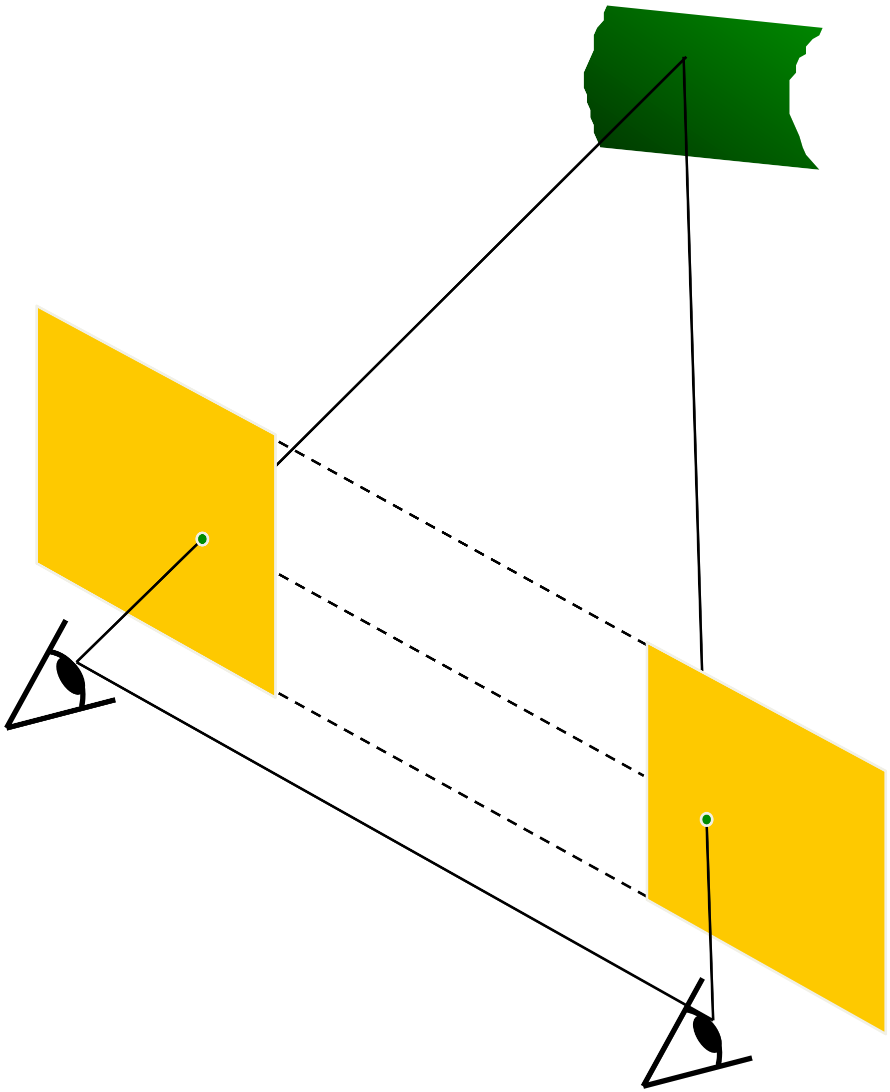
Simplest Case: Parallel images



- Image planes of cameras are parallel to each other and to the baseline
- Camera centers are at same height
- Focal lengths are the same

Slide credit: J. Hayes

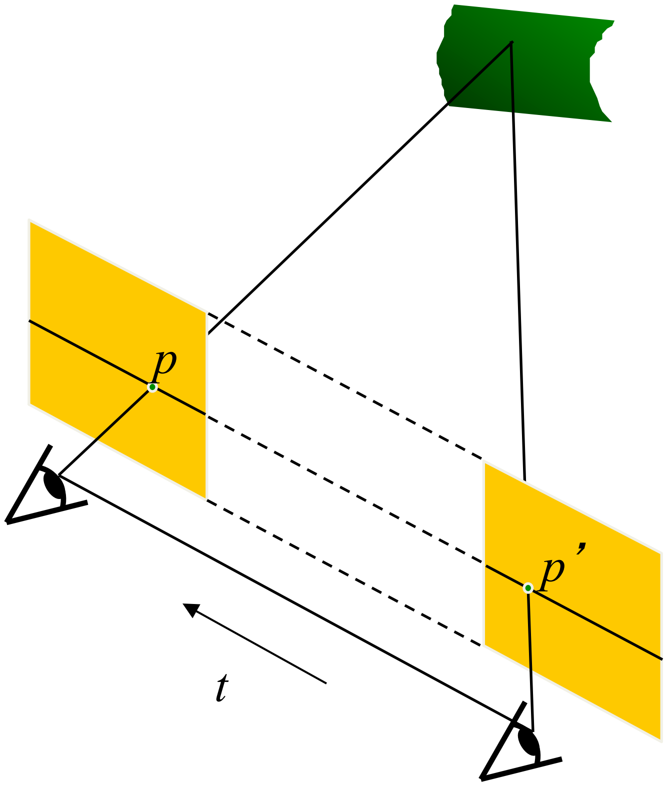
Simplest Case: Parallel images



- Image planes of cameras are parallel to each other and to the baseline
- Camera centers are at same height
- Focal lengths are the same
- Then, epipolar lines fall along the horizontal scan lines of the images

Slide credit: J. Hayes

Essential matrix for parallel images



Epipolar constraint:

$$R = I \qquad t = (T, 0, 0)$$

$$p^T E p' = 0, \quad E = [t_\times]R$$

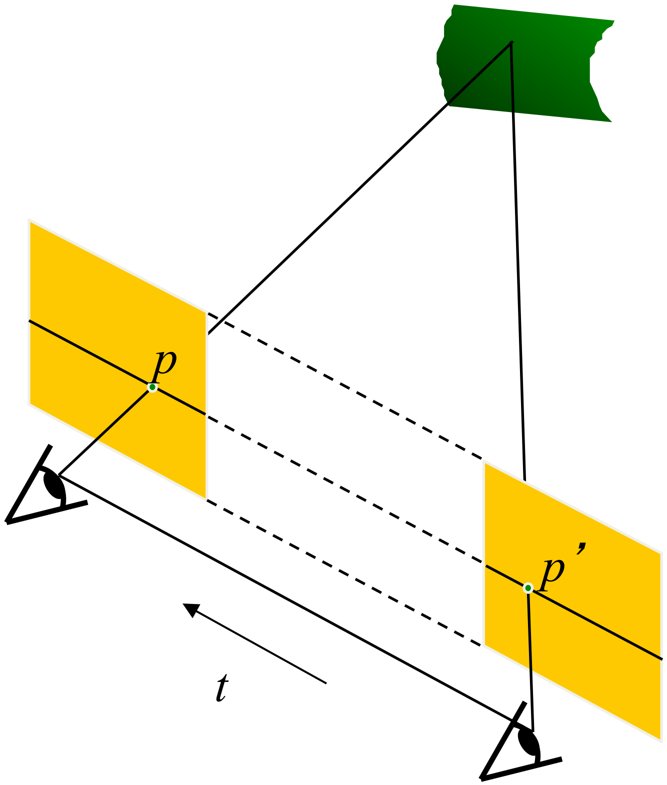
$$E = [t_\times]R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

Reminder: skew symmetric matrix

$$[a_\times] = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

Slide credit: J. Hayes

Essential matrix for parallel images



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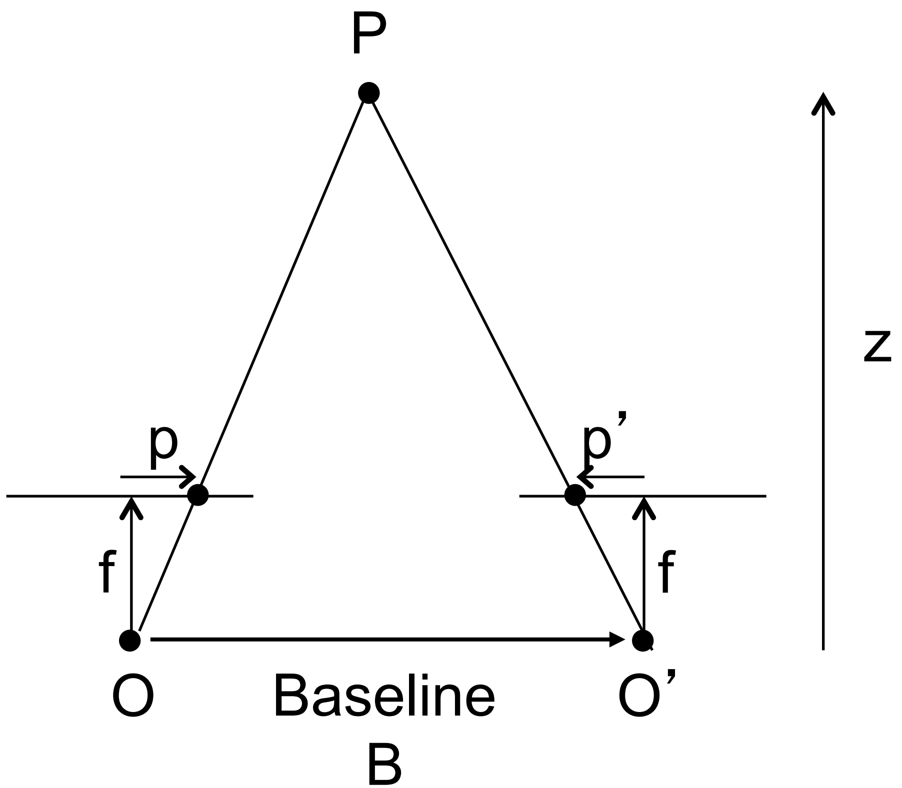
$$\begin{pmatrix} u & v & 1 \end{pmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \qquad \begin{pmatrix} u & v & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -T \\ Tv' \end{pmatrix} = 0 \qquad Tv = Tv'$$

The y-coordinates of corresponding points are the same!

Slide credit: J. Hayes

Triangulation -- depth from disparity

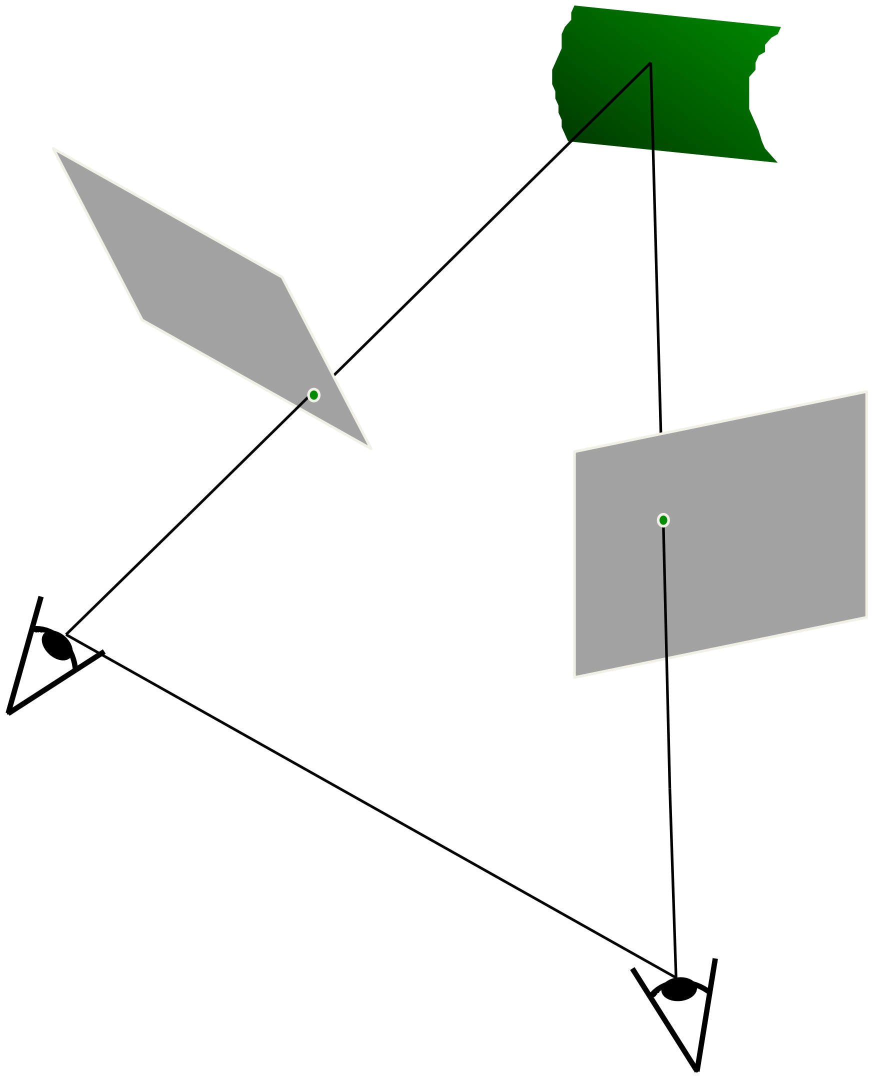


$$disparity = u - u' = \frac{B \cdot f}{z}$$

Disparity is inversely proportional to depth!

