Lecture 8:
Camera Models

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Stanford Vision Lab

What we will learn today?

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
  - Projection matrix
  - Intrinsic parameters
  - Extrinsic parameters

Reading:
[FP] Chapters 1 – 3
[HZ] Chapter 6

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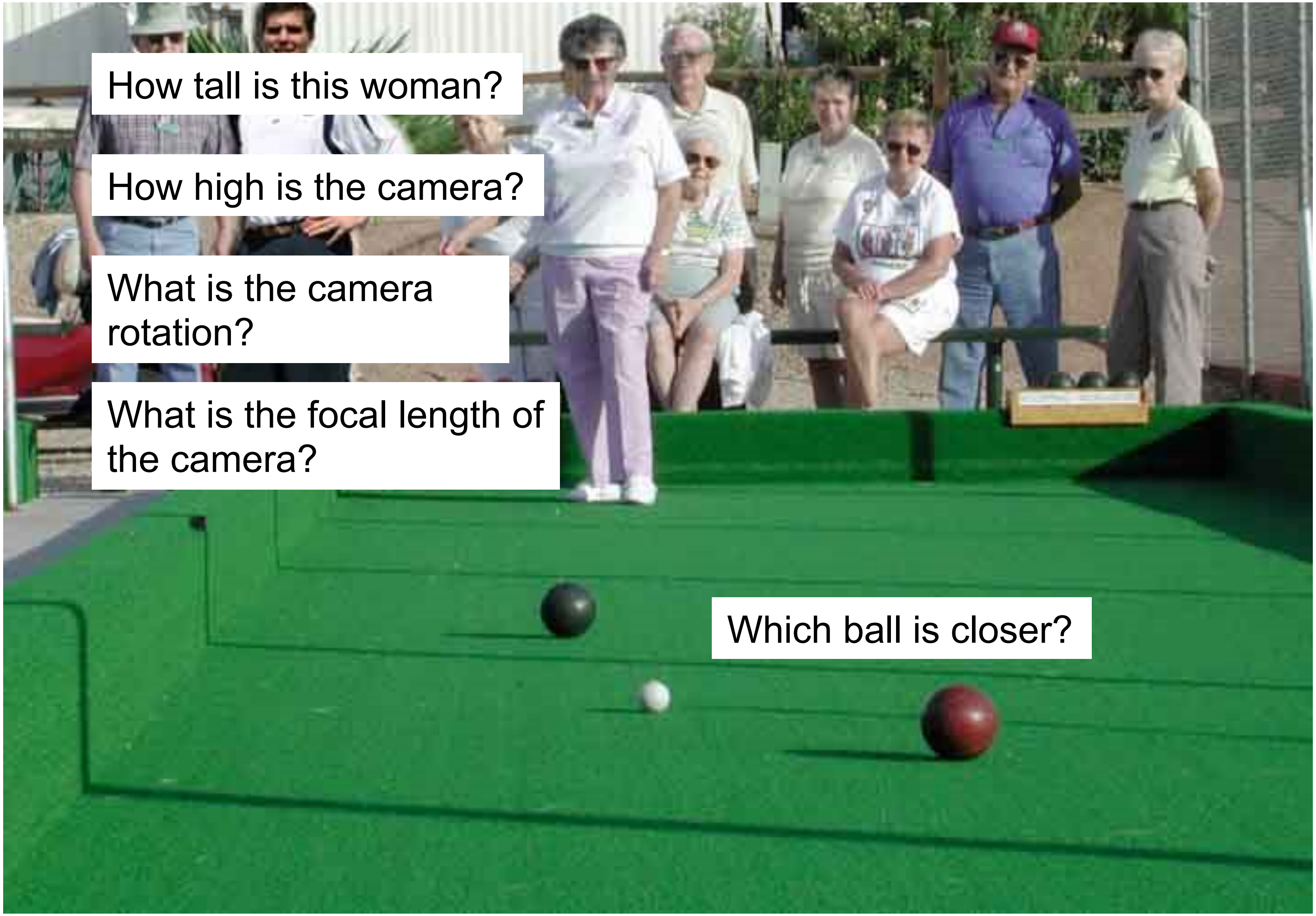
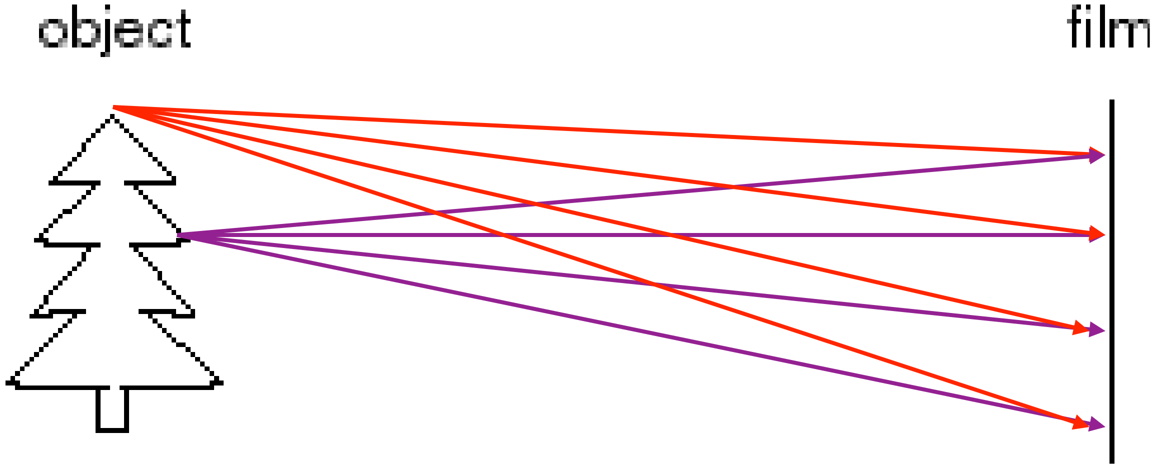
Camera and World Geometry



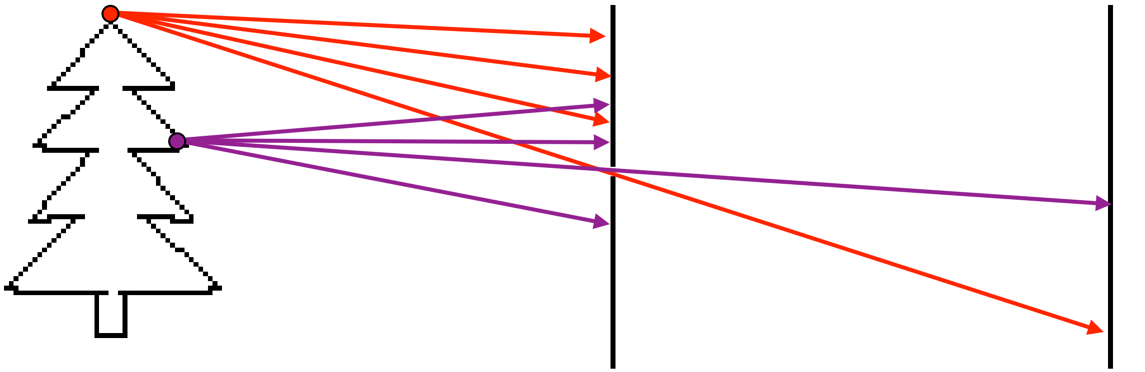
Slide credit: J. Hayes

How do we see the world?



- Let's design a camera
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

Pinhole camera



- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening known as the **aperture**

Camera obscura: the pre-camera

- Known during classical period in China and Greece (e.g. Mo-Tsi, China, 470BC to 390BC)