Slide credit: J. Hayes

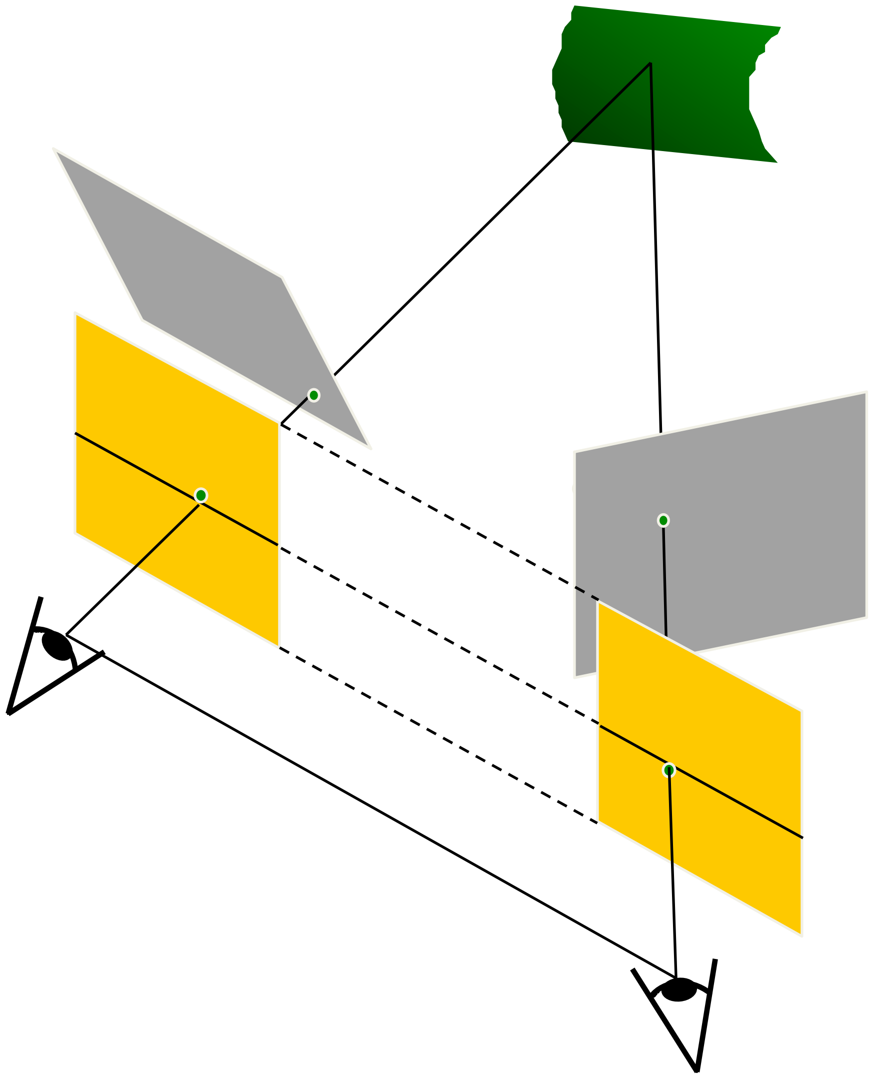
Stereo image rectification



Slide credit: J. Hayes

Stereo image rectification

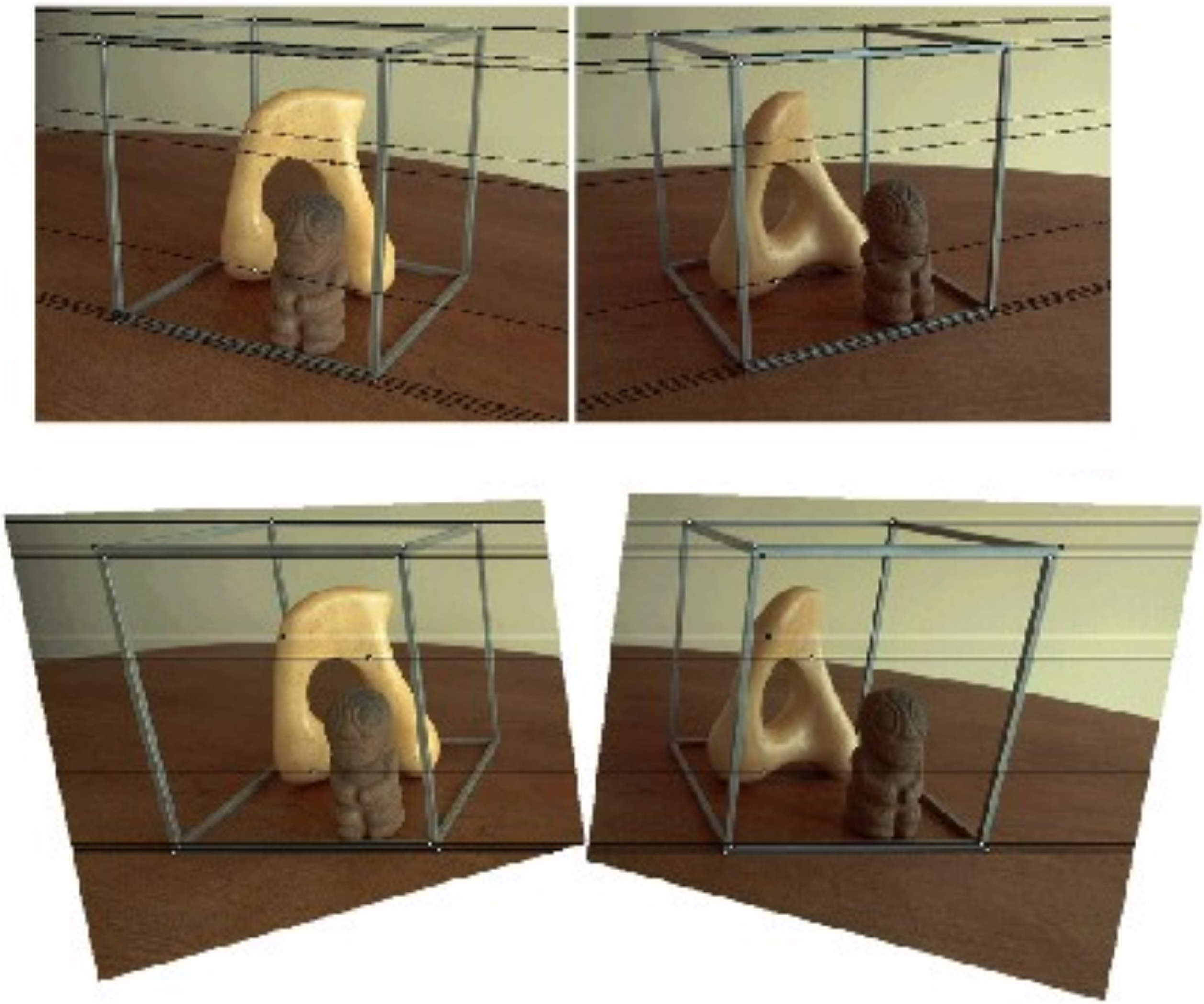
Algorithm:
- Re-project image planes onto a common plane parallel to the line between optical centers
- Pixel motion is horizontal after this transformation
- Two transformation matrices, one for each input image reprojection



➢ C. Loop and Z. Zhang. Computing Rectifying Homographies for Stereo Vision. IEEE Conf. Computer Vision and Pattern Recognition, 1999.

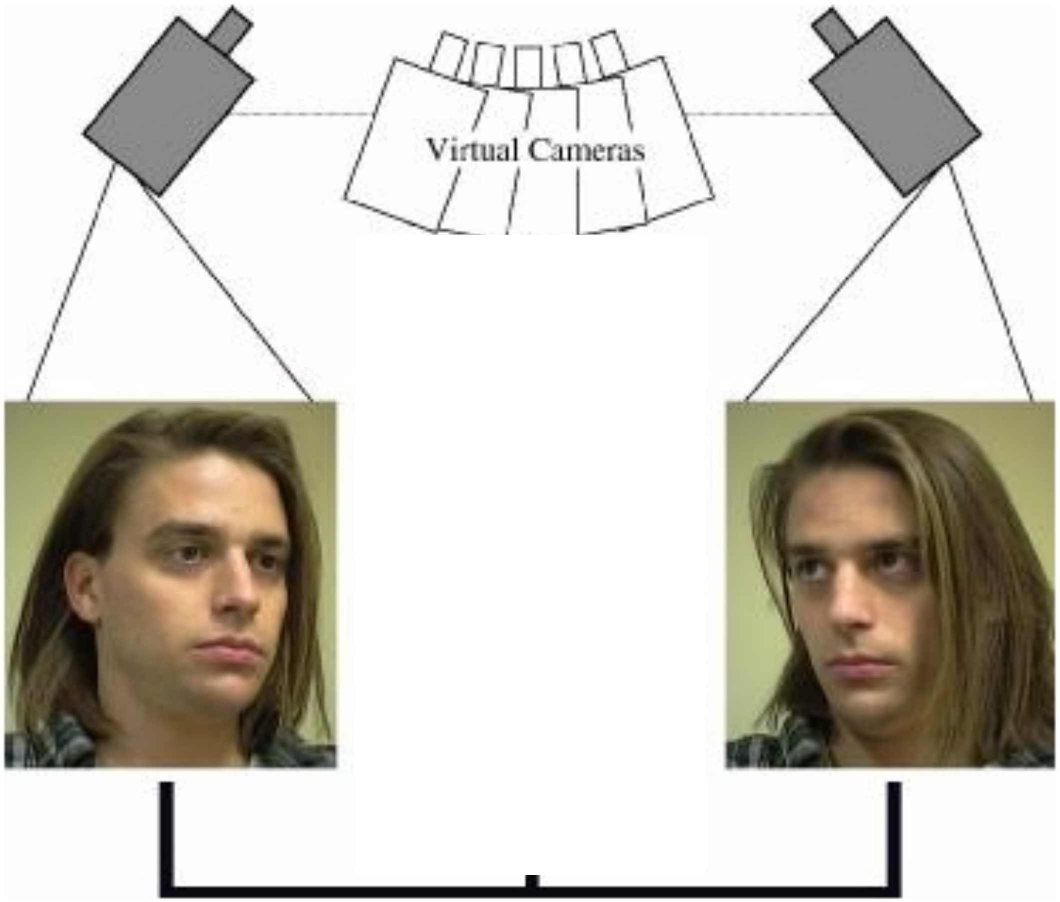
Slide credit: J. Hayes

Rectification example

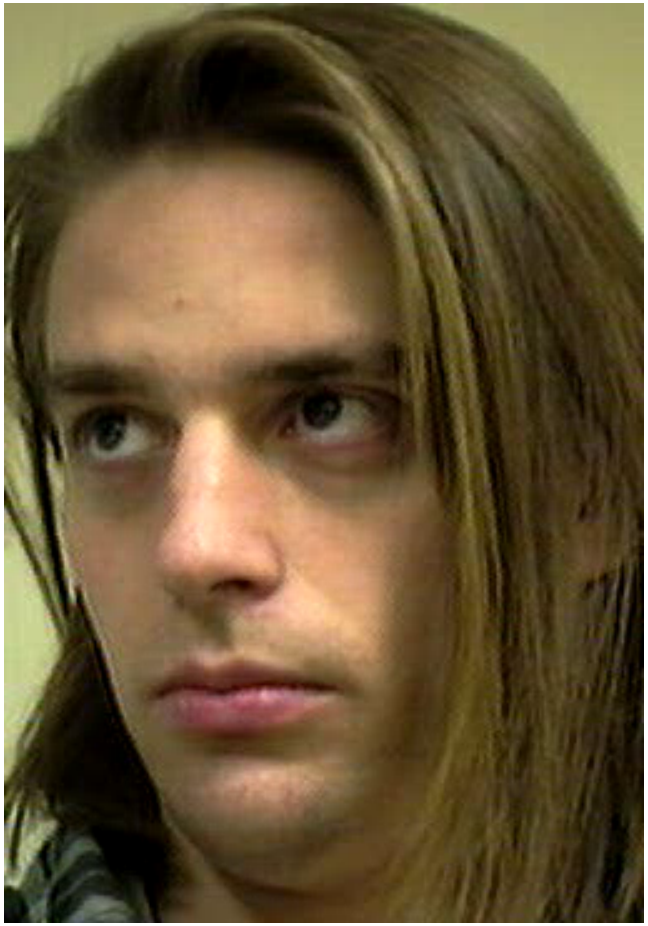


Application: view morphing

S. M. Seitz and C. R. Dyer, *Proc. SIGGRAPH 96*, 1996, 21-30



Application: view morphing



Application: view morphing

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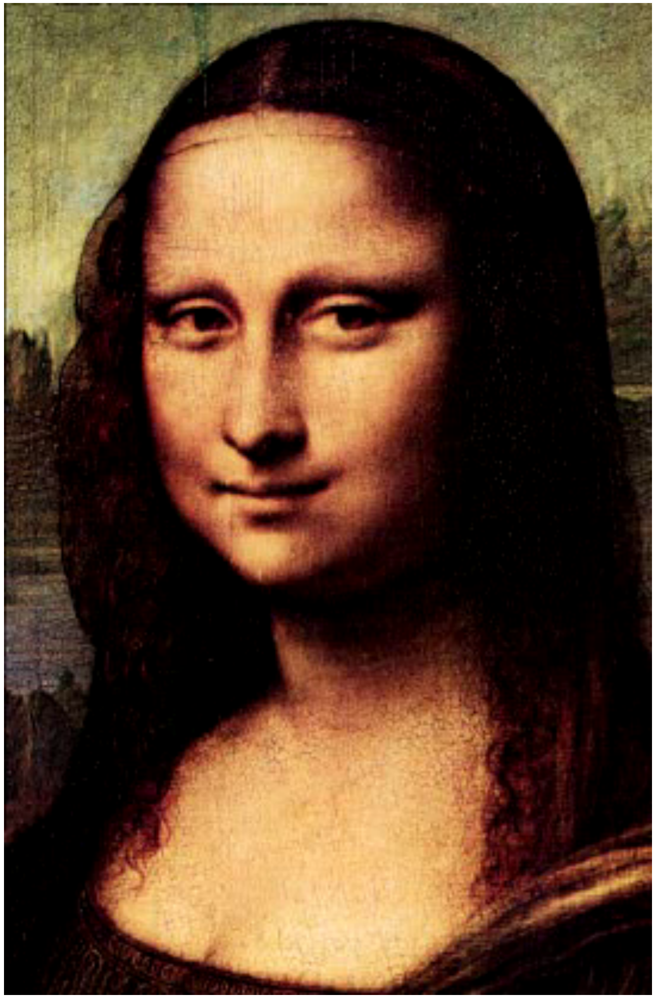
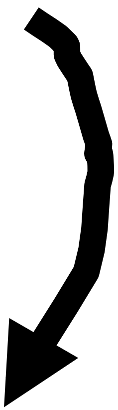