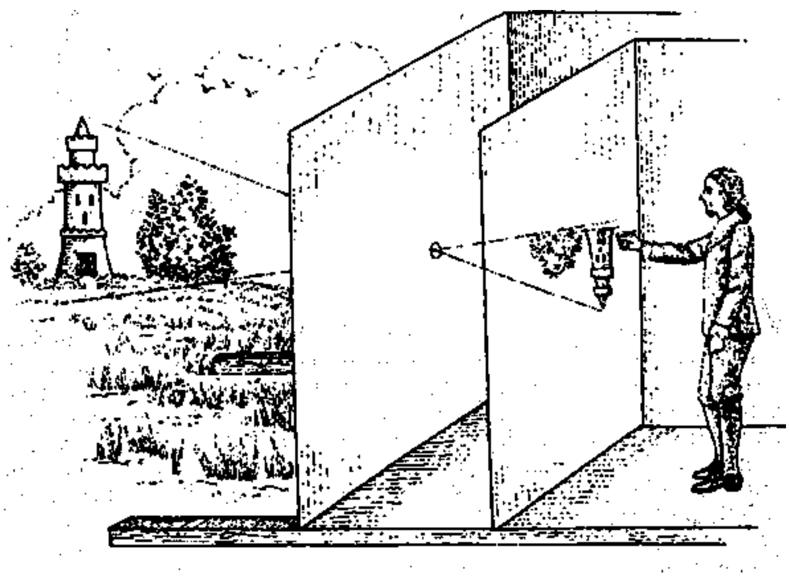


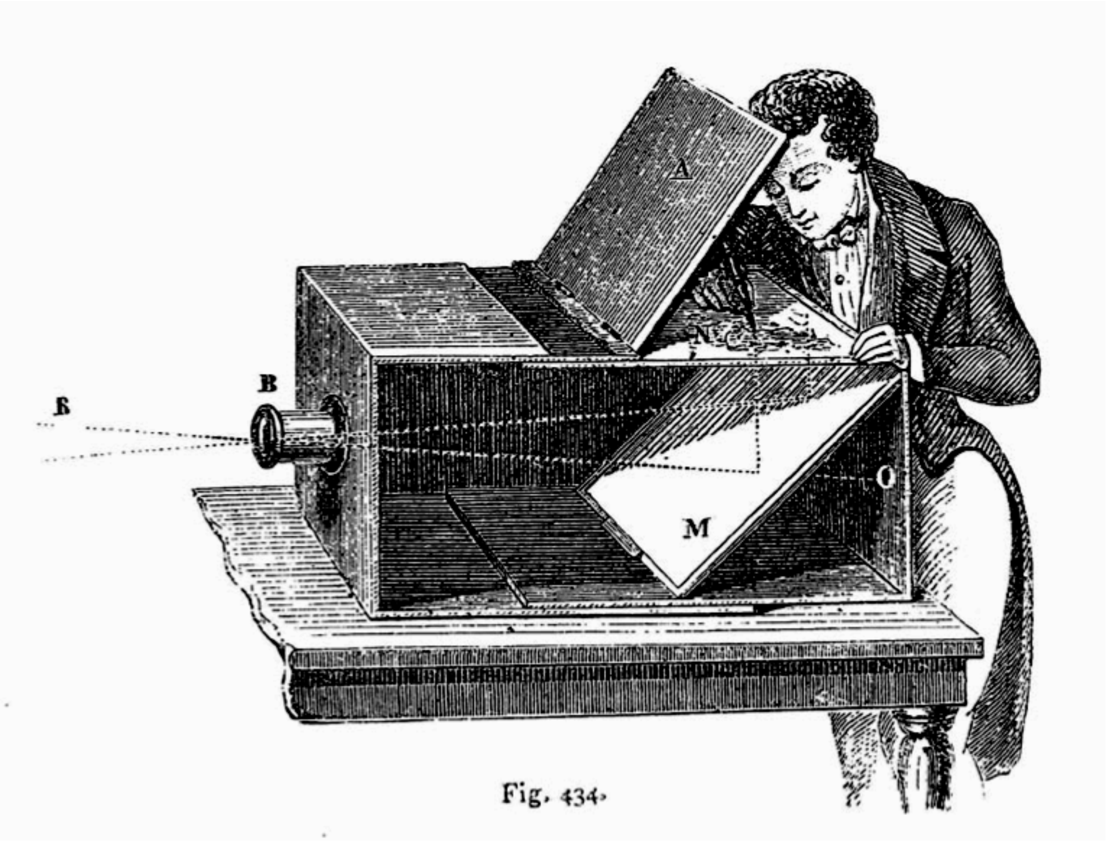
Illustration of Camera Obscura

Freestanding camera obscura at UNC Chapel Hill

Photo by Seth Ilys

Slide credit: J. Hayes

Camera Obscura used for Tracing



Lens Based Camera Obscura, 1568

Slide credit: J. Hayes

First Photograph

Oldest surviving photograph

- Took 8 hours on pewter plate

Joseph Niepce, 1826

Photograph of the first photograph

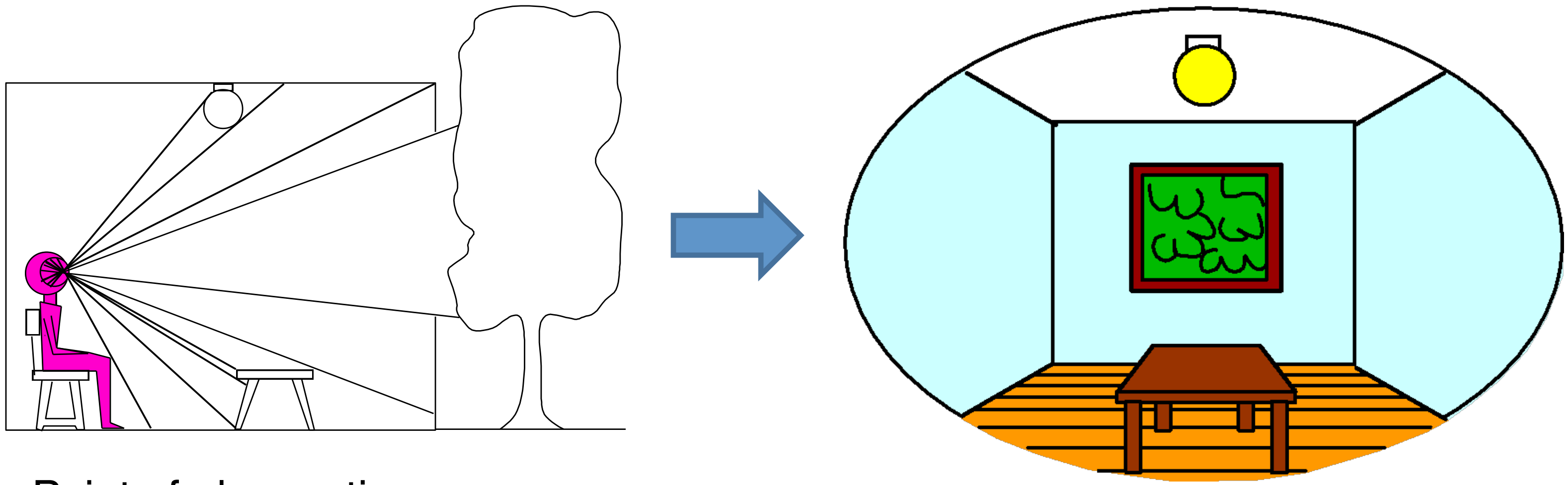
Stored at UT Austin

Niepce later teamed up with Daguerre, who eventually created Daguerrotypes

Slide credit: J. Hayes

Dimensionality Reduction Machine (3D to 2D)

3D world *2D image*



Point of observation

Projection can be tricky…

Slide source: Seitz

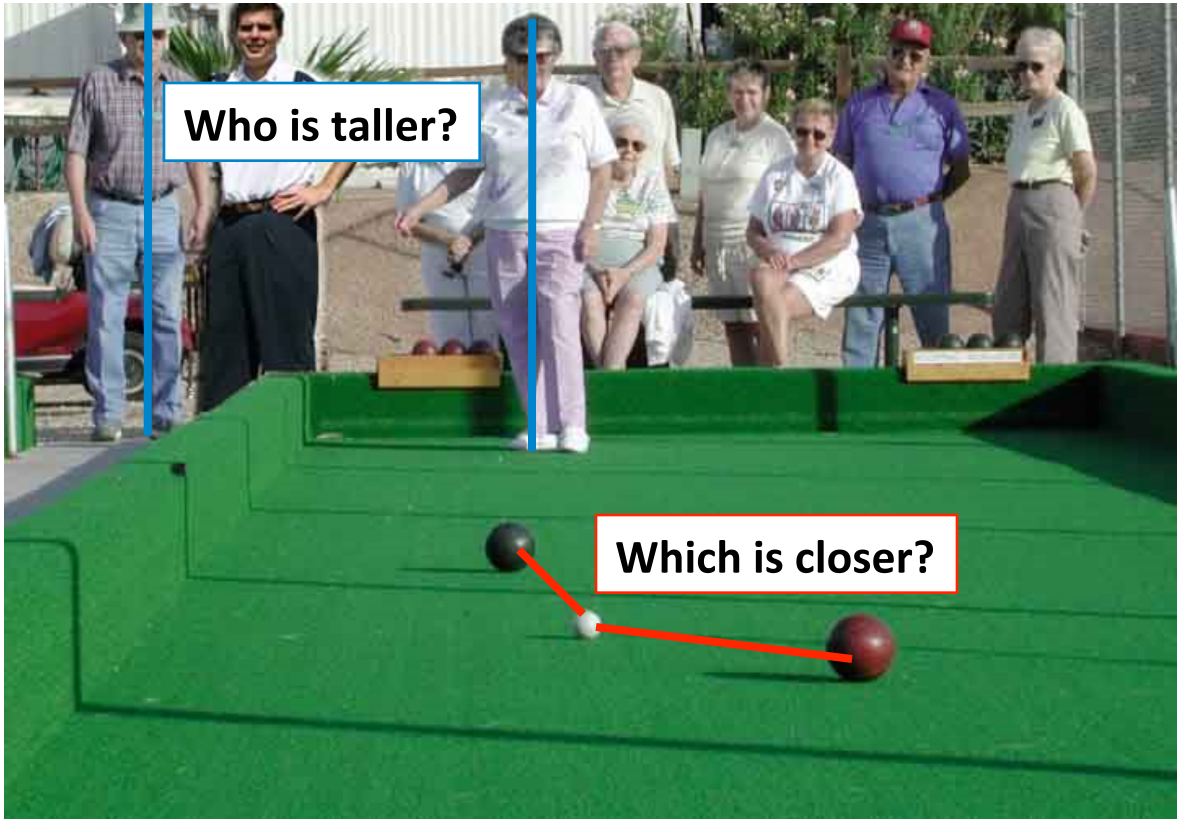
Projection can be tricky…

Slide source: Seitz

Projective Geometry

What is lost?

- Length



Slide credit: J. Hayes

Length is not preserved

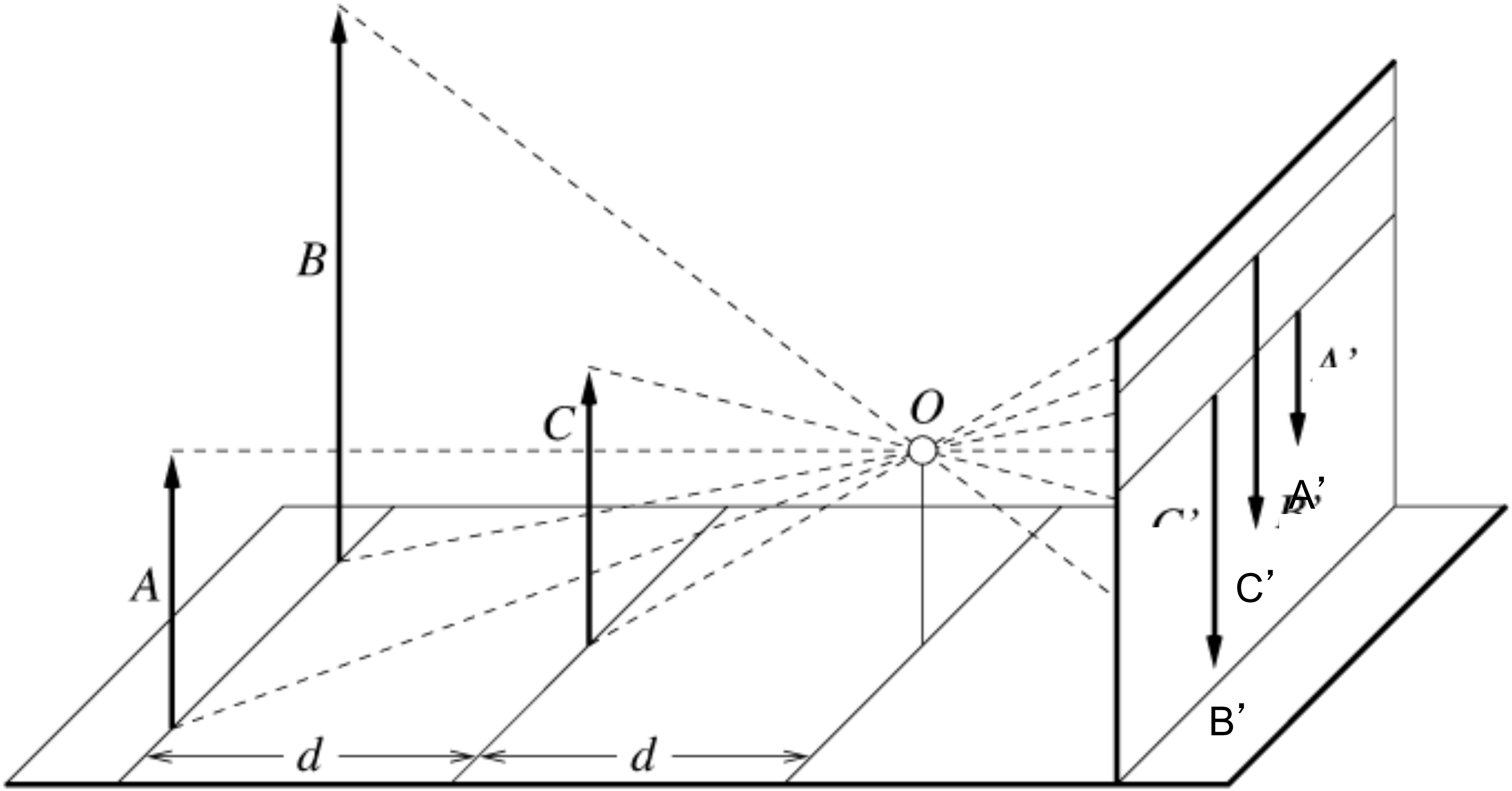
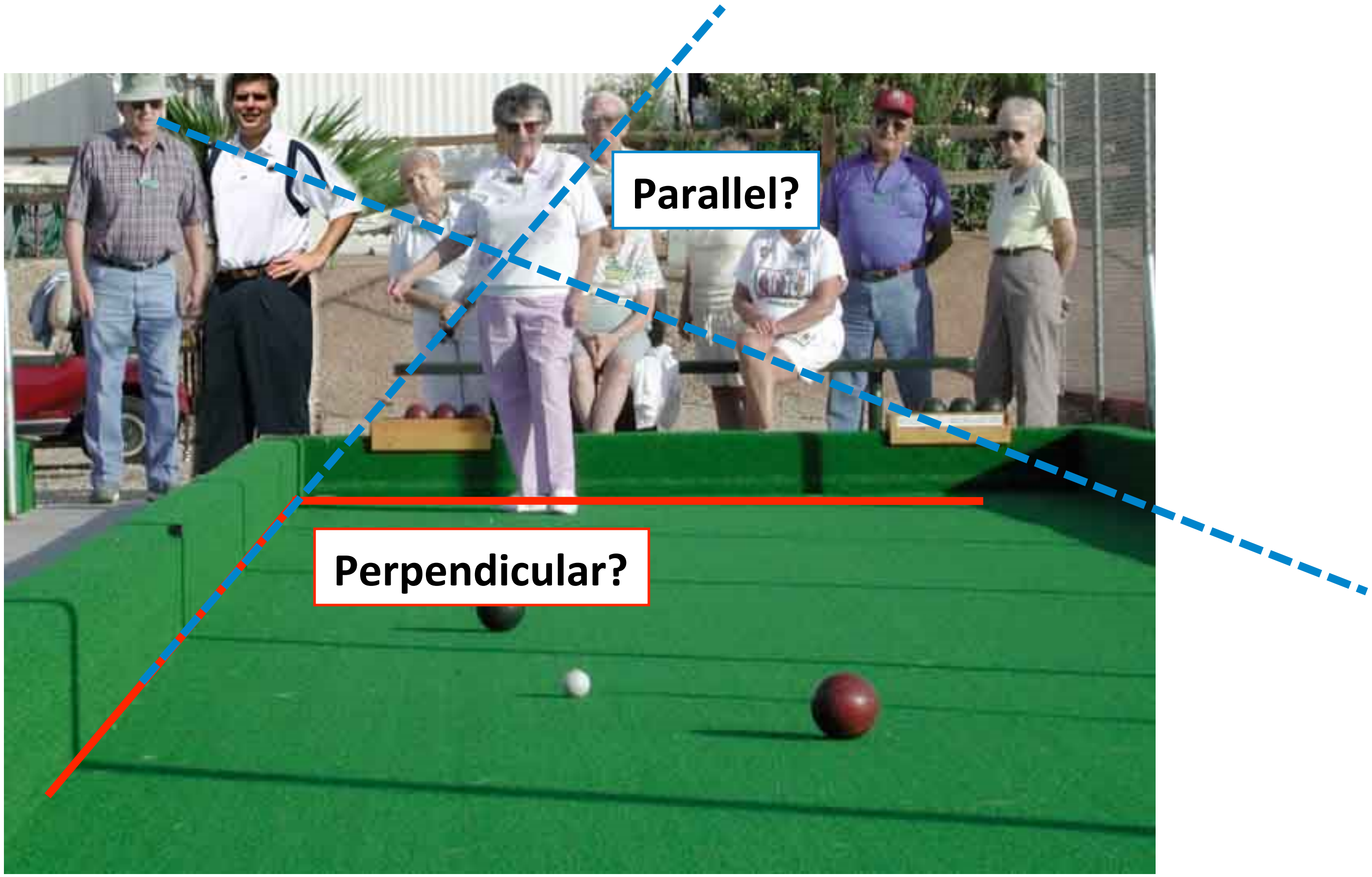


Figure by David Forsyth

Projective Geometry

What is lost?