Removing perspective distortion

(rectification)



H_p

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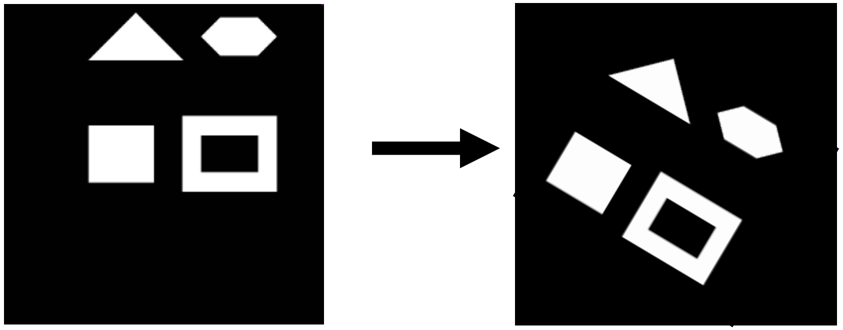
Reminder: transformations in 2D

Special case from lecture 2

(planar rotation & translation)

$$\begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = H_e \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

- 3 DOF
- Preserve distance (areas)
- Regulate motion of rigid object

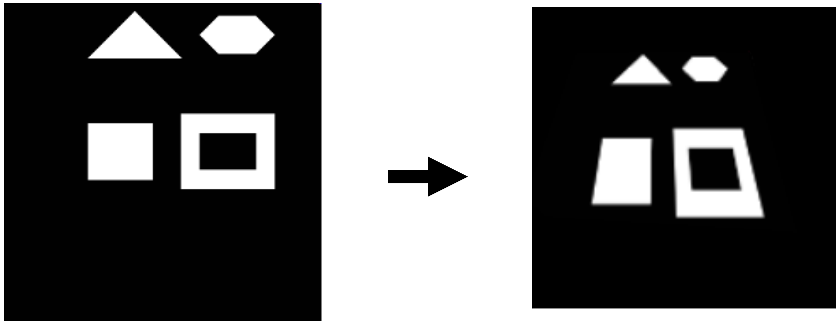


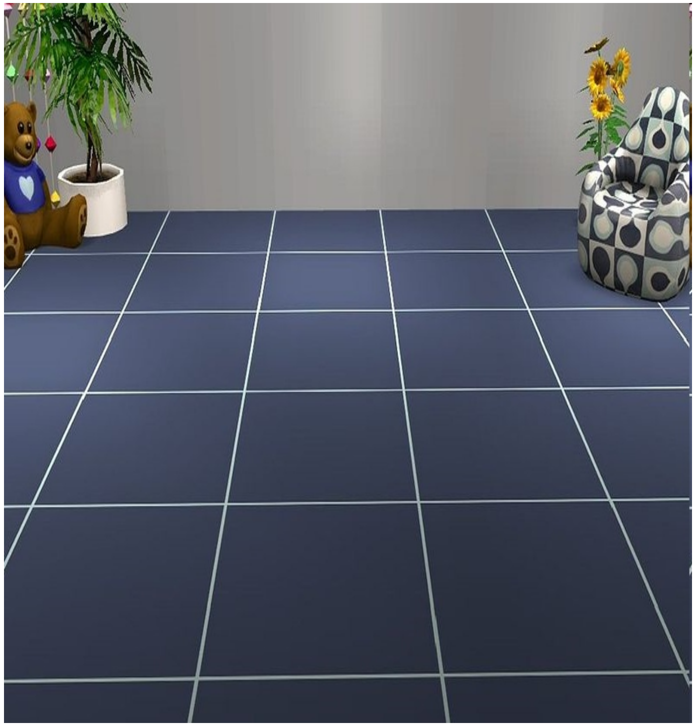
Reminder: transformations in 2D

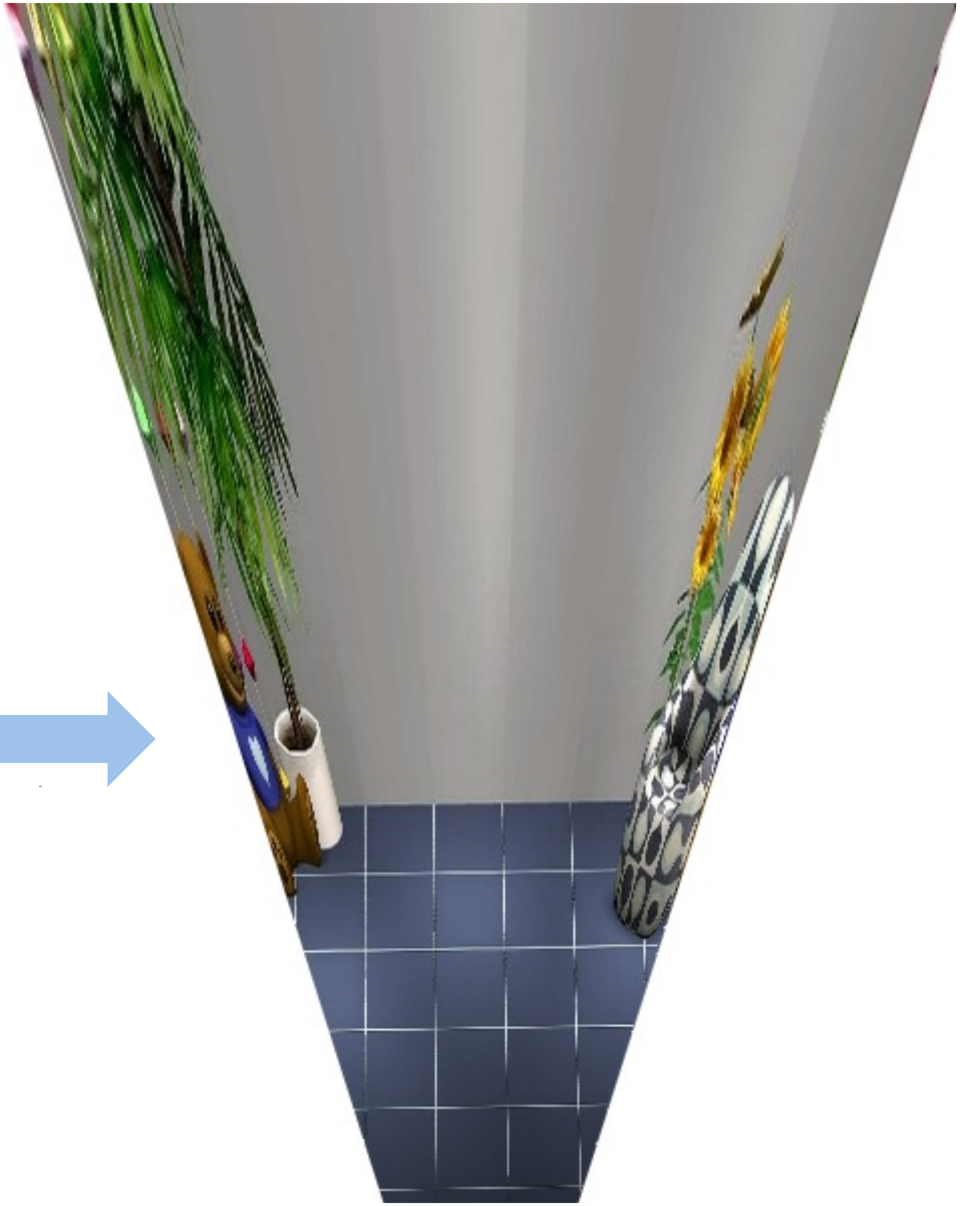
Generic case
(rotation in 3D, scale & translation)

$$\begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = H \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

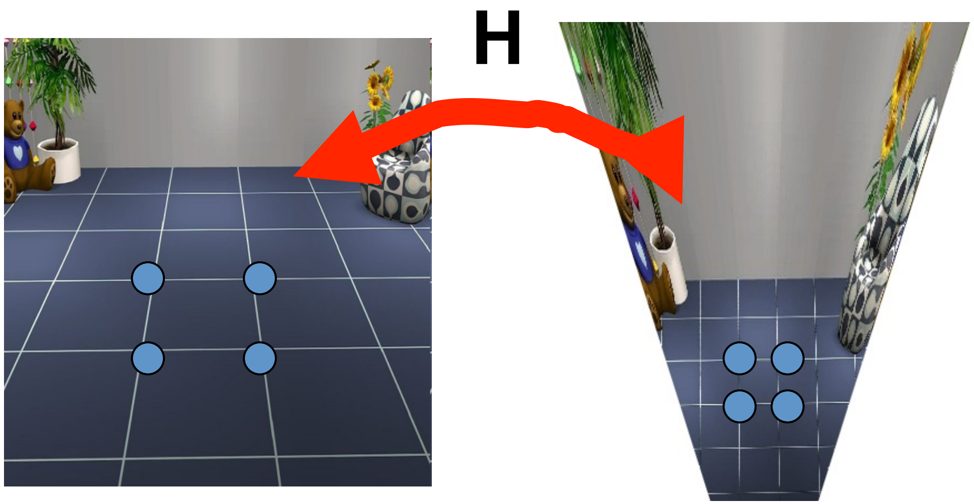
- 8 DOF
- Preserve colinearity







Computing H

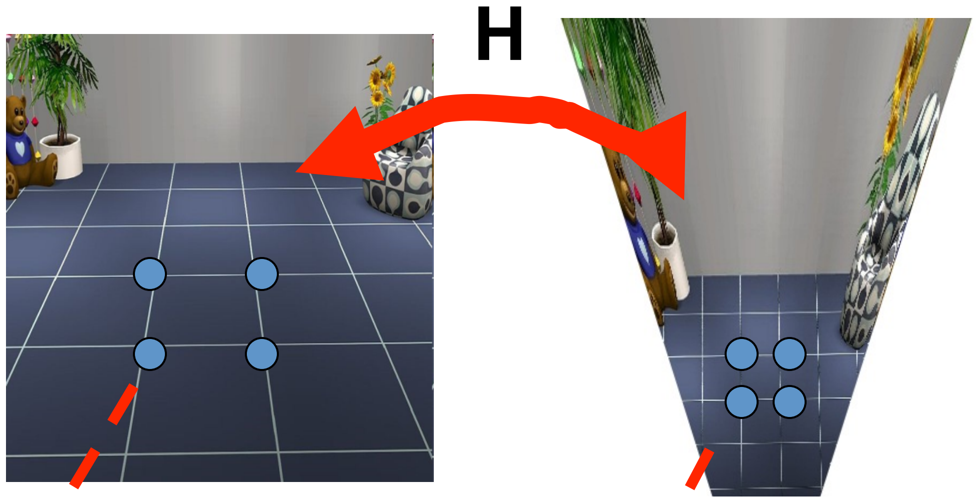


- 8 DOF
- how many points do I need to estimate H?

At least 4 points! (8 equations)

- There are several algorithms…

DLT algorithm (Direct Linear Transformation)



p_i

p_i'

$$p_i' = H\ p_i$$

DLT algorithm (direct Linear Transformation)

$$p_i' \times H\, p_i = 0 \longrightarrow \underbrace{\mathrm{A_i}}_{\text{Function of measurements [2x9]}} \overbrace{\mathbf{h}}^{\text{Unknown [9x1]}} = 0$$

Unknown [9x1]

Function of measurements [2x9]

$$H = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \rightarrow \mathbf{h} = \underbrace{\begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_9 \end{bmatrix}}_{9\text{x}1}$$

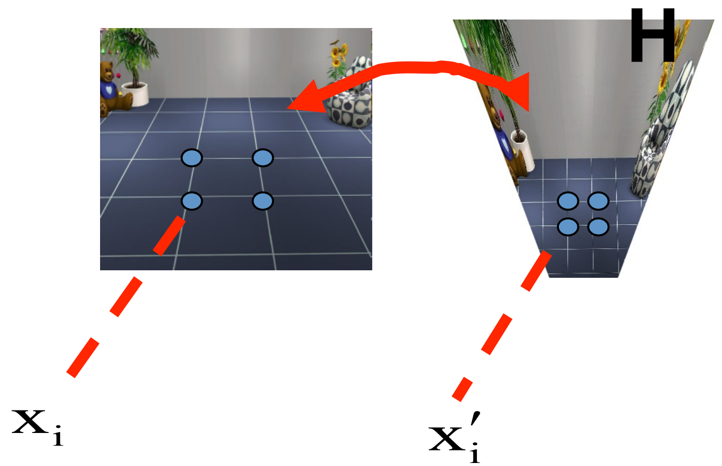
9x1

2 independent equations

DLT algorithm (direct Linear Transformation)

$$A_{2\times 9} \quad h_{9\times 1}$$

$$A_i \, h = 0$$



$$\begin{cases} A_1 \, h = 0 \\ A_2 \, h = 0 \\ \\ A_N \, h = 0 \end{cases} \rightarrow A_{2N\times 9} \, h_{9\times 1} = 0$$

Over determined

Homogenous system

DLT algorithm (direct Linear Transformation)

How to solve $A_{2N \times 9} \, h_{9 \times 1} = 0$?

Singular Value Decomposition (SVD)!

DLT algorithm (direct Linear Transformation)

How to solve $\mathbf{A}_{2N\times 9}\,\mathbf{h}_{9\times 1} = \mathbf{0}$?

Singular Value Decomposition (SVD)!

↓

$$U_{2n\times 9}\,\Sigma_{9\times 9}\,V^{T}_{9\times 9}$$

Last column of V gives h! → H!

Why? See pag 593 of AZ

DLT algorithm (direct Linear Transformation)

How to solve $A_{2N\times 9}\, h_{9\times 1} = 0$?

```
[U,D,V] = svd(A,0);
x = V(:,end);
```

Clarification about SVM

$$P_{m\times n} = U_{m\times n} \; D_{n\times n} \; V^{T}_{n\times n}$$

Has n orthogonal columns (U)

Orthogonal matrix (V)

- This is one of the possible SVD decompositions
- This is typically used for efficiency
- The classic SVD is actually:

$$P_{m\times n} = U_{m\times m} \; D_{m\times n} \; V^{T}_{n\times n}$$

orthogonal (U)

Orthogonal (V)

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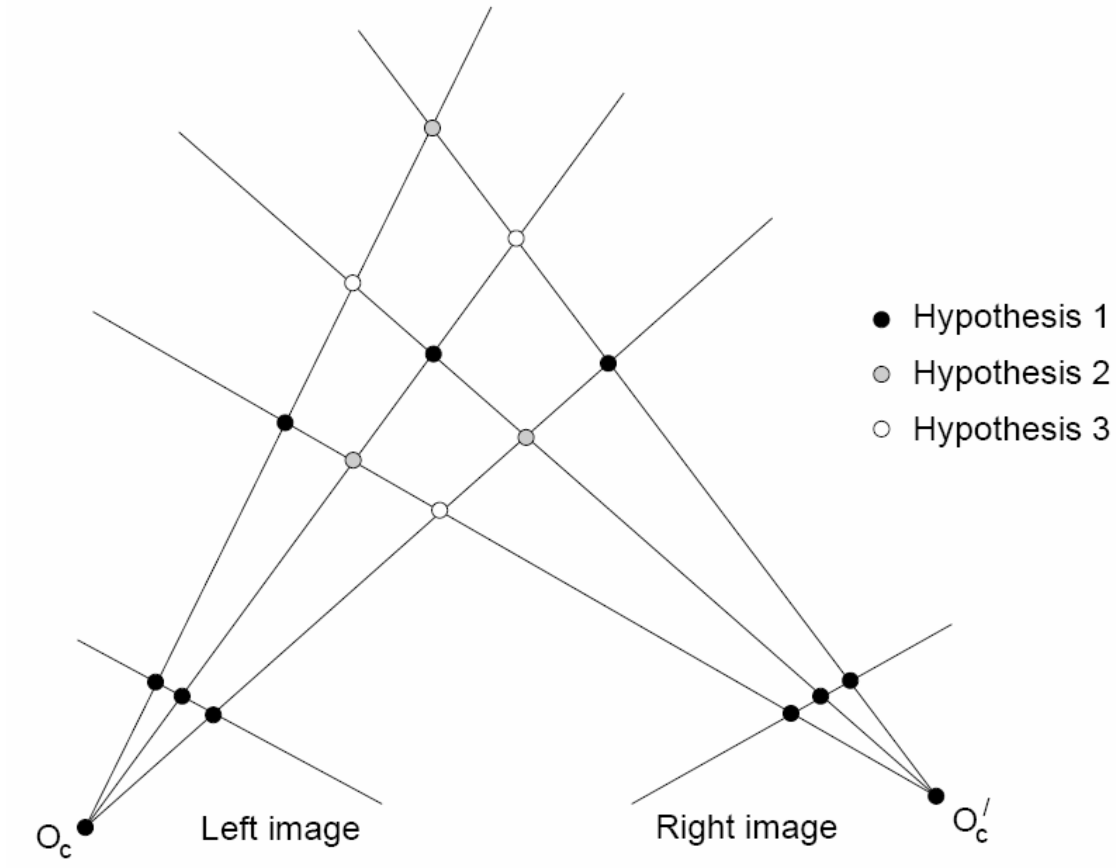
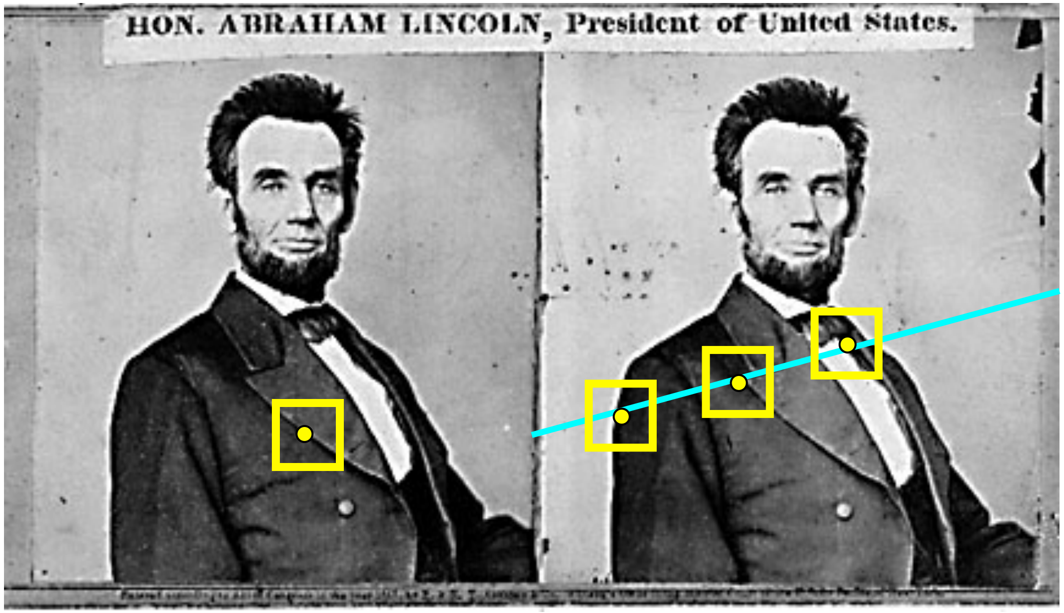
Stereo matching: solving the correspondence problem



- Multiple matching hypotheses satisfy the epipolar constraint, but which one is correct?

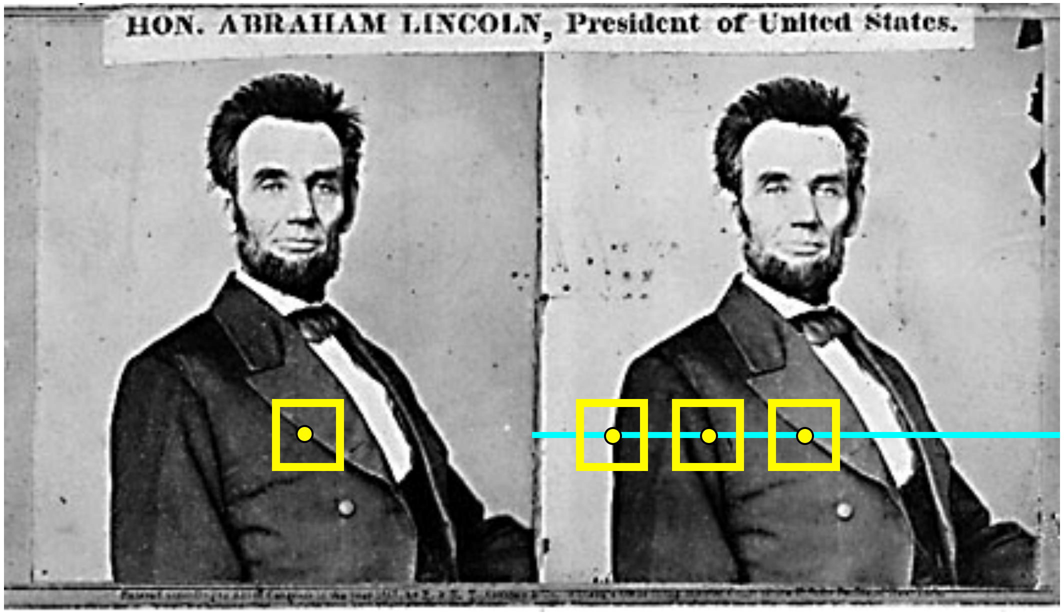
Basic stereo matching algorithm



- For each pixel in the first image
  - Find corresponding epipolar line in the right image
  - Examine all pixels on the epipolar line and pick the best match
  - Triangulate the matches to get depth information
- Simplest case: epipolar lines are scanlines
  - When does this happen?

Slide credit: J. Hayes

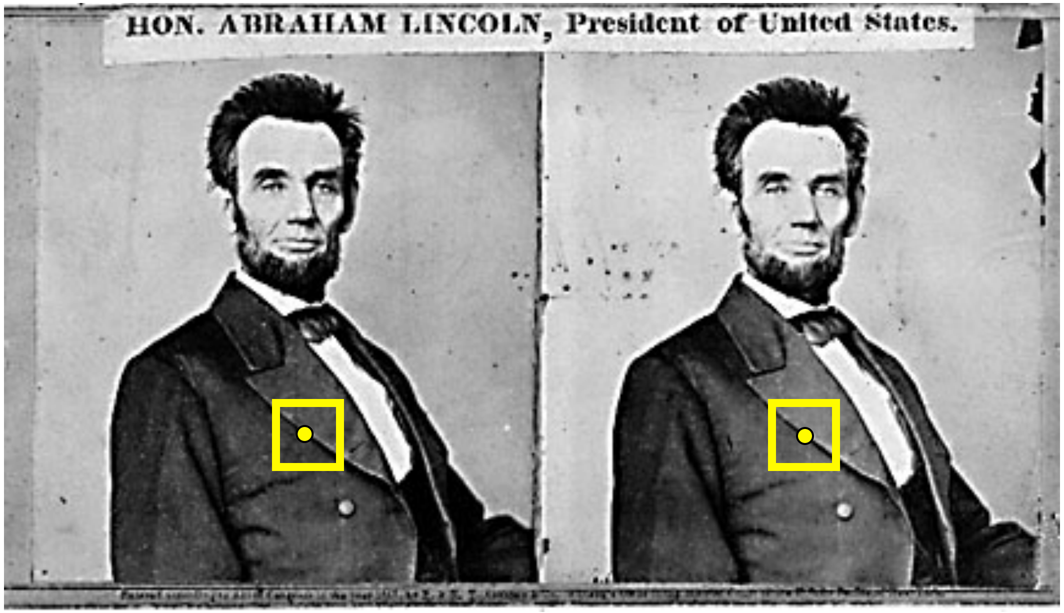
Basic stereo matching algorithm



- If necessary, rectify the two stereo images to transform epipolar lines into scanlines
- For each pixel x in the first image
  - Find corresponding epipolar scanline in the right image
  - Examine all pixels on the scanline and pick the best match x'
  - Compute disparity x-x' and set depth(x) = 1/(x-x')

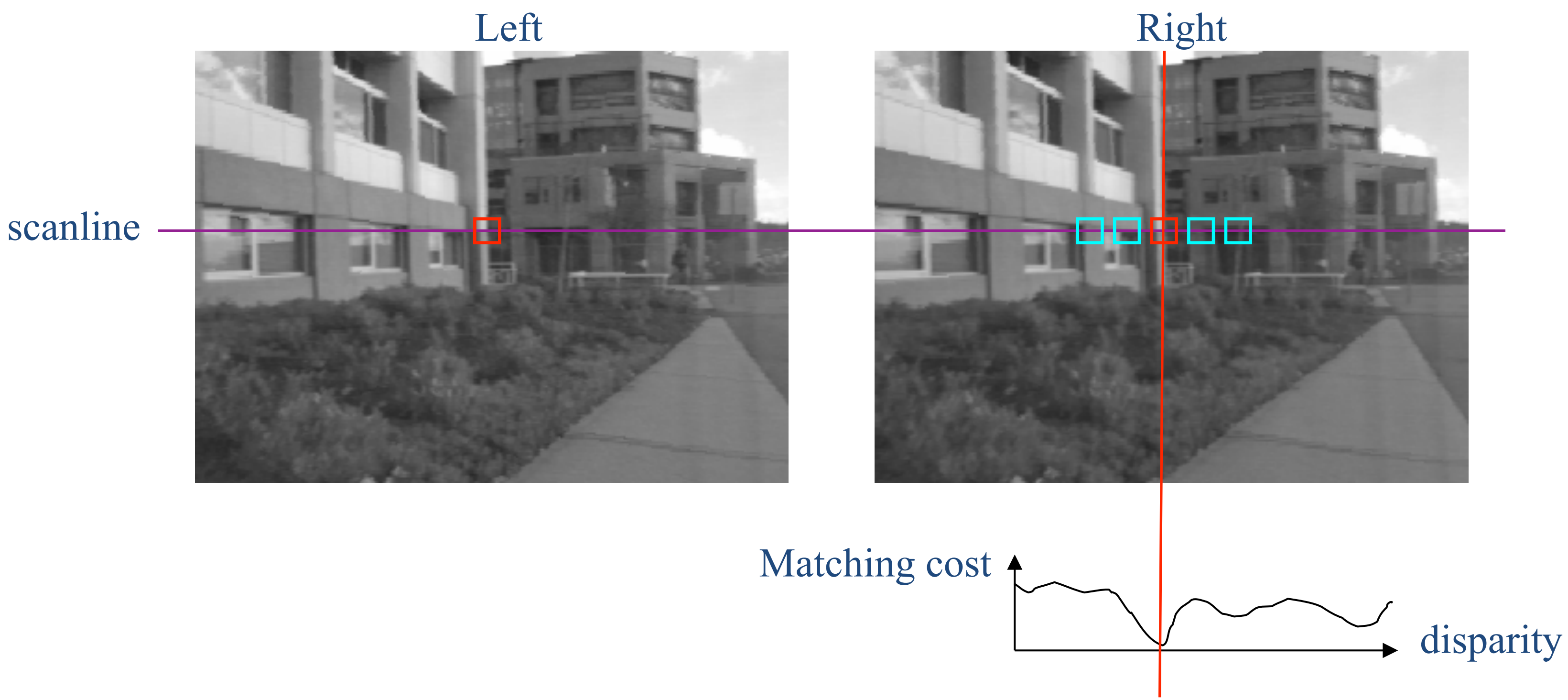
Correspondence problem

- Let’s make some assumptions to simplify the matching problem
  - The baseline is relatively small (compared to the depth of scene points)
  - Then most scene points are visible in both views
  - Also, matching regions are similar in appearance