- Length
- Angles

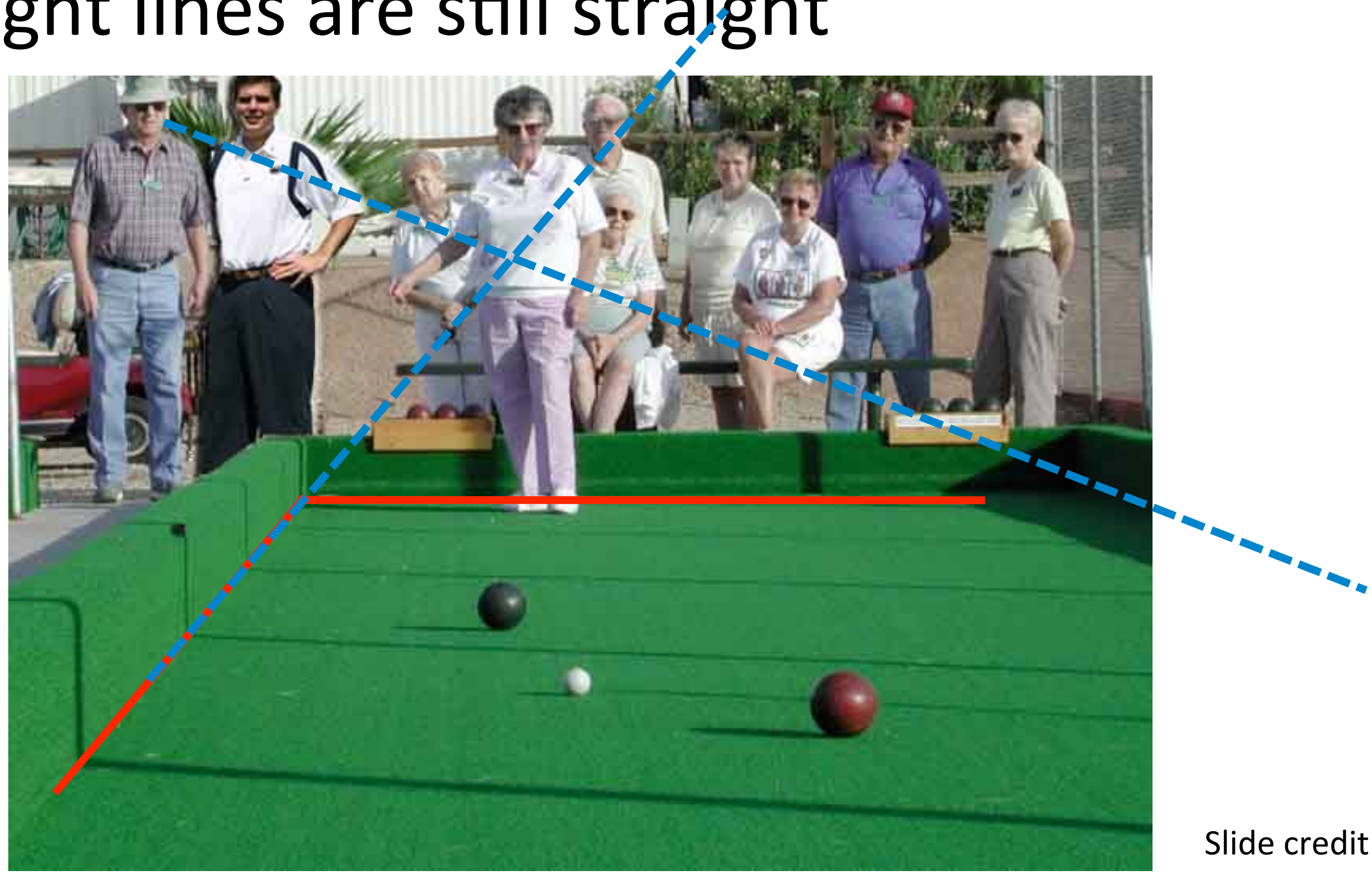


Slide credit: J. Hayes

Projective Geometry

What is preserved?

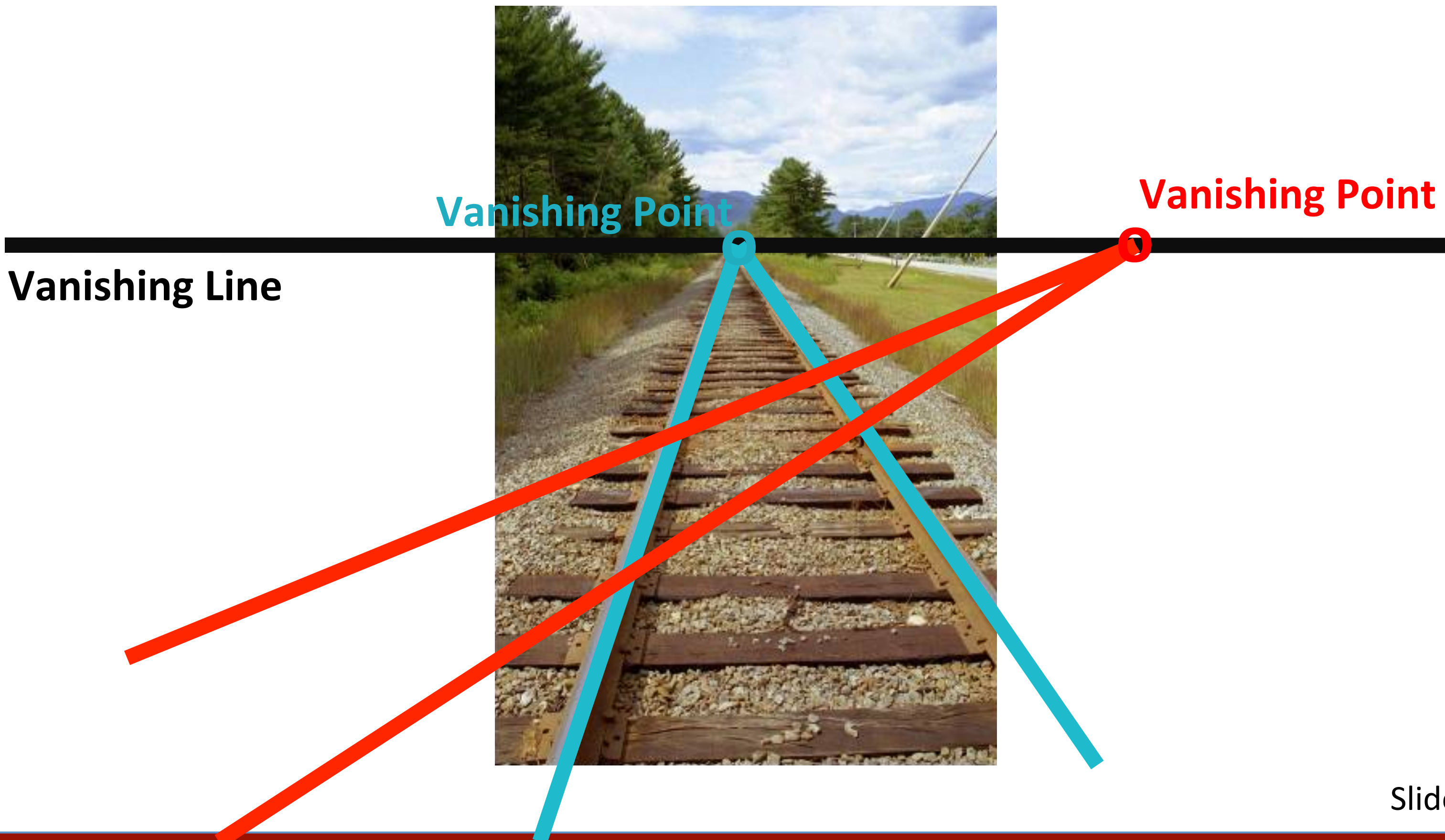
- Straight lines are still straight



Slide credit: J. Hayes

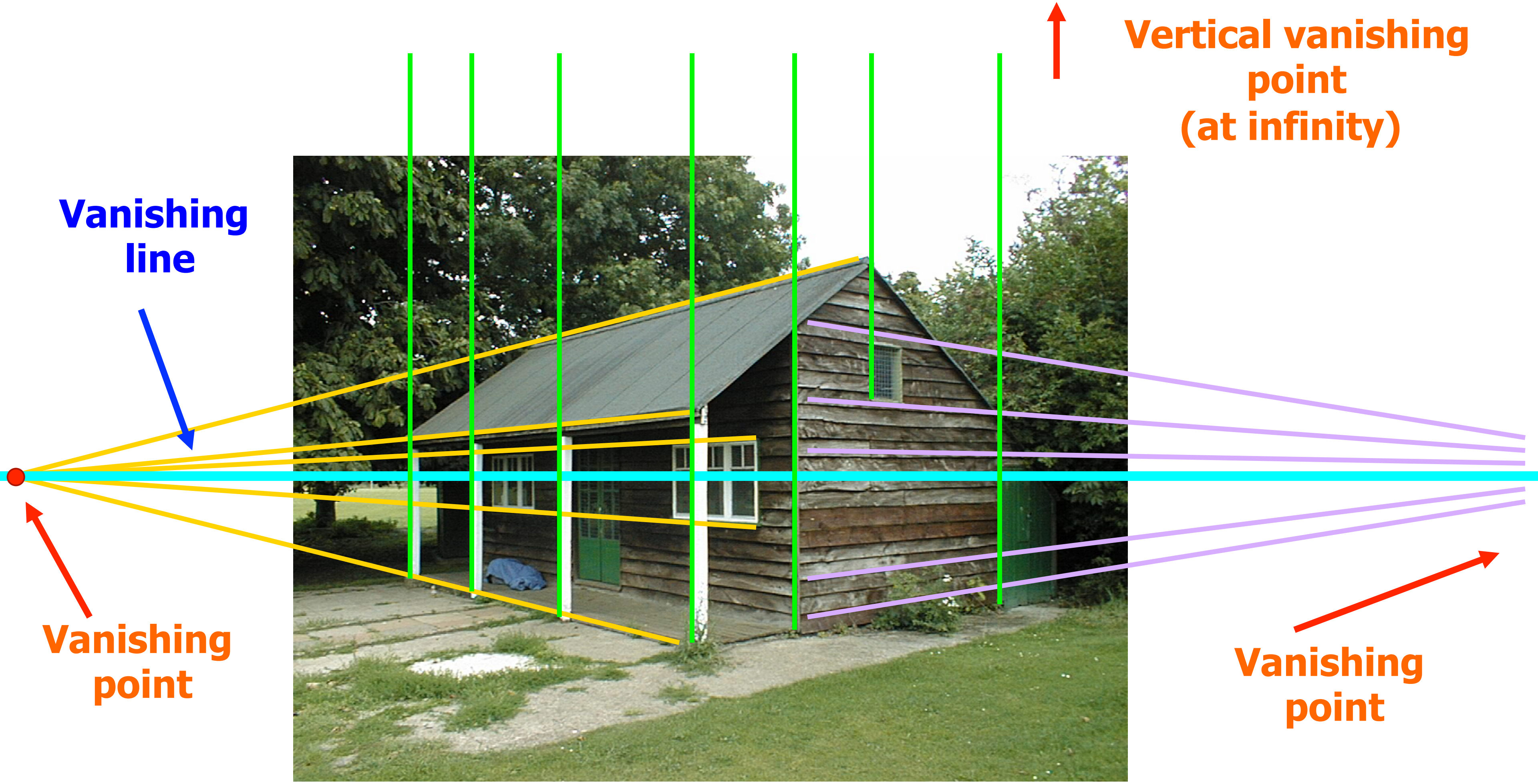
Vanishing points and lines

Parallel lines in the world intersect in the image at a "vanishing point"