Slide credit: J. Hayes

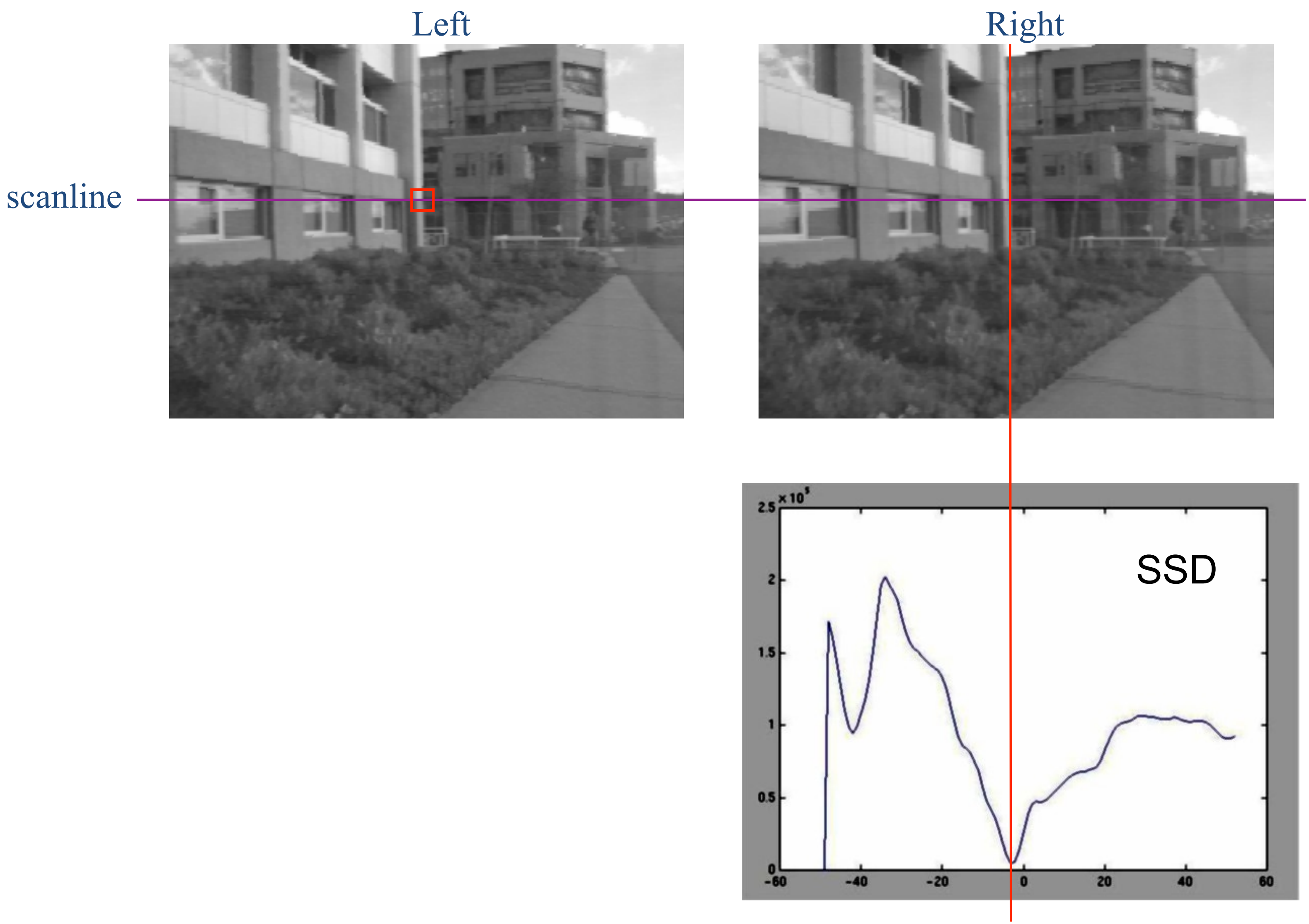
Correspondence search with similarity constraint



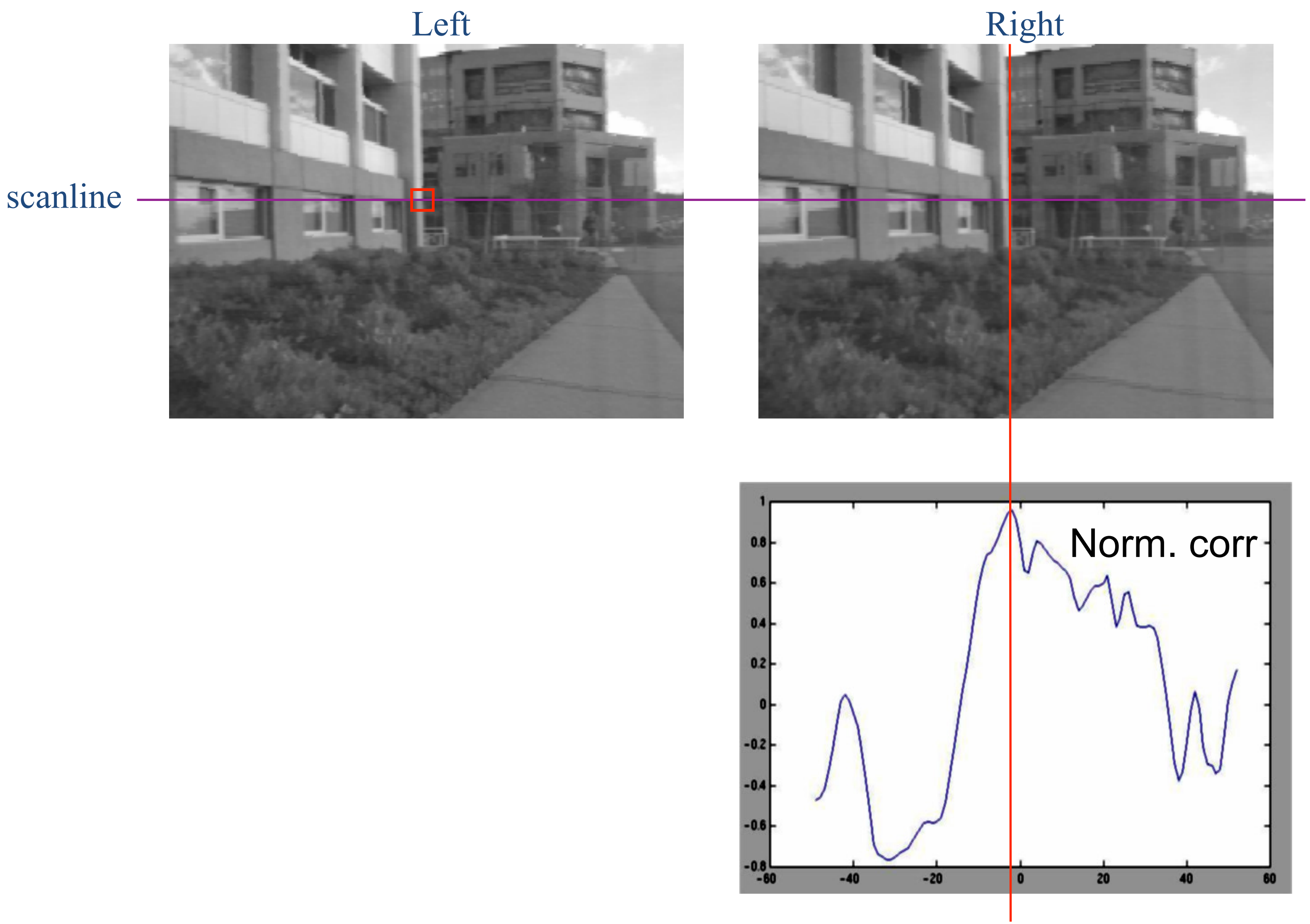
- Slide a window along the right scanline and compare contents of that window with the reference window in the left image
- Matching cost: SSD or normalized correlation

Slide credit: J. Hayes

Correspondence search with similarity constraint

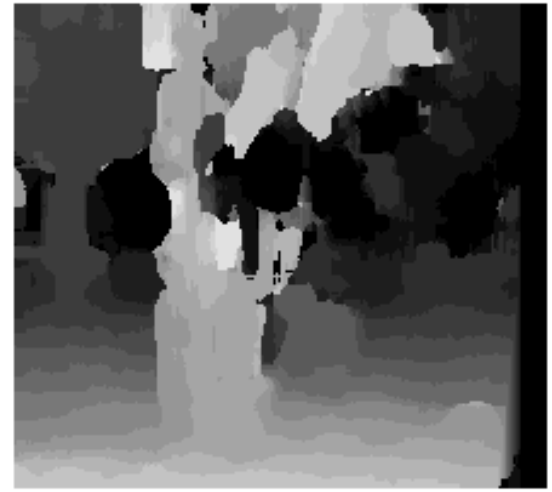


Correspondence search with similarity constraint



Effect of window size

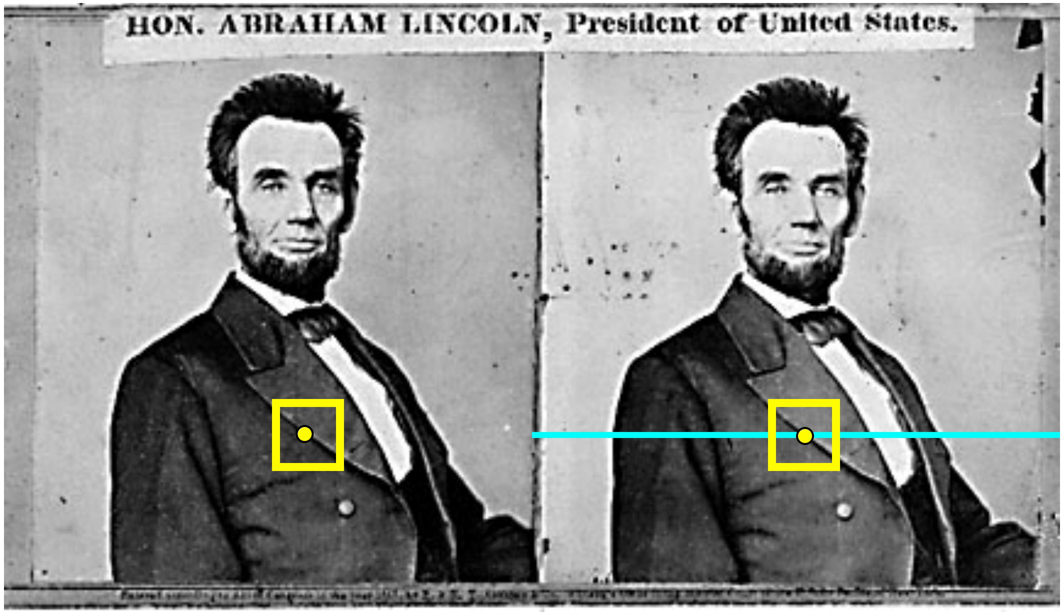
W = 3



W = 20

- Smaller window
  - + More detail
  - • More noise
- Larger window
  - + Smoother disparity maps
  - • Less detail

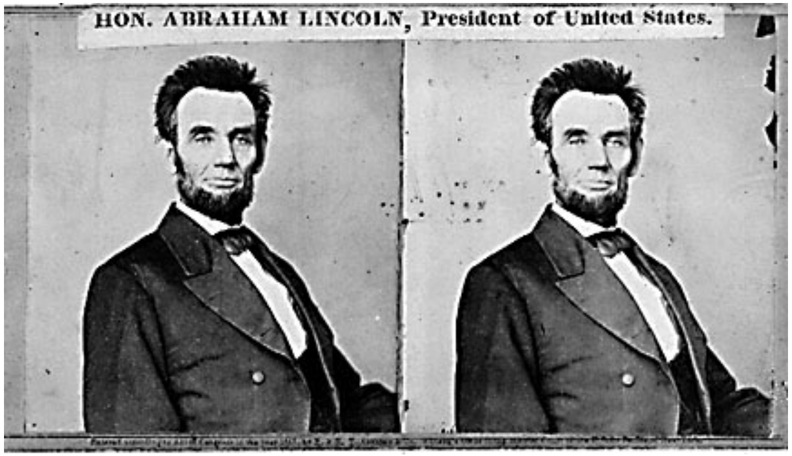
The similarity constraint



- Corresponding regions in two images should be similar in appearance
- …and non-corresponding regions should be different
- When will the similarity constraint fail?

Slide credit: J. Hayes

Limitations of similarity constraint



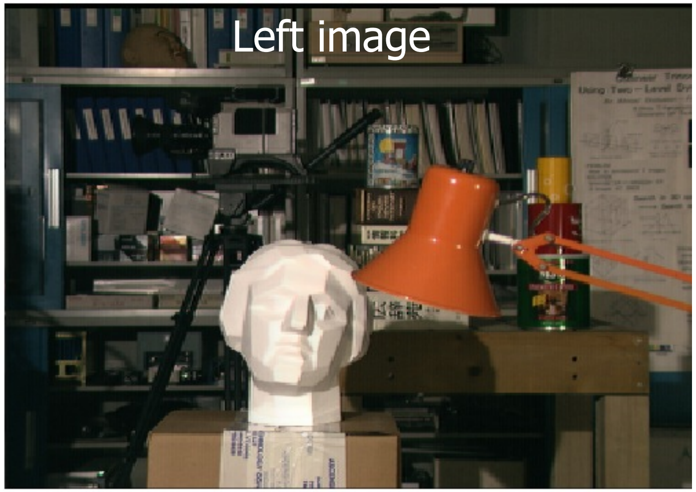
Textureless surfaces

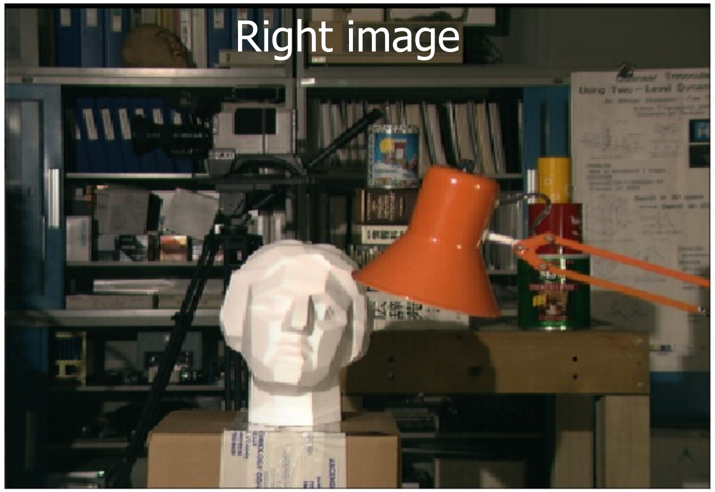
Occlusions, repetition

Specular surfaces

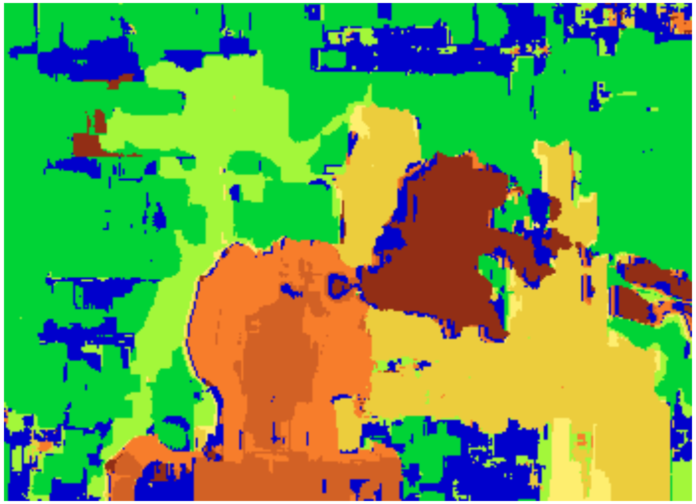
Slide credit: J. Hayes

Results with window search





Window-based matching



Ground truth

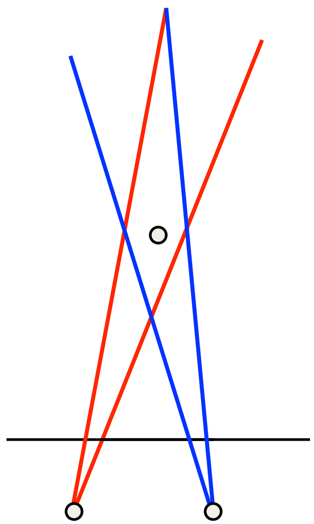
Better methods exist... (CS231a)

Graph cuts

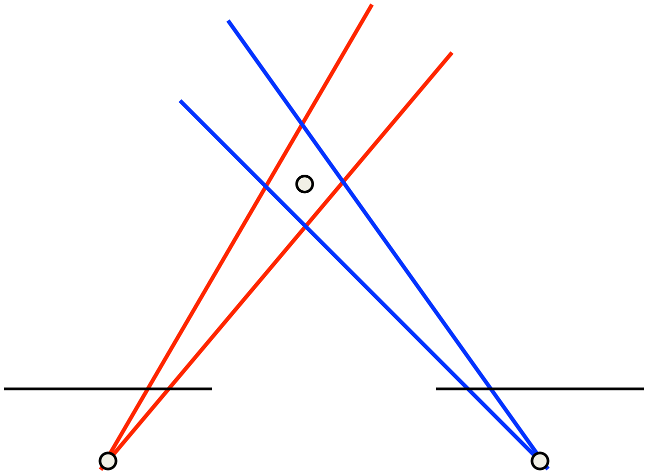
Ground truth

Y. Boykov, O. Veksler, and R. Zabih,
[Fast Approximate Energy Minimization via Graph Cuts](), PAMI 2001

The role of the baseline



Small Baseline

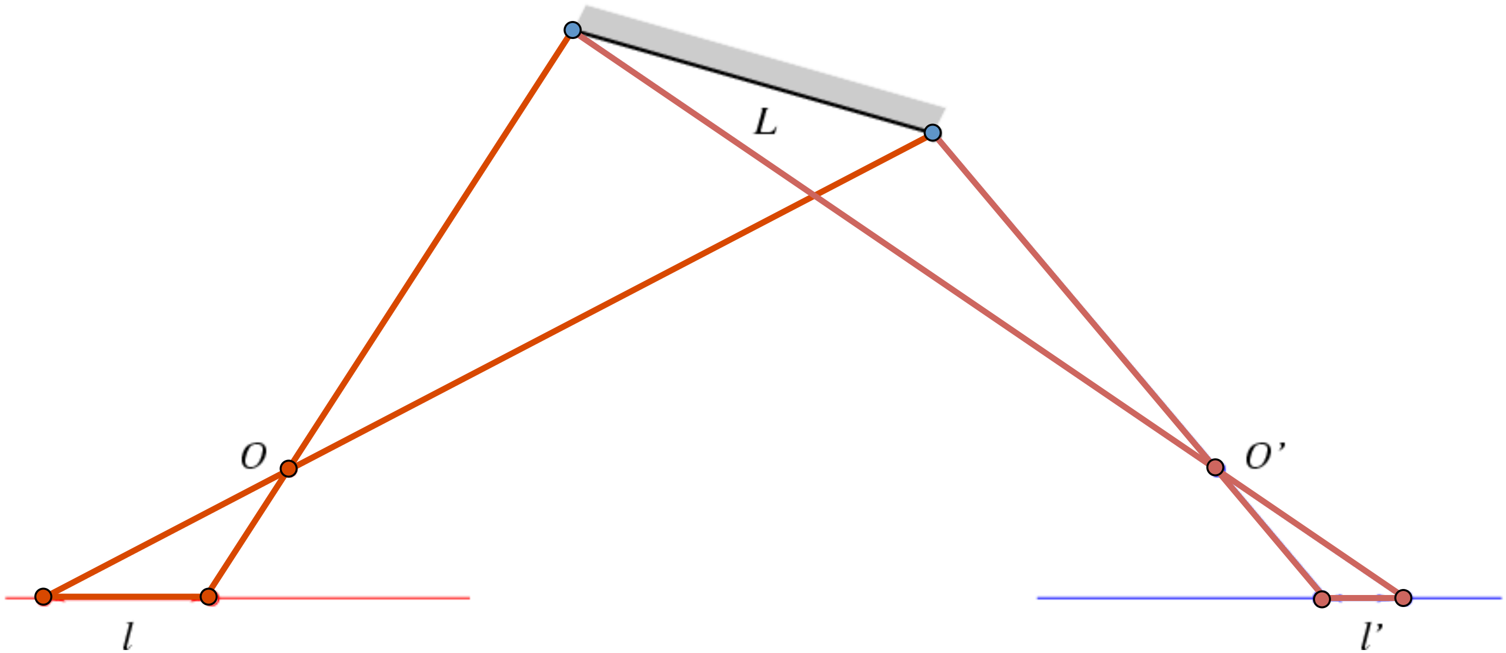


Large Baseline

- Small baseline: large depth error
- Large baseline: difficult search problem

Slide credit: S. Seitz

Problem for wide baselines: Foreshortening



- Matching with fixed-size windows will fail!
- Possible solution: adaptively vary window size
- Another solution: *model-based stereo* (CS231a)

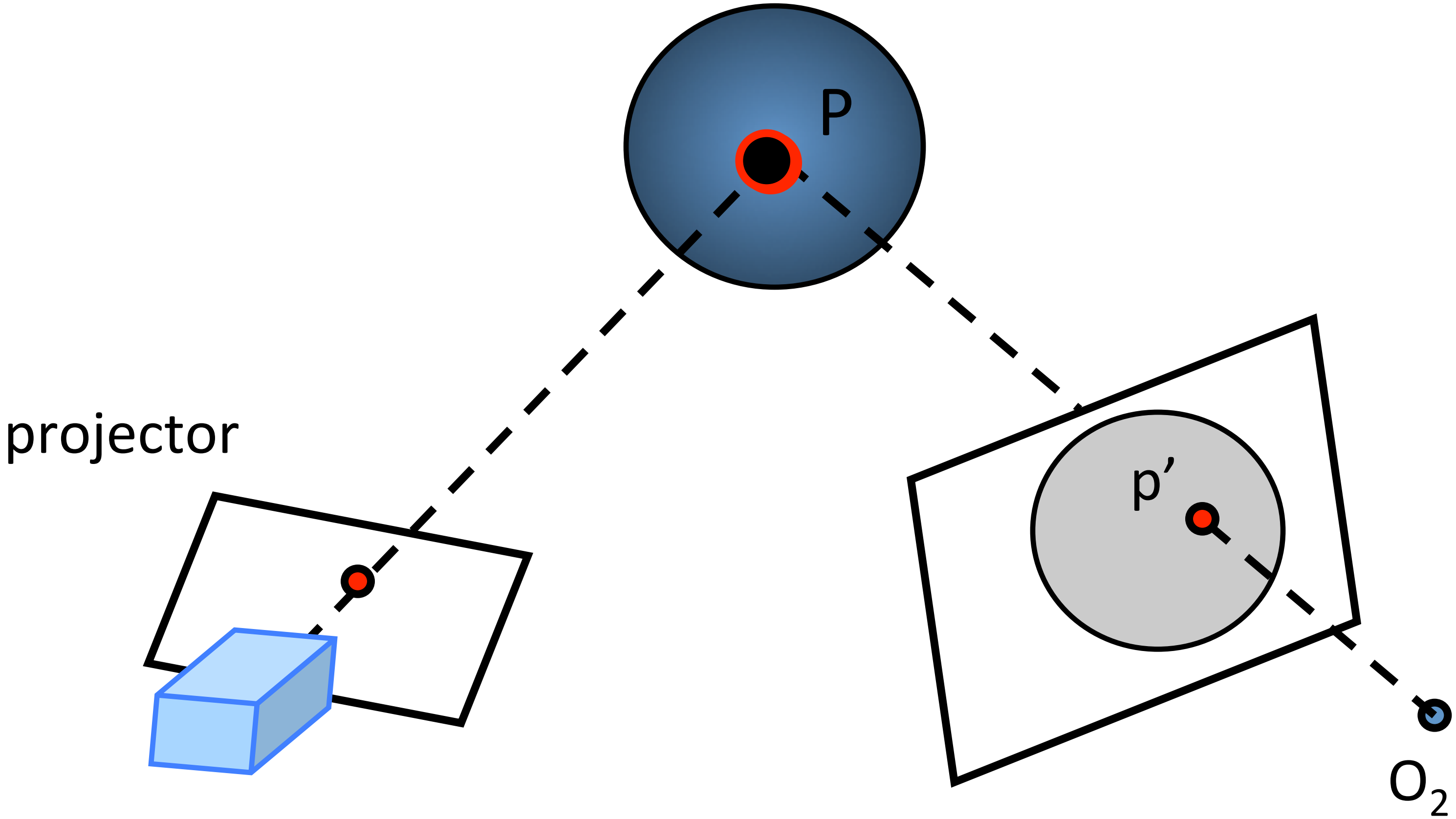
Slide credit: J. Hayes

What we will learn today?

- Introduction to stereo vision
- Epipolar geometry: a gentle intro
- Parallel images
- Image rectification
- Solving the correspondence problem
- Active stereo vision system

Reading:
[HZ] Chapters: 4, 9, 11
[FP] Chapters: 10

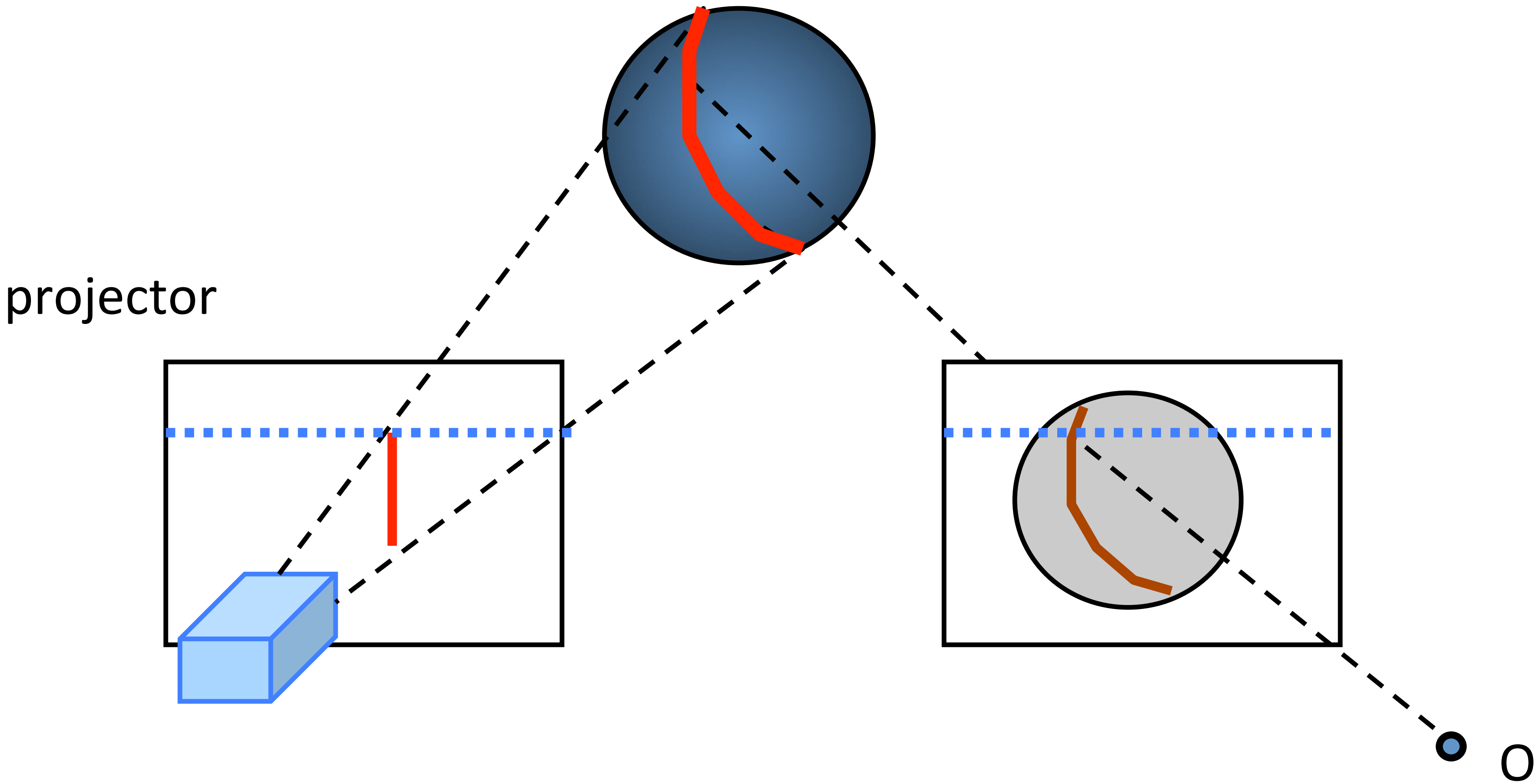
Active stereo (point)



Replace one of the two cameras by a projector

- Single camera
- Projector geometry calibrated
- What's the advantage of having the projector? Correspondence problem solved!