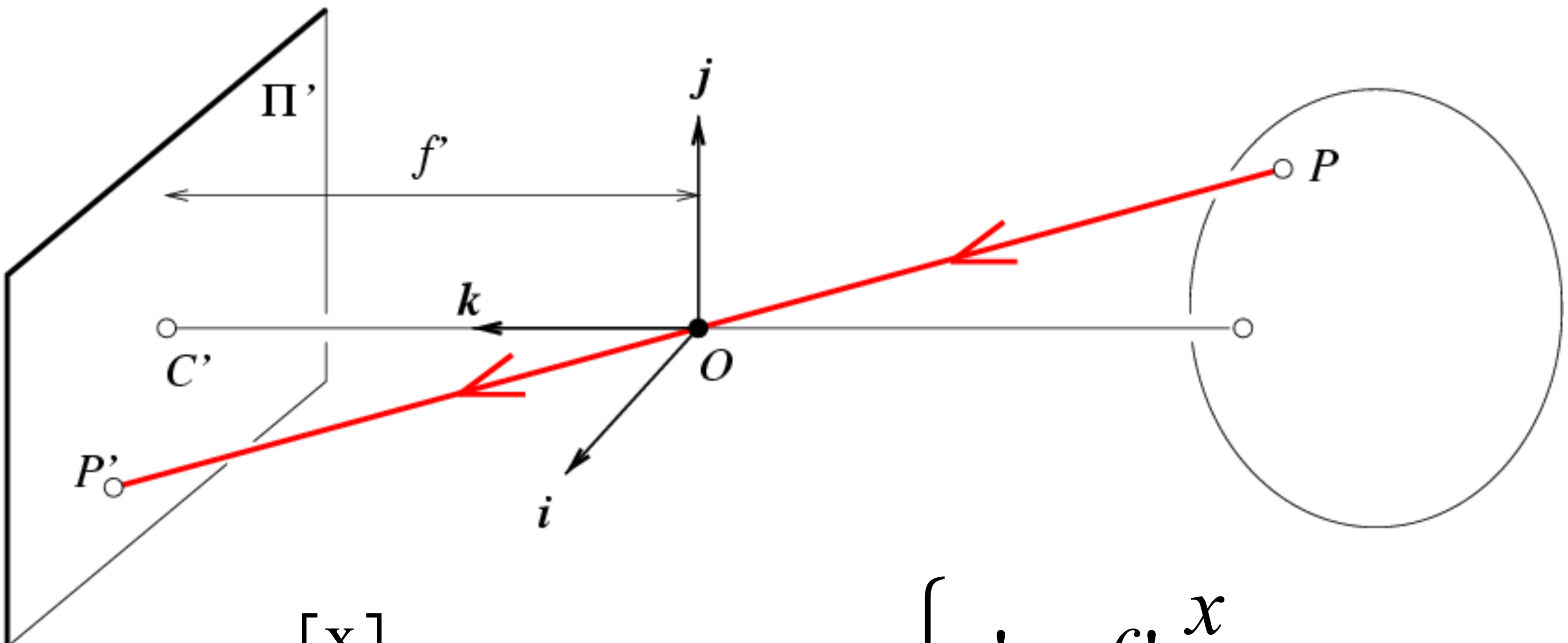
Slide credit: J. Hayes

Vanishing points and lines



Slide from Efros, Photo from Criminisi

Pinhole camera



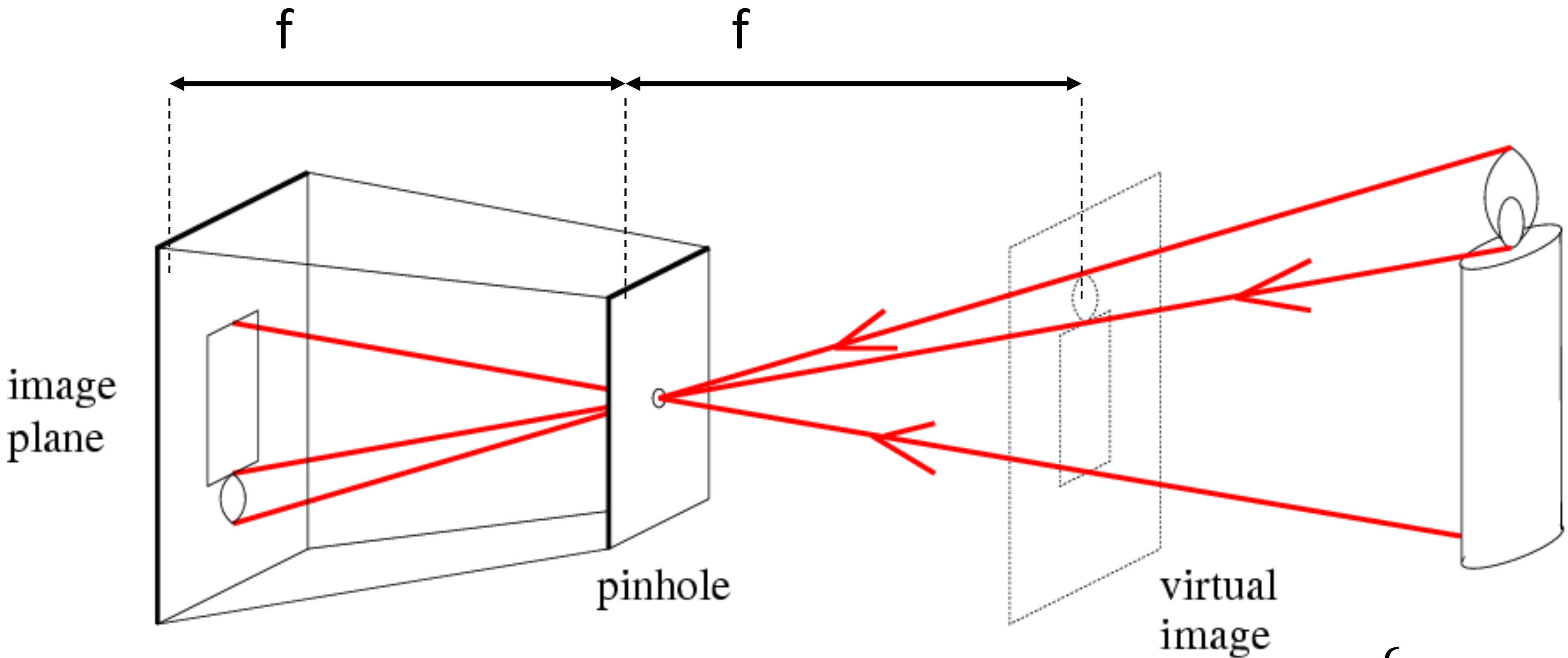
$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{cases} x' = f' \dfrac{x}{z} \\ y' = f' \dfrac{y}{z} \end{cases}$$

Note: z is always negative.

Derived using similar triangles

Pinhole camera

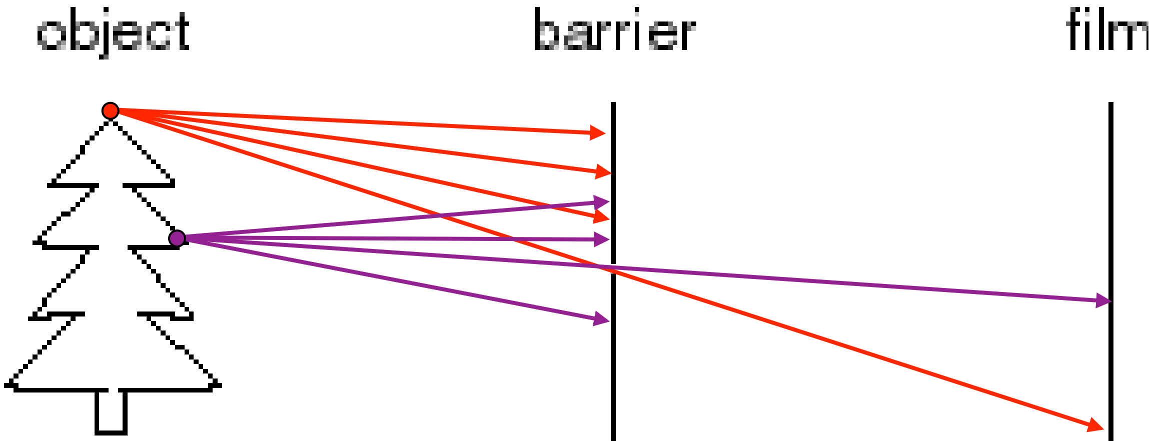


$$\begin{cases} x' = f\dfrac{x}{z} \\ y' = f\dfrac{y}{z} \end{cases}$$

- Common to draw image plane *in front* of the focal point
- Moving the image plane merely scales the image.

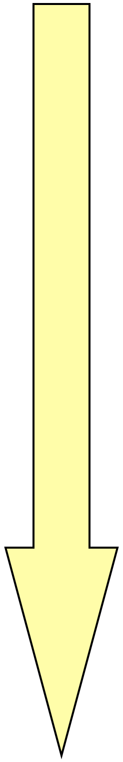
Pinhole camera

Is the size of the aperture important?

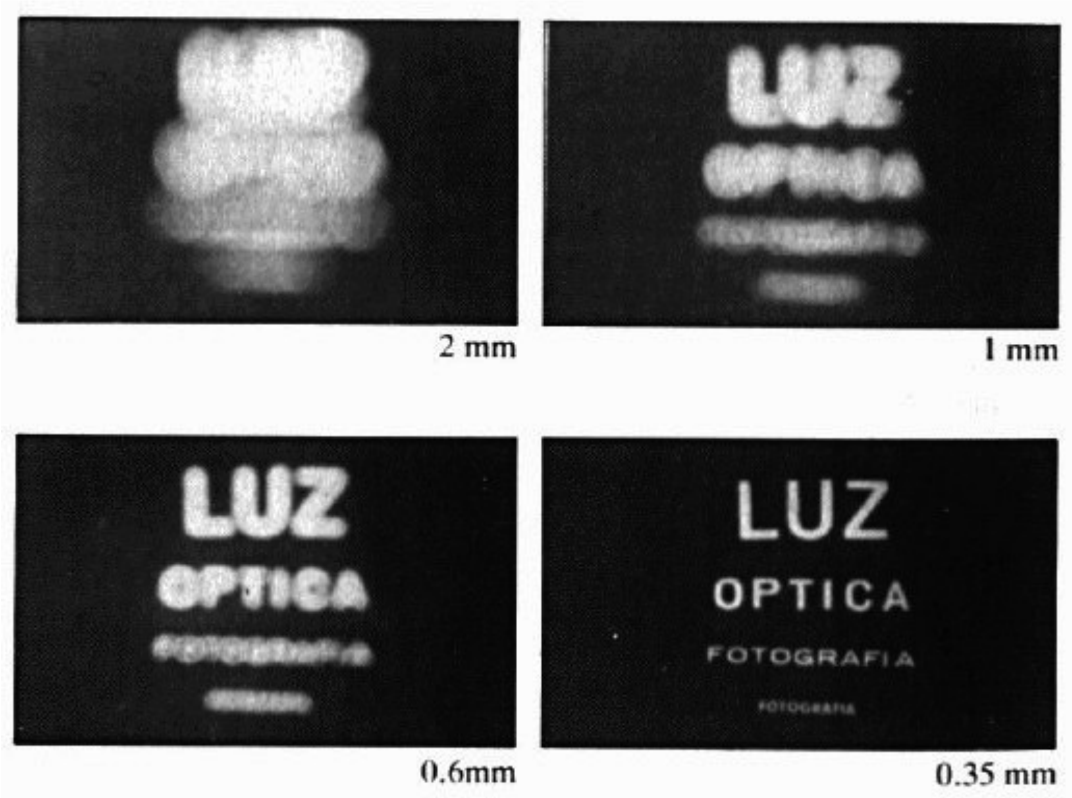


Cameras & Lenses

Shrinking aperture size



- Rays are mixed up



-Why the aperture cannot be too small?

- -Less light passes through
- -Diffraction effect

Adding lenses!

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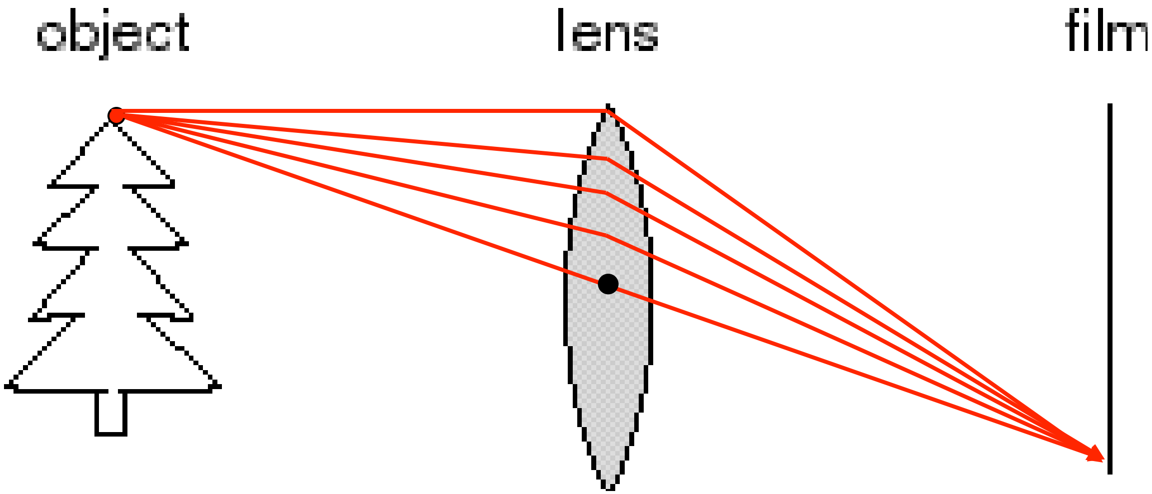
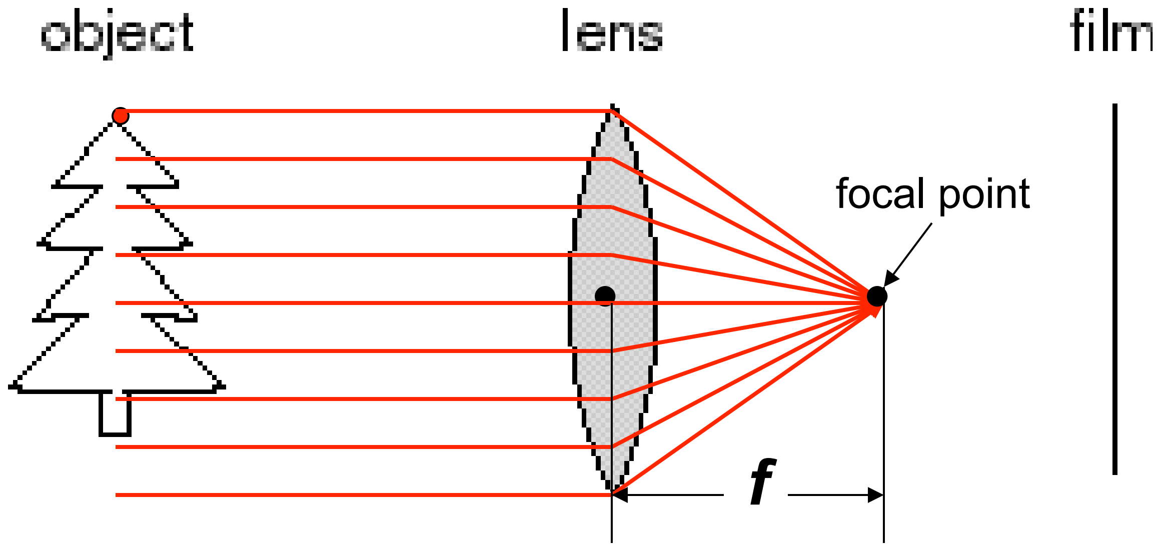
Cameras & Lenses



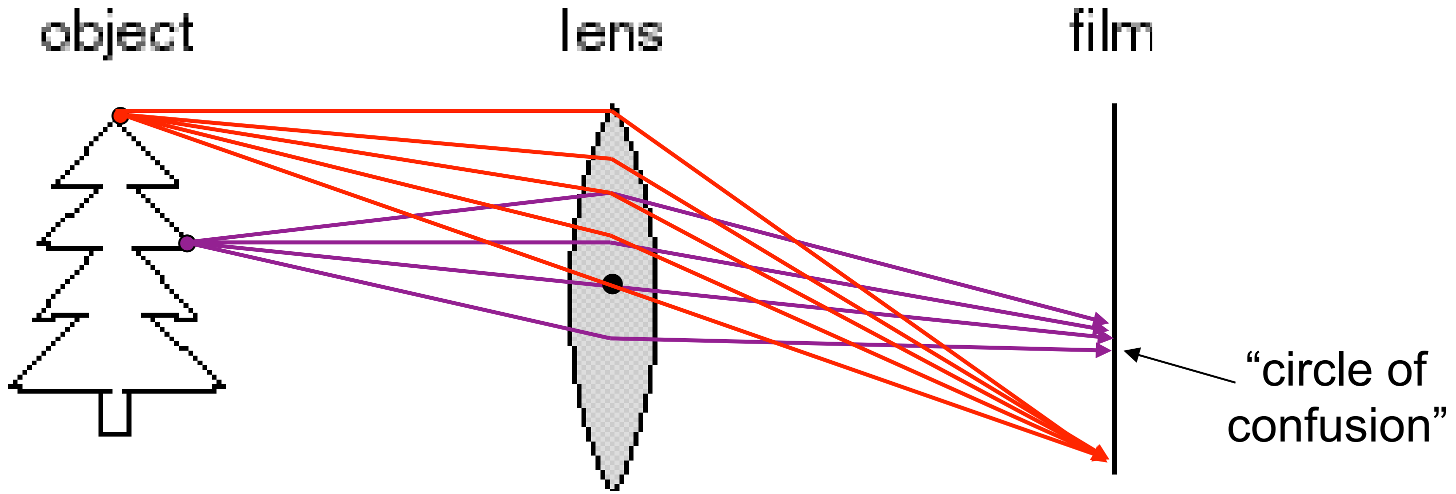
- A lens focuses light onto the film

Cameras & Lenses



- A lens focuses light onto the film
  - Rays passing through the center are not deviated
  - All parallel rays converge to one point on a plane located at the *focal length f*

Cameras & Lenses



- A lens focuses light onto the film
  - There is a specific distance at which objects are "in focus"
    [other points project to a "circle of confusion" in the image]

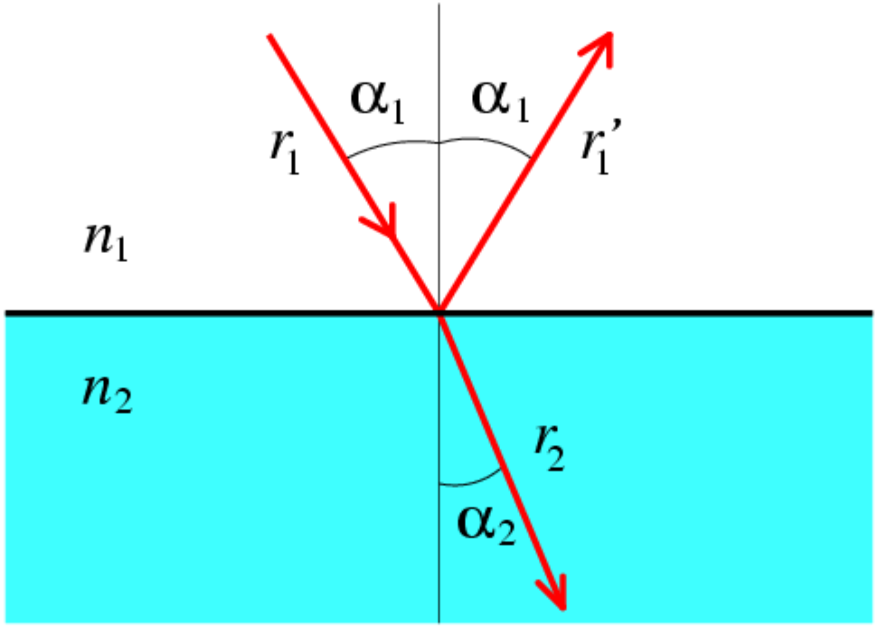
Cameras & Lenses

- Laws of geometric optics
  - Light travels in straight lines in homogeneous medium
  - Reflection upon a surface: incoming ray, surface normal, and reflection are co-planar
  - Refraction: when a ray passes from one medium to another

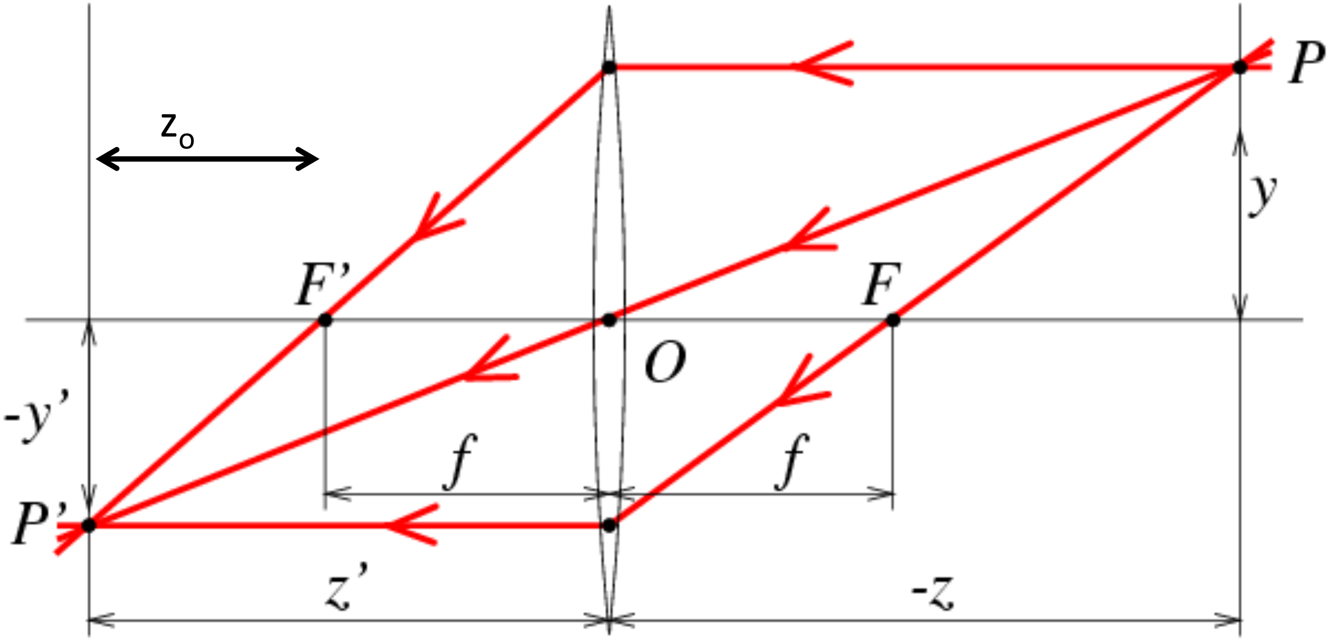
Snell's law

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

α_1 = incident angle
α_2 = refraction angle
n_i = index of refraction



Thin Lenses



$$z' = f + z_o$$

$$f = \frac{R}{2(n-1)}$$

Snell's law:

$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$

Small angles:

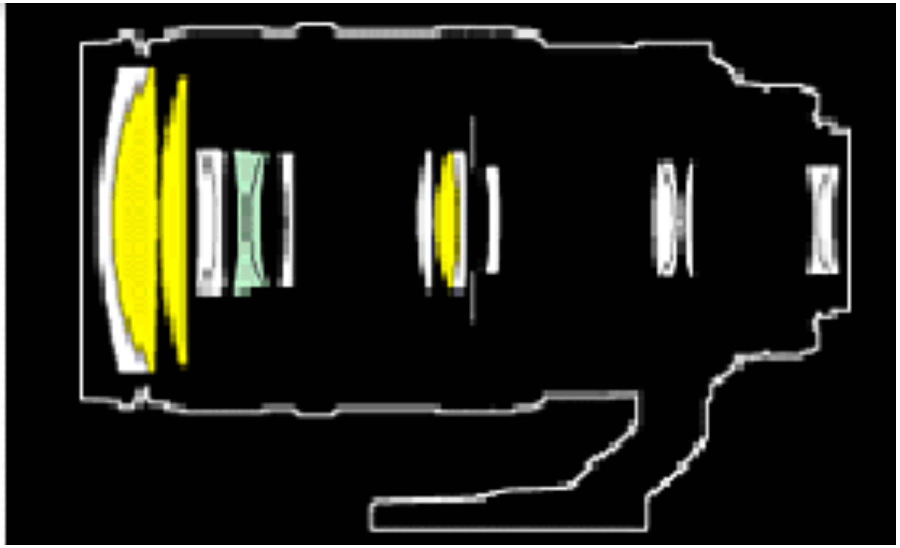
$n_1 \alpha_1 \approx n_2 \alpha_2$

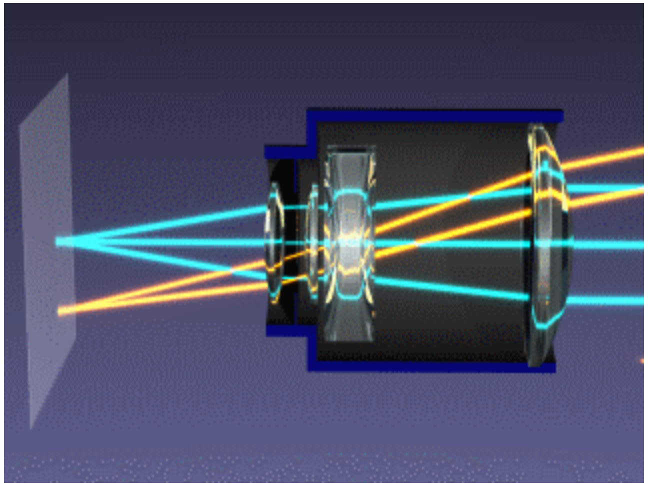
$n_1 = n$ (lens)

$n_1 = 1$ (air)

$$\begin{cases} x' = z' \dfrac{x}{z} \\ y' = z' \dfrac{y}{z} \end{cases}$$

Cameras & Lenses



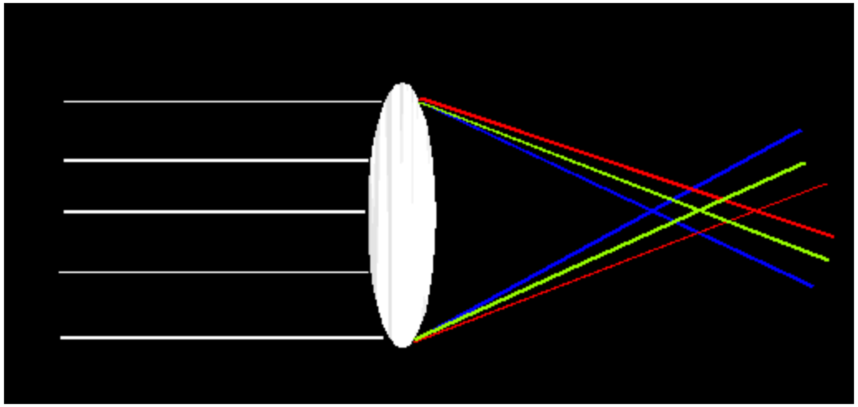


Source wikipedia

Issues with lenses: Chromatic Aberration

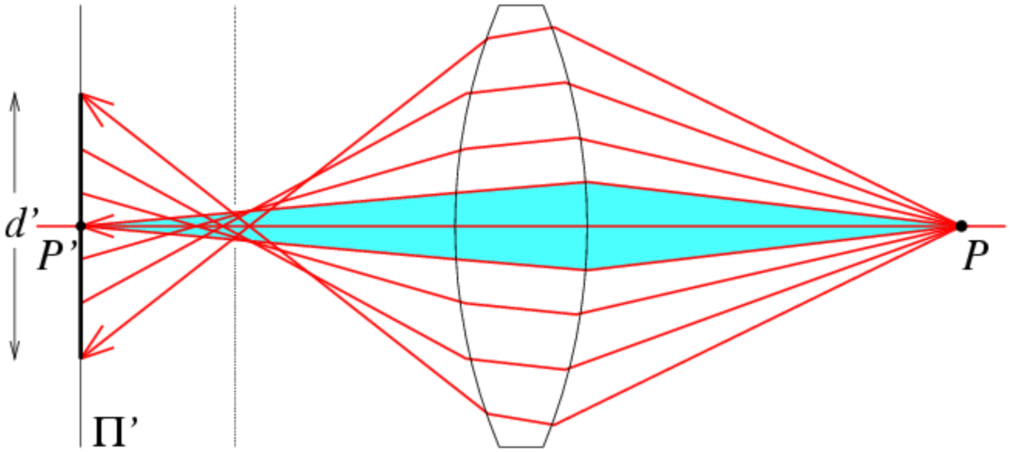
- Lens has different refractive indices for different wavelengths: causes color fringing

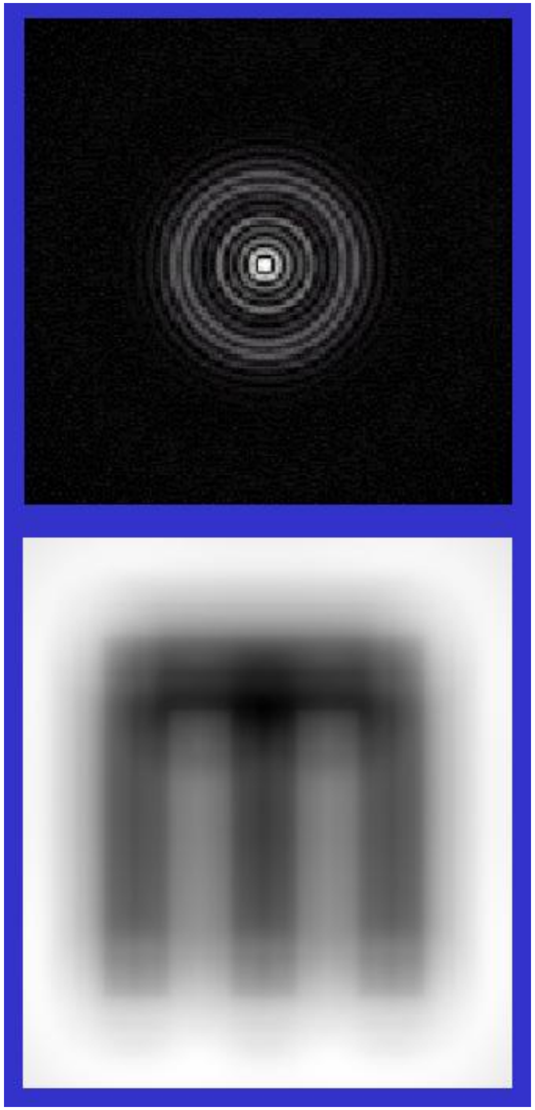
$$f = \frac{R}{2(n-1)}$$



Issues with lenses: Chromatic Aberration

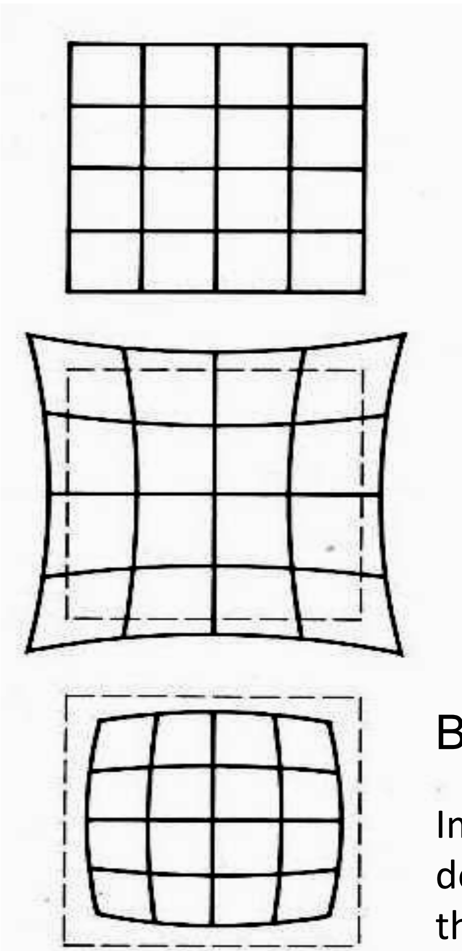
- Rays farther from the optical axis focus closer





Issues with lenses: Chromatic Aberration

– Deviations are most noticeable for rays that pass through the edge of the lens



No distortion

Pin cushion

Barrel (fisheye lens)

Image magnification decreases with distance from the optical axis

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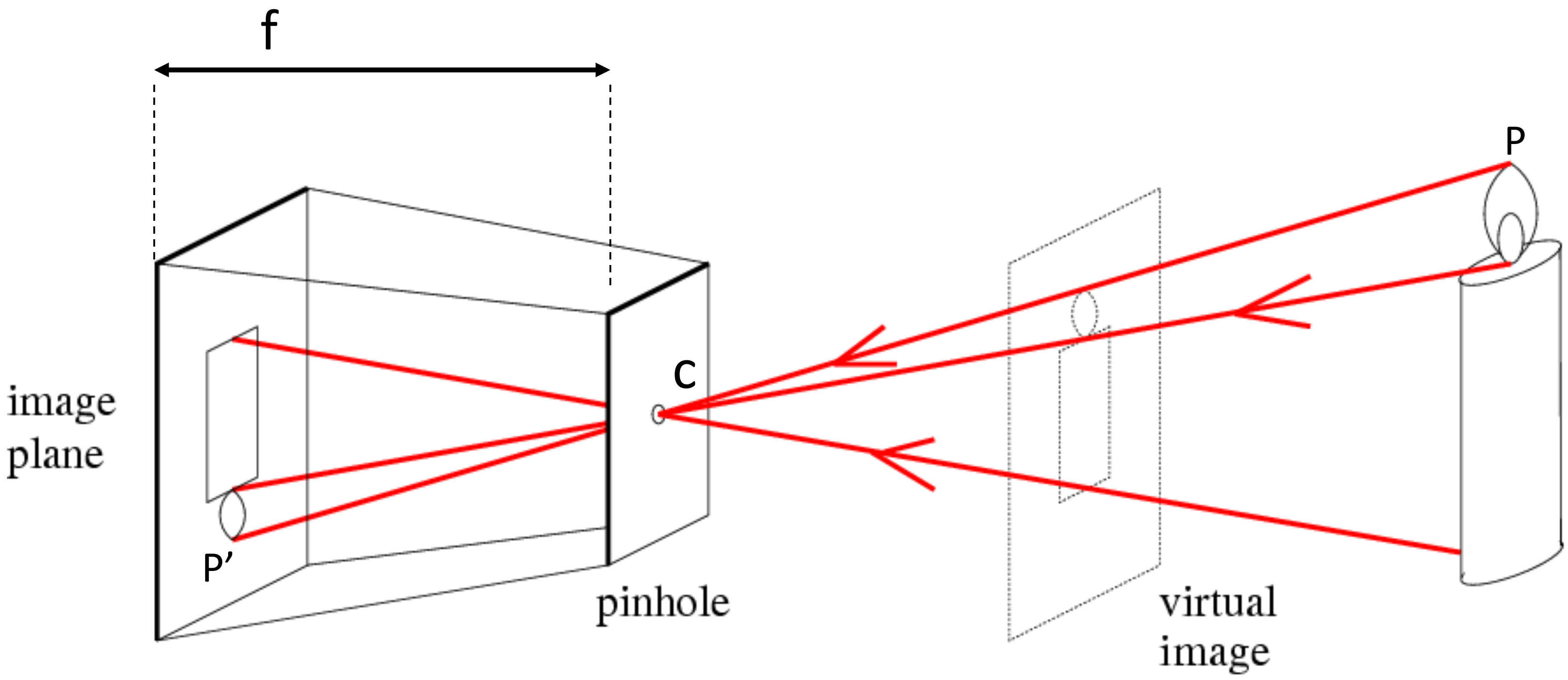
Relating real-world point to a point on a camera



$$P = (x, y, z) \rightarrow P' = (f\frac{x}{z}, f\frac{y}{z})$$

f = focal length

c = center of the camera

$$\Re^3 \xrightarrow{E} \Re^2$$

Relating real-world point to a point on a camera

Is this a linear transformation?

$$P = (x, y, z) \rightarrow P' = (f\frac{x}{z}, f\frac{y}{z})$$

No — division by z is nonlinear!