Active stereo (stripe)

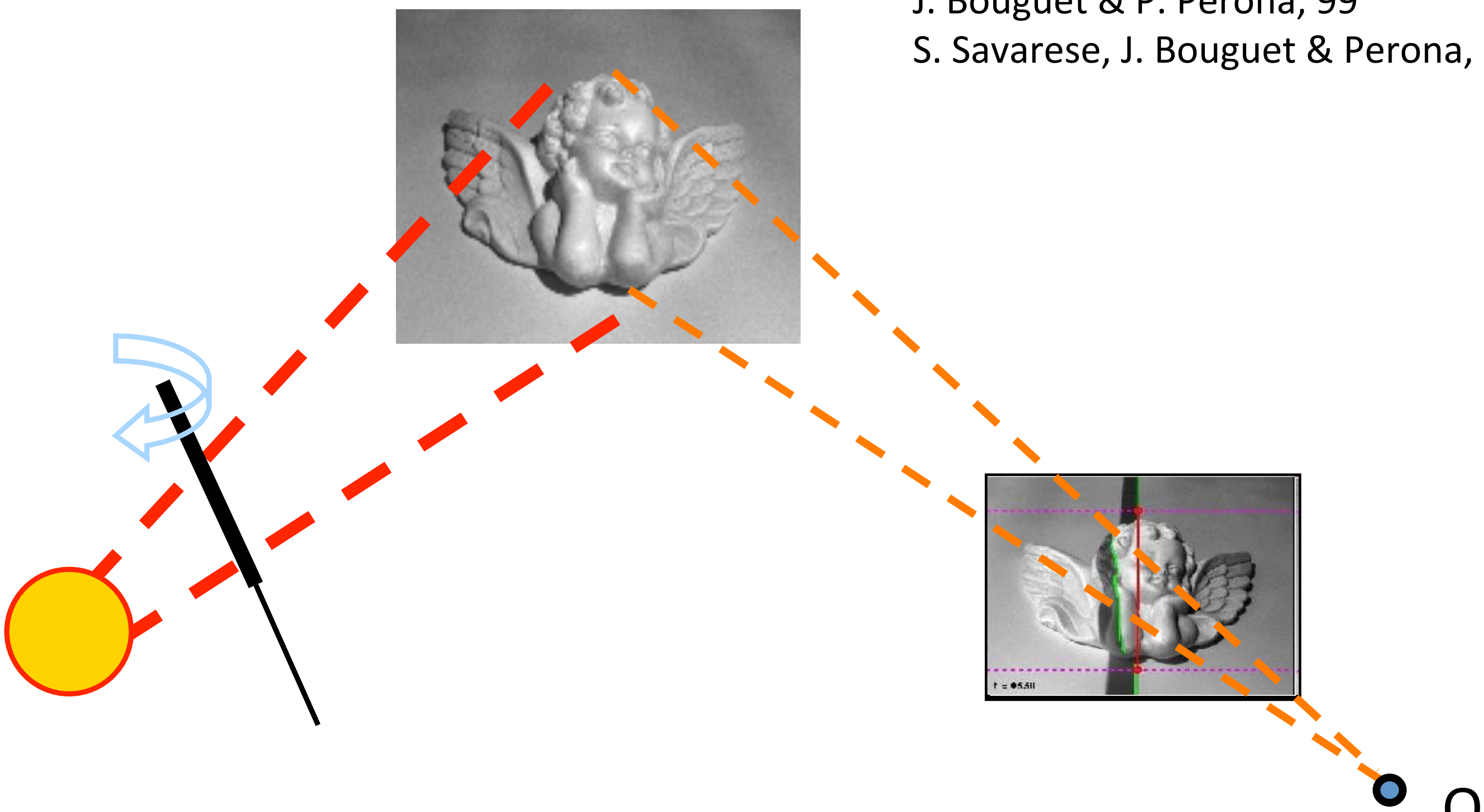


-Projector and camera are parallel

- Correspondence problem solved!

Active stereo (shadows)

J. Bouguet & P. Perona, 99
S. Savarese, J. Bouguet & Perona, 00

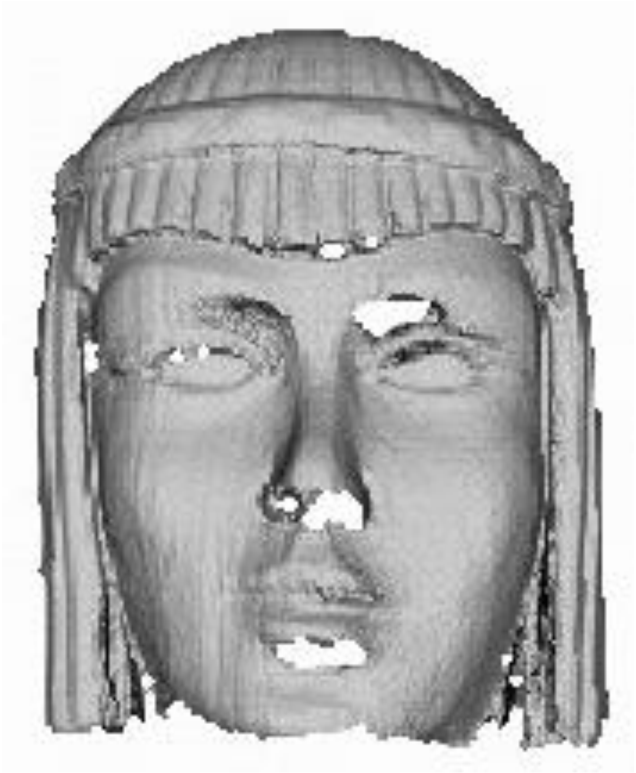


Light source

- 1 camera,1 light source
- very cheap setup
- calibrated light source

Active stereo (shadows)

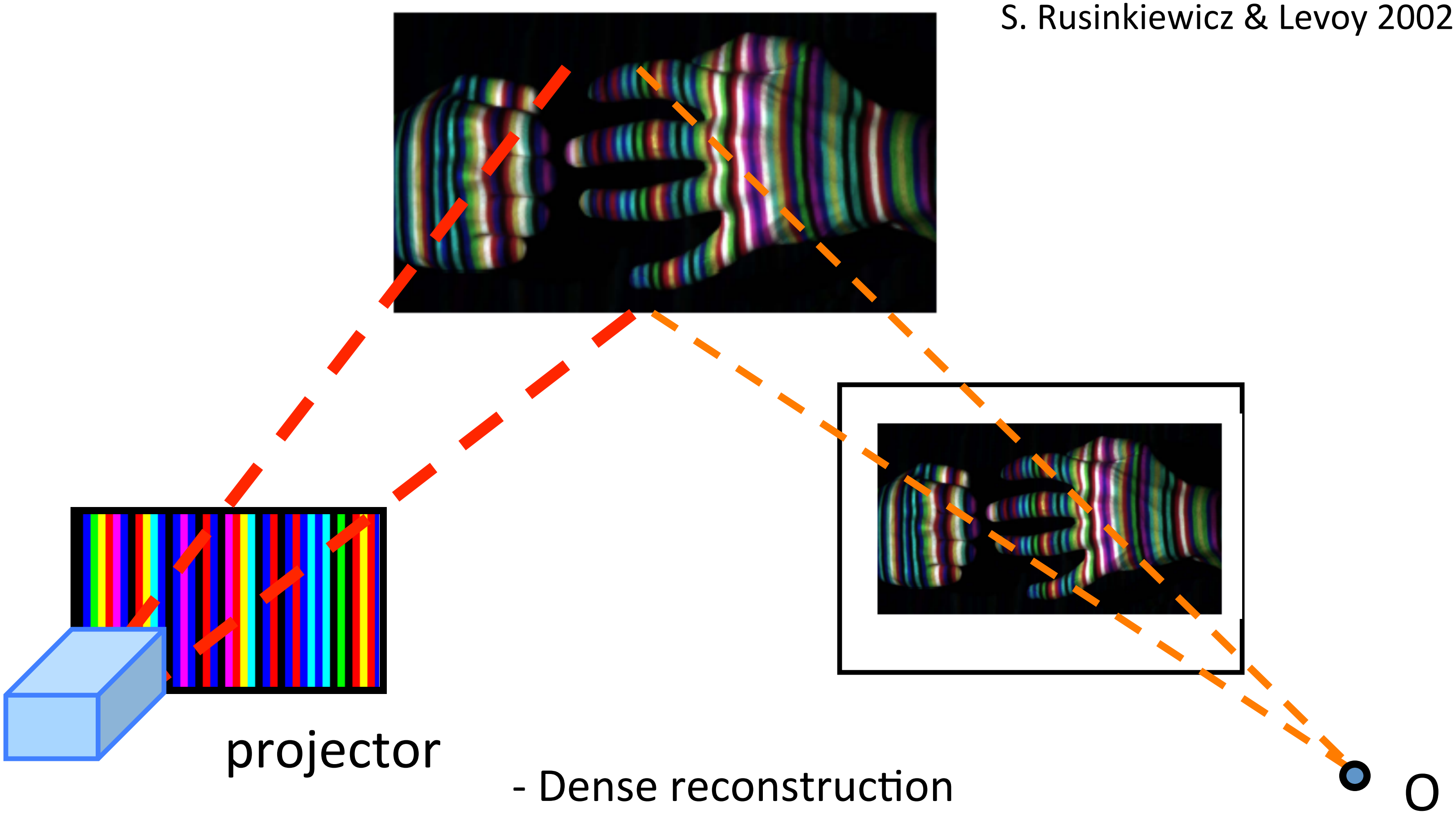
J. Bouguet & P. Perona, 99
S. Savarese, J. Bouguet & Perona, 00



Active stereo (color-coded stripes)

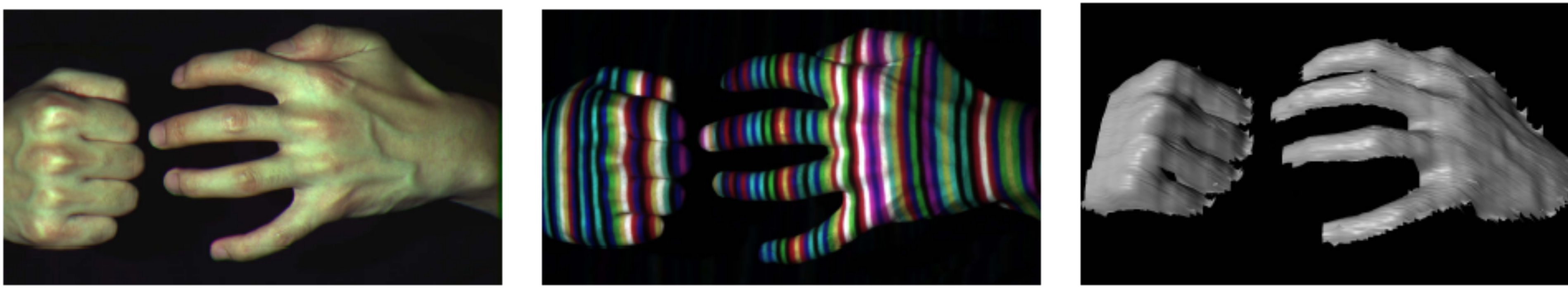
L. Zhang, B. Curless, and S. M. Seitz 2002

S. Rusinkiewicz & Levoy 2002



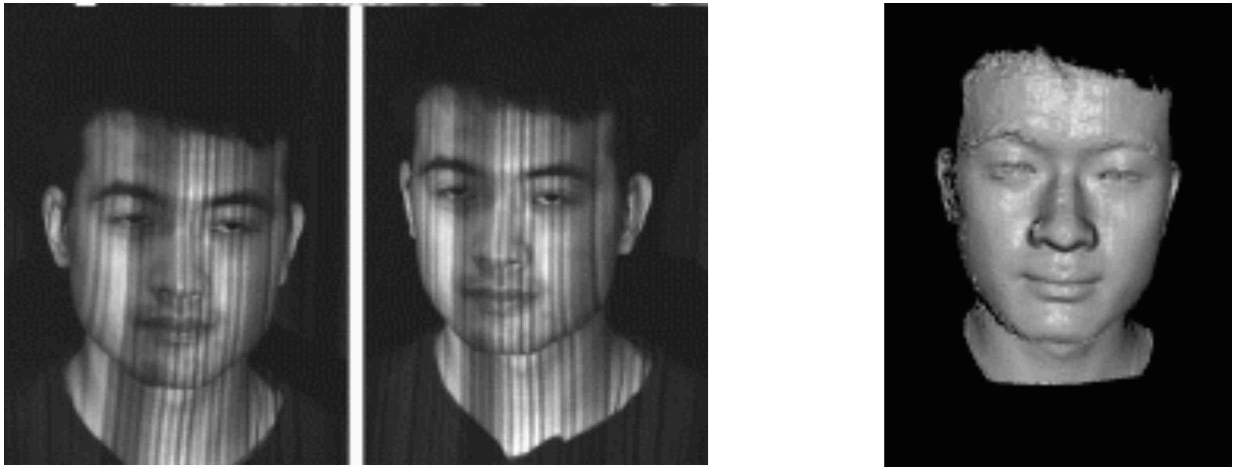
- Dense reconstruction
- Correspondence problem again
- Get around it by using color codes