How to make it linear?

Homogeneous coordinates – a reminder

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

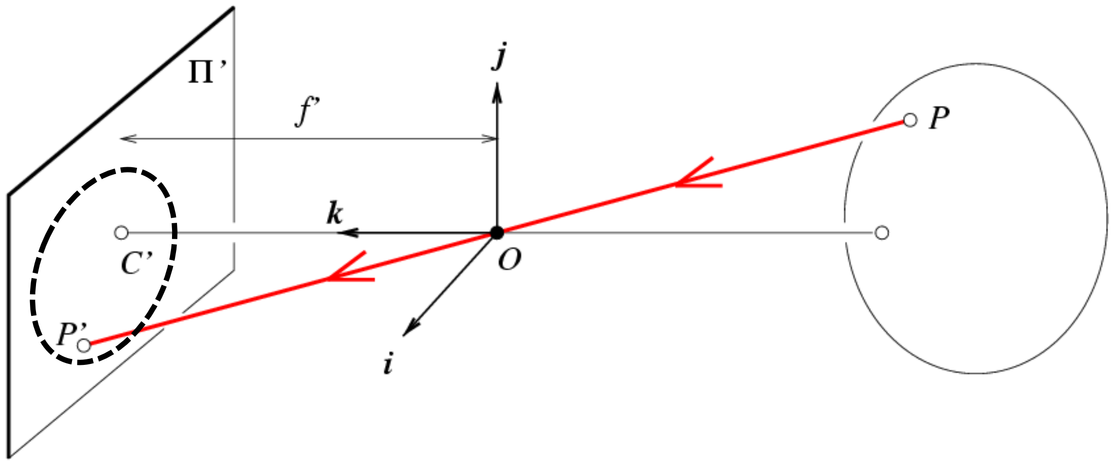
homogeneous scene coordinates

- Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Relating a real-world point to a point on the camera



In Cartesian coordinates:

$$P = (x, y, z) \rightarrow P' = (f\frac{x}{z}, f\frac{y}{z})$$

In homogeneous coordinates:

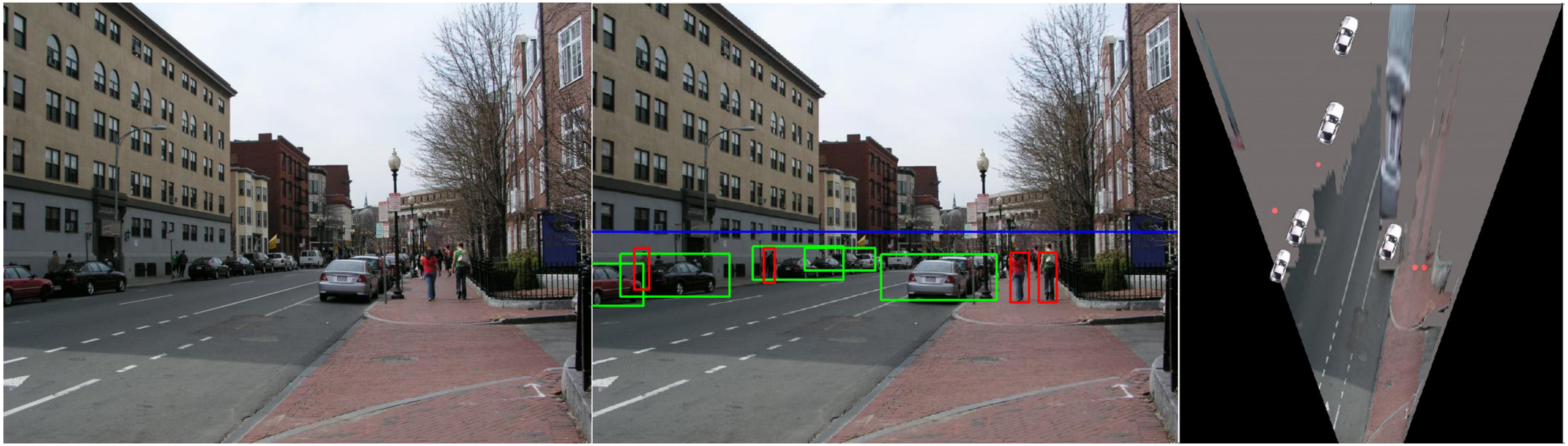
$$P' = \begin{bmatrix} f\,x \\ f\,y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{M}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

"Projection matrix"

$$P' = M\ P \qquad \Re^4 \xrightarrow{H} \Re^3$$

Interlude: why does this matter?

Object Recognition (CVPR 2006)



Slide credit: J. Hayes

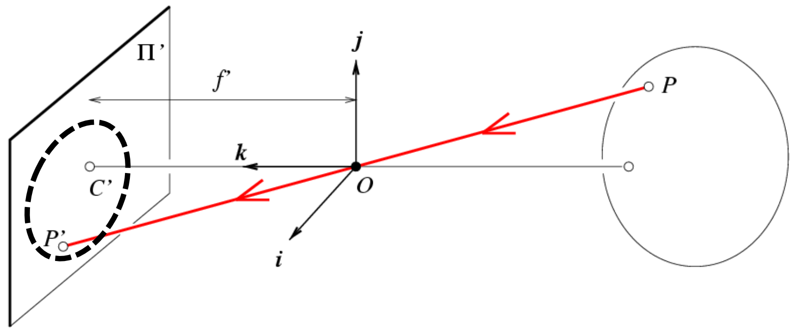
Inserting photographed objects into images (SIGGRAPH 2007)

Original

Created

Slide credit: J. Hayes

Relating a real-world point to a point on the camera



In homogeneous coordinates:

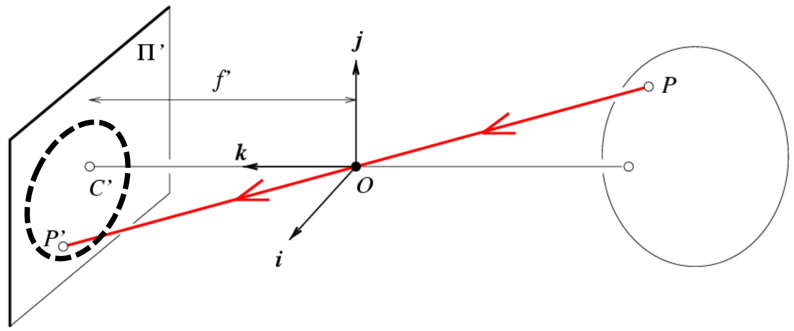
$$P' = \begin{bmatrix} f\,x \\ f\,y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{M} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

M ← ideal world

Intrinsic Assumptions
- Unit aspect ratio
- Optical center at (0,0)
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

Relating a real-world point to a point on the camera



In homogeneous coordinates:

$$P' = \begin{bmatrix} f\,x \\ f\,y \\ z \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = K\begin{bmatrix} I & 0 \end{bmatrix} P$$

K

Intrinsic Assumptions
- Optical center at (0,0)
- Square pixels
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

Remove assumption: known optical center

Intrinsic Assumptions
- ~~Optical center at (0,0)~~
- Optical center at (u_0, v_0)
- Square pixels
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$P' = K\begin{bmatrix} I & 0 \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & u_0 & 0 \\ 0 & f & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide inspiration: S. Savarese

Remove assumption: square pixels

Intrinsic Assumptions

- Optical center at (u_0, v_0)
- ~~Square pixels~~
- Rectangular pixels
- No skew

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$P' = K\begin{bmatrix} I & 0 \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

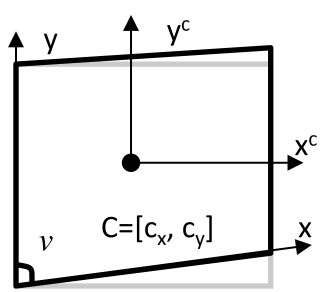
Slide inspiration: S. Savarese

Remove assumption: non-skewed pixels

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- ~~No skew~~
- Small skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$P' = K\begin{bmatrix} I & 0 \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Slide inspiration: S. Savarese

Remove assumption: non-skewed pixels

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

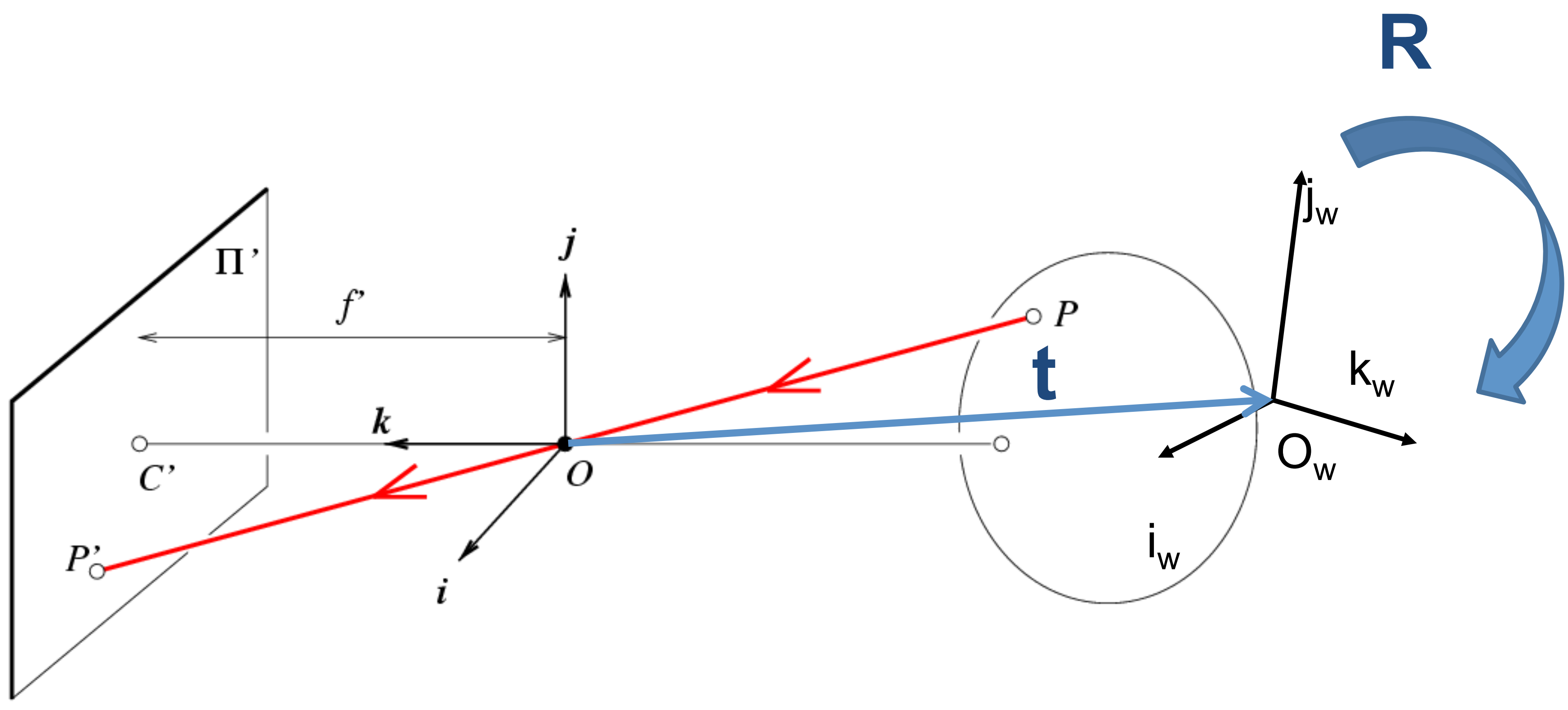
Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

Intrinsic parameters

$$P' = K\begin{bmatrix} I & 0 \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide inspiration: S. Savarese

Real world camera: Translate + Rotate



Slide inspiration: S. Savarese, J. Hayes

Remove assumption: allow translation

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- No rotation
- Camera at ~~(0,0,0)~~ -> (t_x,t_y,t_z)

$$P' = K\begin{bmatrix} I & \overline{t} \end{bmatrix} P \quad \Rightarrow \quad w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide inspiration: S. Savarese

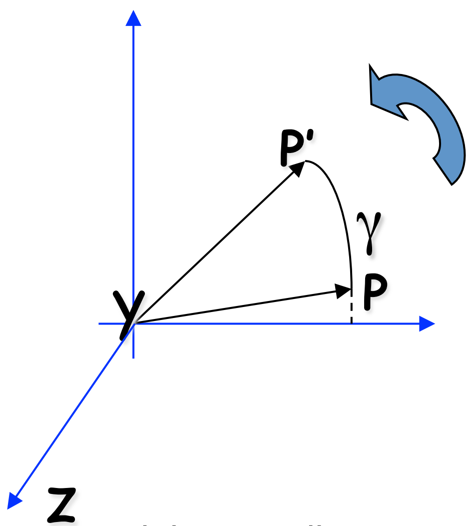
Remove assumption: allow rotation

Intrinsic Assumptions

- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions

- ~~No~~ rotation
- Camera at (t_x,t_y,t_z)



Rotation around the coordinate axes, counter-clockwise

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Slide inspiration: S. Savarese

Remove assumption: allow rotation

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- ~~No~~ rotation
- Camera at (t_x,t_y,t_z)

$$P' = K\begin{bmatrix} R & \bar{t} \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide inspiration: S. Savarese

A generic projection matrix

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- Allow rotation
- Camera at (t_x, t_y, t_z)

$$P' = K\begin{bmatrix} R & \overline{t} \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide inspiration: S. Savarese

A generic projection matrix

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- Allow rotation
- Camera at (t_x, t_y, t_z)

$$P' = K\begin{bmatrix} R & \overline{t} \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Degrees of freedom??

Slide inspiration: S. Savarese

A generic projection matrix

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- Allow rotation
- Camera at (t_x, t_y, t_z)

$$P' = K\begin{bmatrix} R & \overline{t} \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

5

6

Degrees of freedom??

Slide inspiration: S. Savarese

CS231a: Camera Calibration
estimate all intrinsic and extrinsic parameters

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- Allow rotation
- Camera at (t_x,t_y,t_z)

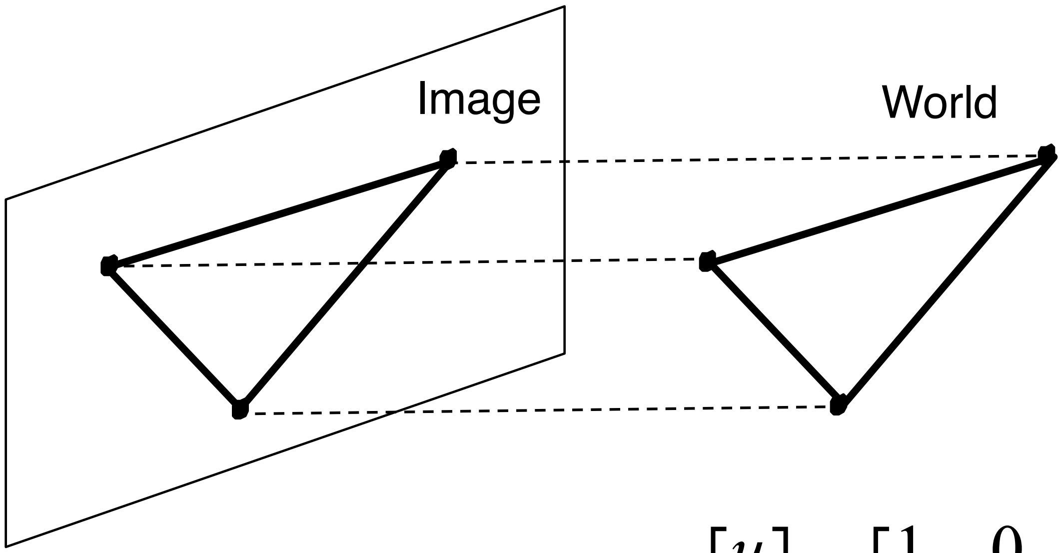
$$P' = K\begin{bmatrix} R & \bar{t} \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

5 6

Slide inspiration: S. Savarese

Orthographic Projection

- Special case of perspective projection
  - Distance from the COP to the image plane is infinite



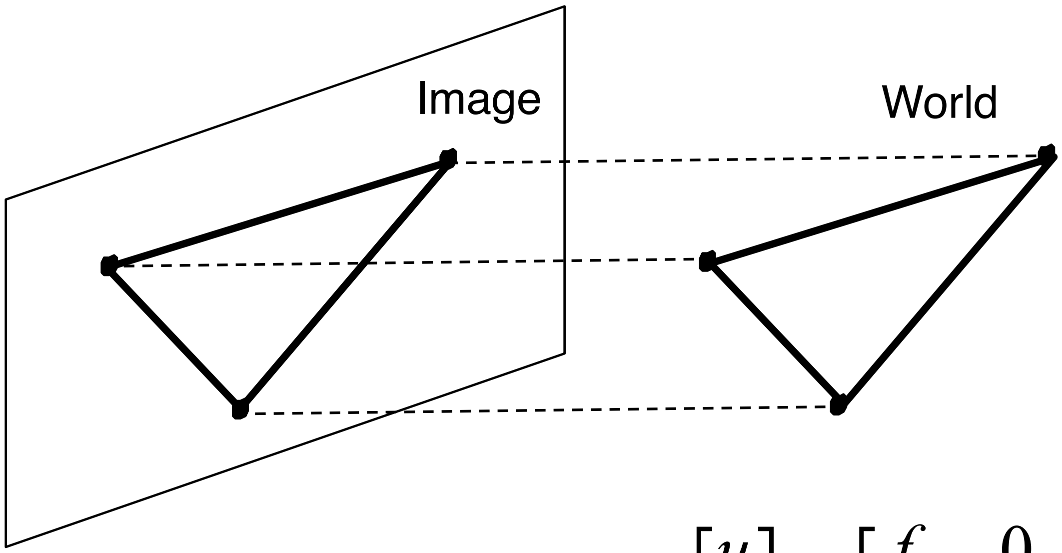
  - Also called “parallel projection”
  - What’s the projection matrix?

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide credit: Steve Seitz

Scaled Orthographic Projection

- Special case of perspective projection
  - Object dimensions are small compared to distance to camera

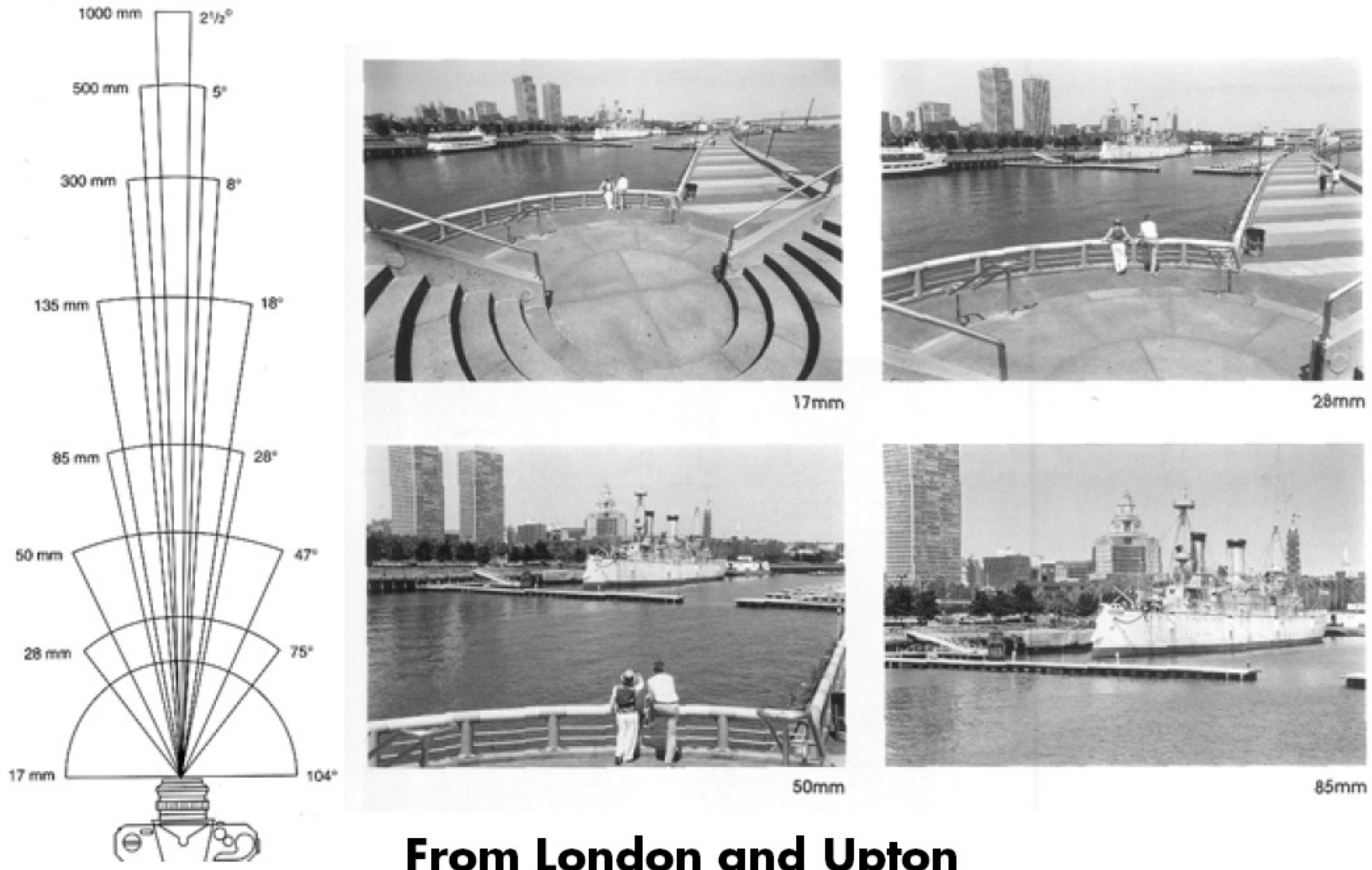


  - Also called "weak perspective"
  - What's the projection matrix?

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 0 & s \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide credit: Steve Seitz

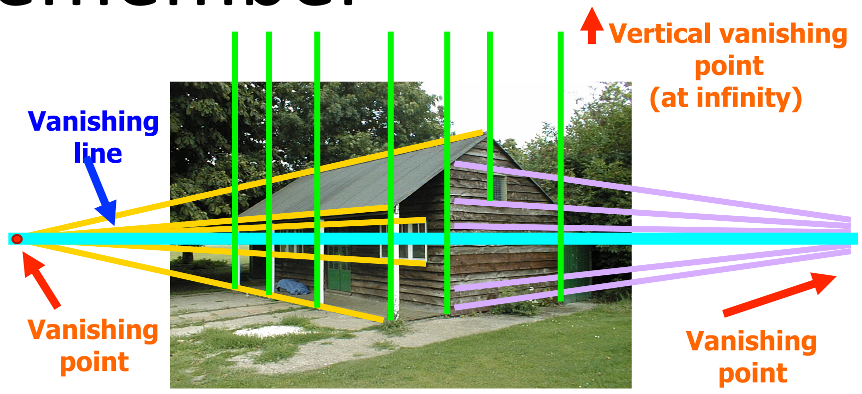
Field of View (Zoom)



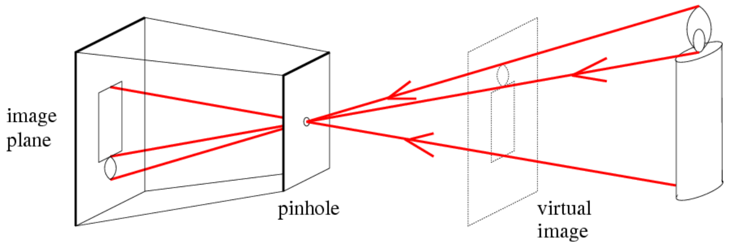
From London and Upton

Things to remember

- Vanishing points and vanishing lines



- Pinhole camera model and camera projection matrix M
  - Intrinsic parameters
  - Extrinsic parameters



$$P' = K\begin{bmatrix} R & \bar{t} \end{bmatrix} P$$

- Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Slide inspiration: J. Hayes

What we have learned today?

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
  - Projection matrix
  - Intrinsic parameters
  - Extrinsic parameters

Reading:
[FP] Chapters 1 – 3
[HZ] Chapter 6