Active stereo (color-coded stripes)

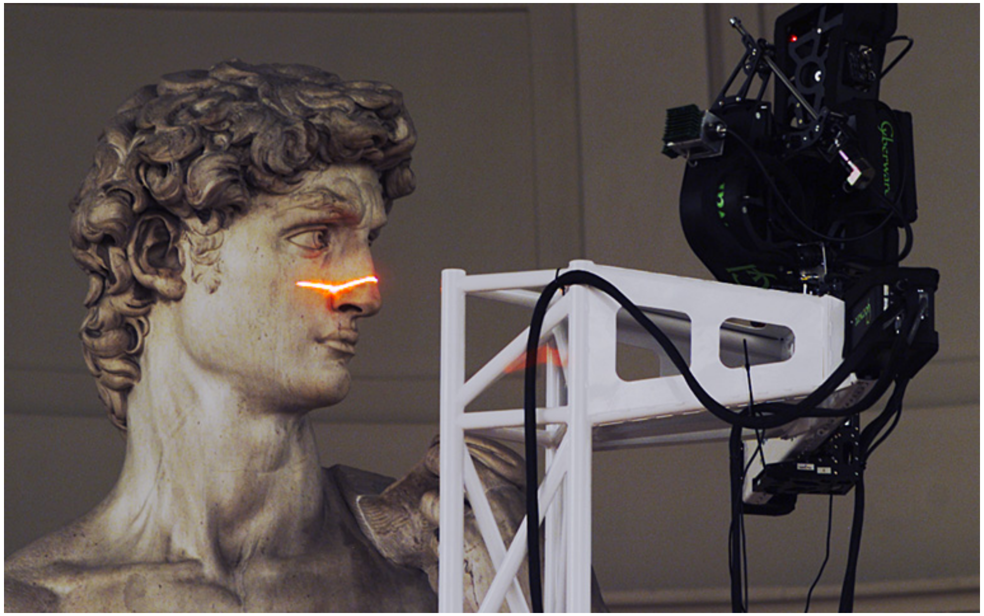


Rapid shape acquisition: Projector + stereo cameras

L. Zhang, B. Curless, and S. M. Seitz. Rapid Shape Acquisition Using Color Structured Light and Multi-pass Dynamic Programming. *3DPVT* 2002



Active stereo (stripe)



Digital Michelangelo Project
http://graphics.stanford.edu/projects/mich/

- Optical triangulation
  - Project a single stripe of laser light
  - Scan it across the surface of the object
  - This is a very precise version of structured light scanning

Laser scanned models

The Digital Michelangelo Project, Levoy et al.

Slide credit: S. Seitz

Laser scanned models

The Digital Michelangelo Project, Levoy et al.

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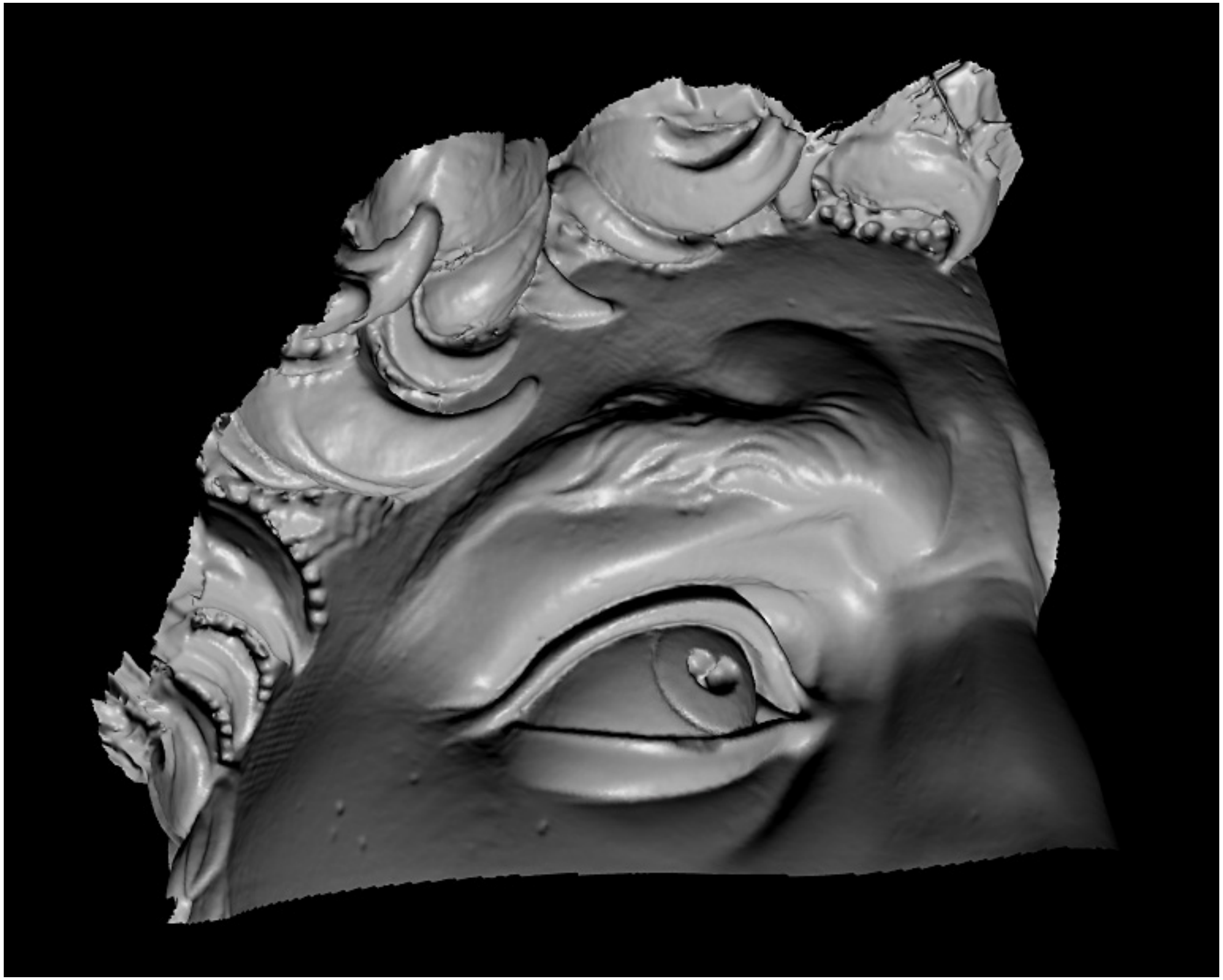
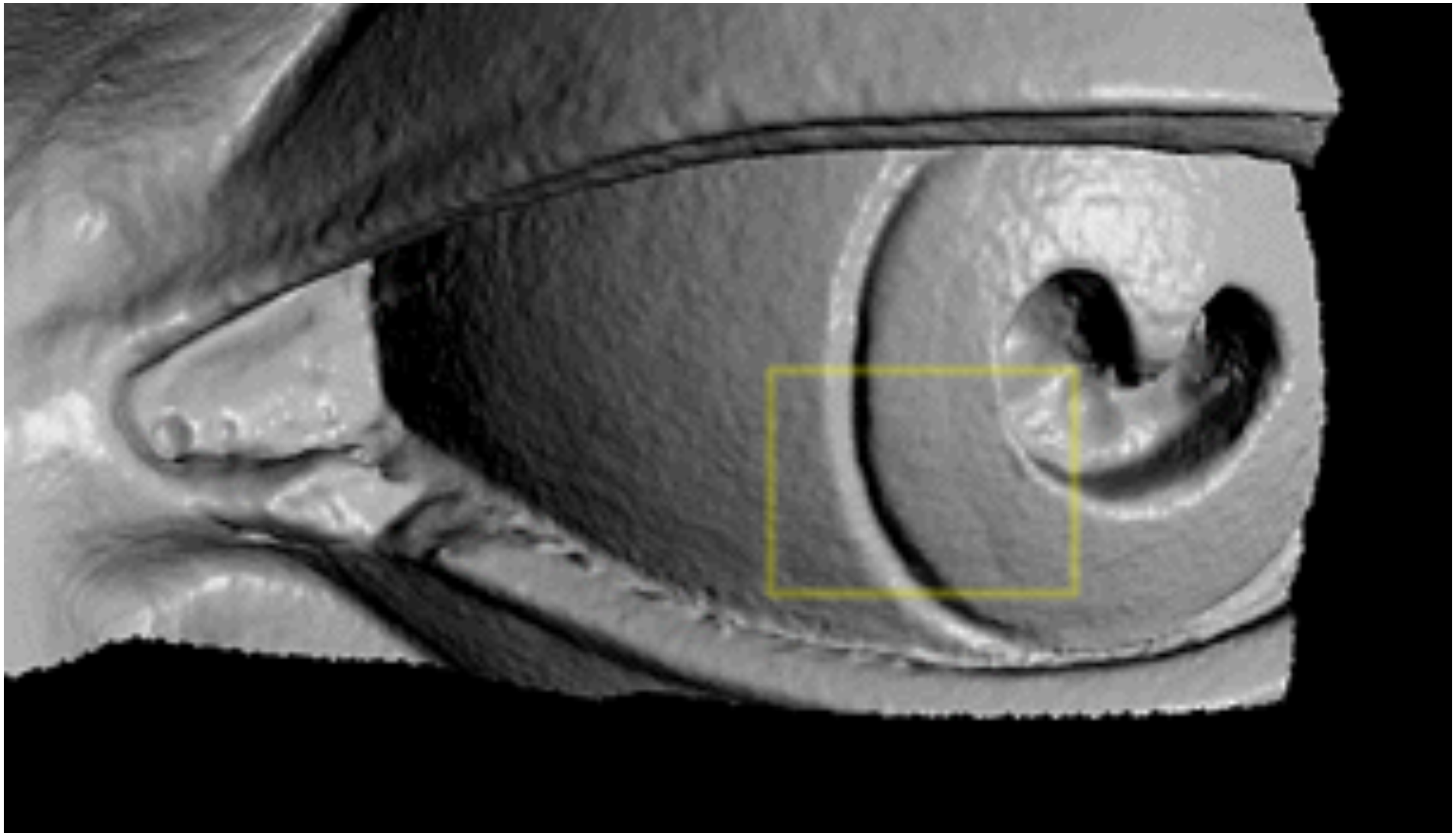
Laser scanned models



The Digital Michelangelo Project, Levoy et al.

Slide credit: S. Seitz

Laser scanned models

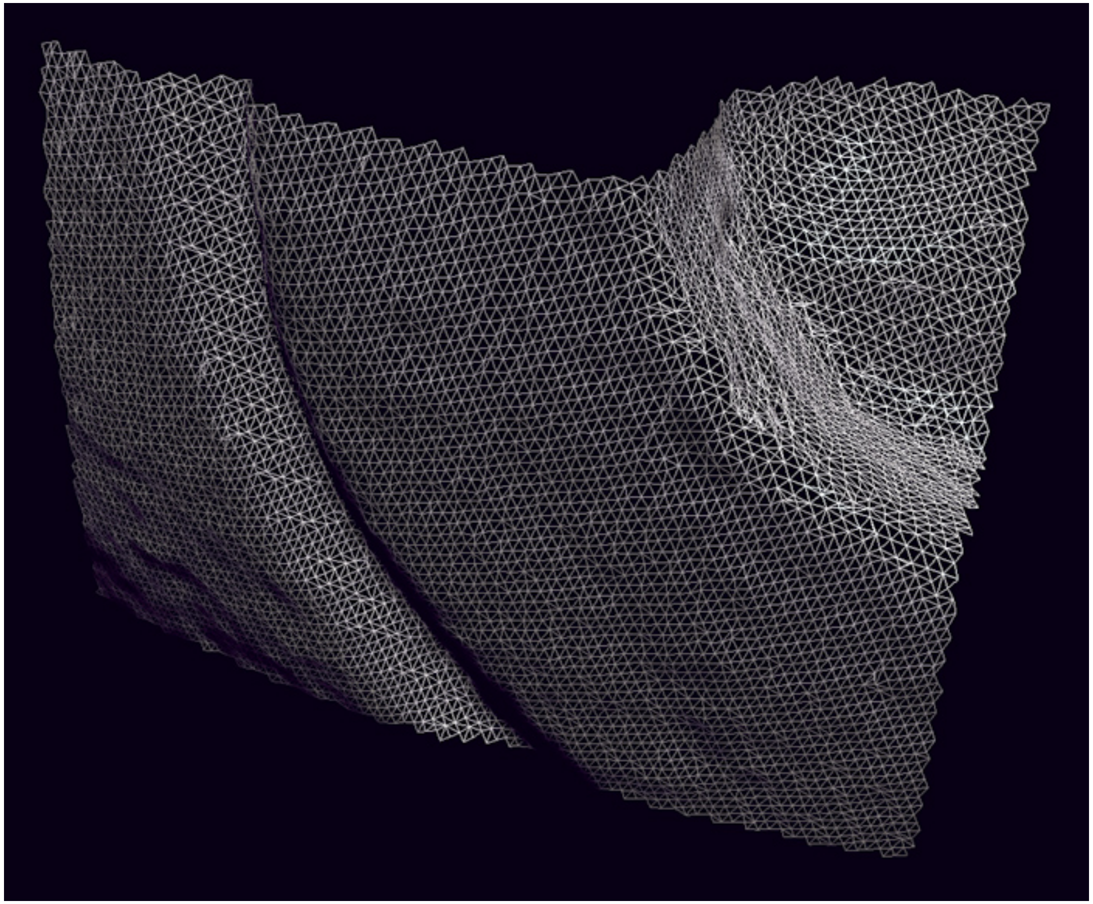


The Digital Michelangelo Project, Levoy et al.

Slide credit: S. Seitz

Laser scanned models

1.0 mm resolution (56 million triangles)



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